Adaptively Sound Zero-Knowledge SNARKs for UP

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All of the above constructions/transformations also **satisfy/preserve zero-knowledge!**

<u>S</u>uccinct <u>N</u>on-interactive <u>ARG</u>ument

Fix NP language L

$\mathscr{P}$

$\mathscr{V}$

Instance x, witness w

Succinct Non-interactive ARGument

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Common Reference String (crs)

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Fix NP language L

Common Reference String (crs)

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$\mathscr{V}$

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$$\Pr_{\mathsf{crs}}[(x^*, \pi^*) \leftarrow \mathscr{P}^*(\mathsf{crs}) \land x^* \notin L \land \mathscr{V}(\mathsf{crs}, x^*, \pi^*) = 1] \leq \mathsf{negl}(\lambda)$$

Designated-Verifier SNARG

Common Reference String (crs)

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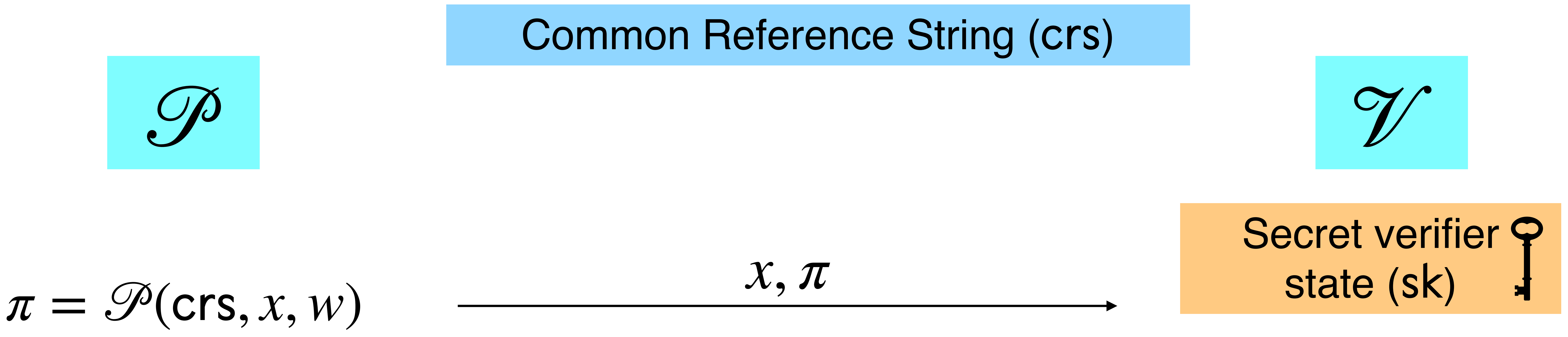
$$\pi = \mathscr{P}(\mathsf{crs}, x, w) \xrightarrow{x,\ \pi}$$

Designated-Verifier SNARG



<u>D</u>esignated-<u>V</u>erifier SNARG

Common Reference String (crs)

$\mathcal{P}$

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Secret verifier state (sk)

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Theorem 1: We construct a dvSNARG for **UP** from **LWE** and **evasive LWE.**

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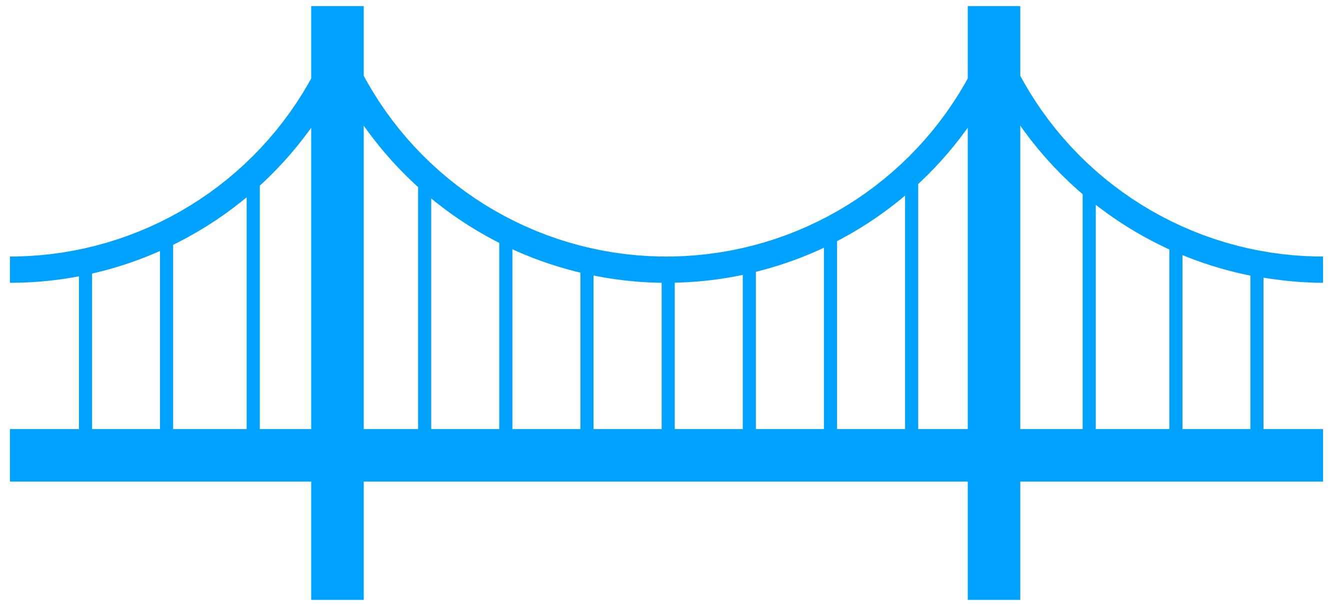
LWE  **Obfuscation**

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- ABE for unbounded depth circuits [HLL23]

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- DDH $= \{(g, a, b, c) \mid \exists x, y$ s.t. $a = g^x, b = g^y, c = g^{xy}\}$

Tool: Evasive LWE

Proposed by Wee (Eurocrypt '22).

Fix distributions $\mathbf{S}, \mathbf{B}$ and $\mathbf{P}$ (possibly correlated).

if $$(\utilde{\mathbf{SB}}, \utilde{\mathbf{SP}}) \approx_c (\mathscr{U}, \mathscr{U})$$

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Throughout this talk, squiggly lines indicate **noise**

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Intuition: Given $\mathbf{SB}$ and $\mathbf{B}^{-1}(\mathbf{P})$, can compute $\mathbf{SB} \cdot \mathbf{B}^{-1}(\mathbf{P}) \approx \mathbf{SP}$, and not much else.

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  - **Idea:** Collect many equations on **low-norm secrets** over low-norm constants. Solve over integers!

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- With $\mathbf{E}$ in the clear, no more LWE guarantees on $\mathbf{SB} + \mathbf{E}$!
- Similar attack works for $\mathbf{SP}$ with correlated rows.
- **Evasive LWE:** This is the only attack! Doesn't work if $\mathbf{SP}$ were uniform.

Main Tool

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- Use this obfuscation to instantiate a "Sahai-Waters"-like SNARG. More details later!

TL;DR

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In this work, we

1. Build a designated-verifier **SNARG for UP** from LWE and evasive LWE

2. Show our dvSNARG, and any “Sahai-Waters”-like dvSNARG can be made **adaptively sound**.

    - **Adaptively sound** SNARGs from **falsifiable assumptions** ([JLS20] iO + OWF)!

3. Transformation from **SNAR<u>G</u> for UP** to **SNAR<u>K</u>** for **UP**.

    - Corollary: **Adaptively sound** dv-SNAR<u>K</u> for UP from **falsifiable assumptions.**

All of the above constructions/transformations also **satisfy/preserve zero-knowledge!**

SNARGs vs. SNARKs

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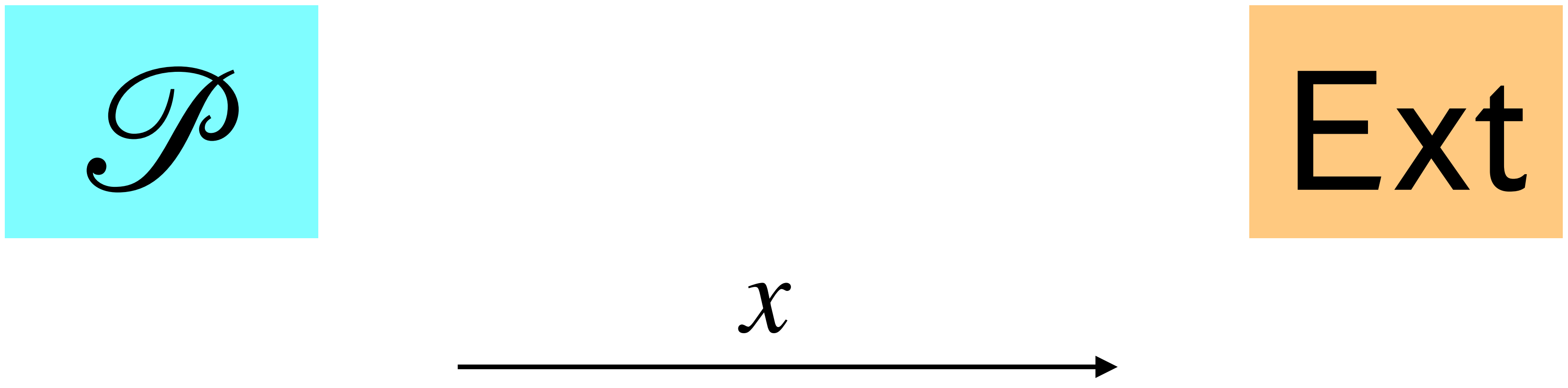

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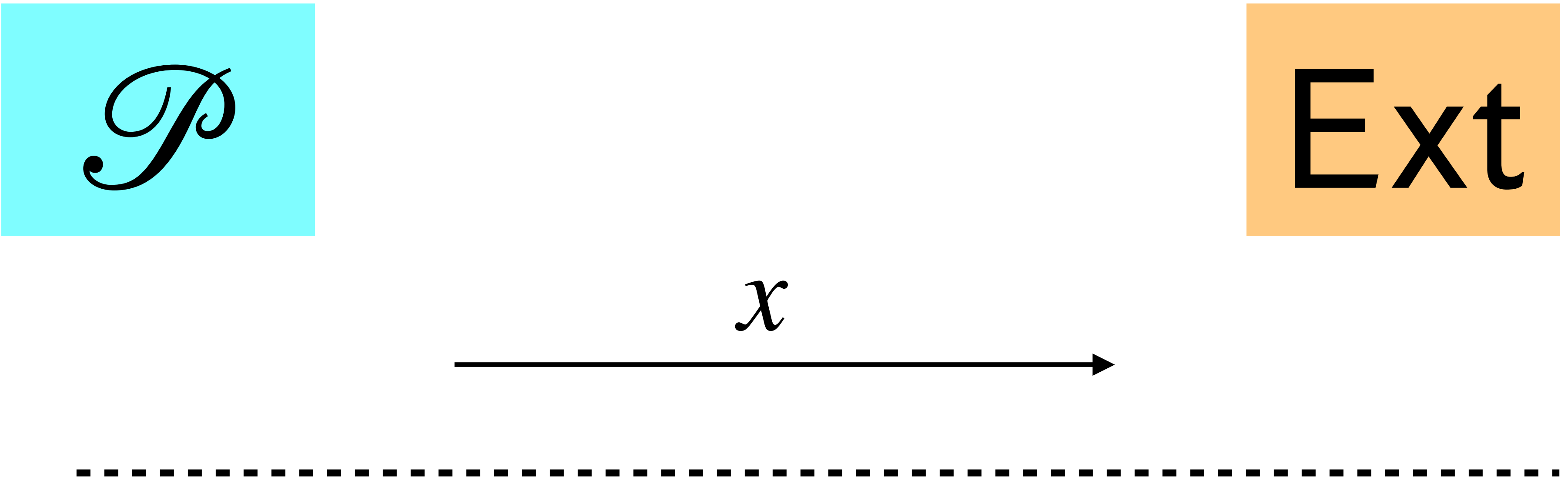

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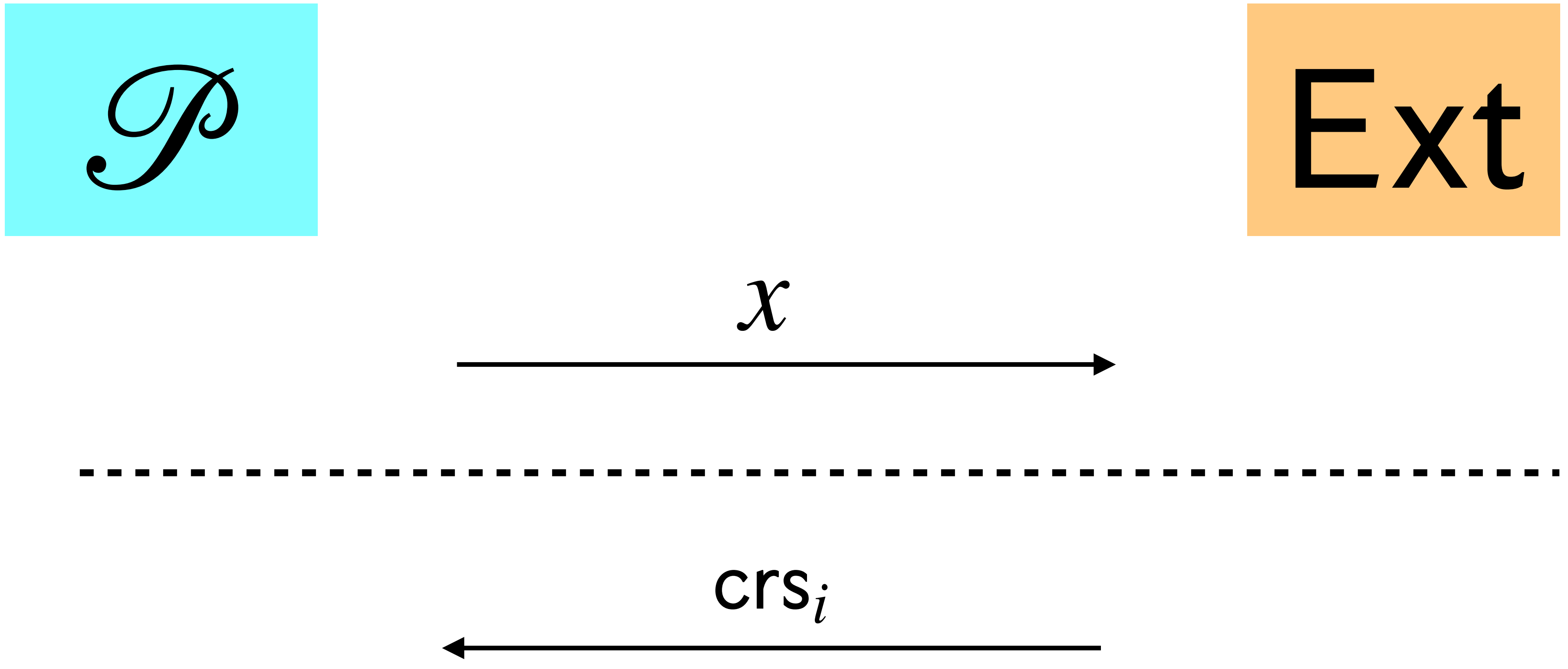

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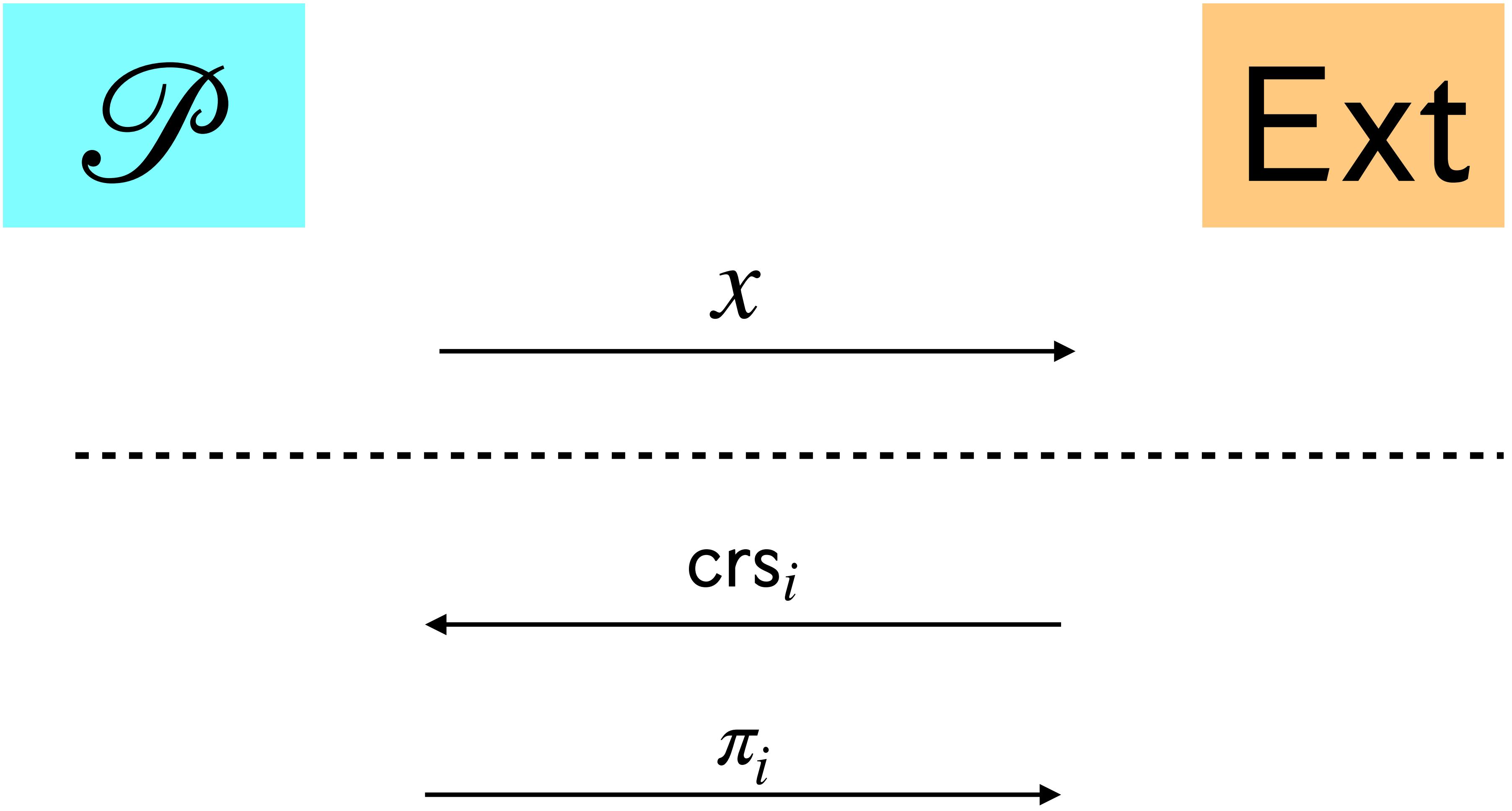

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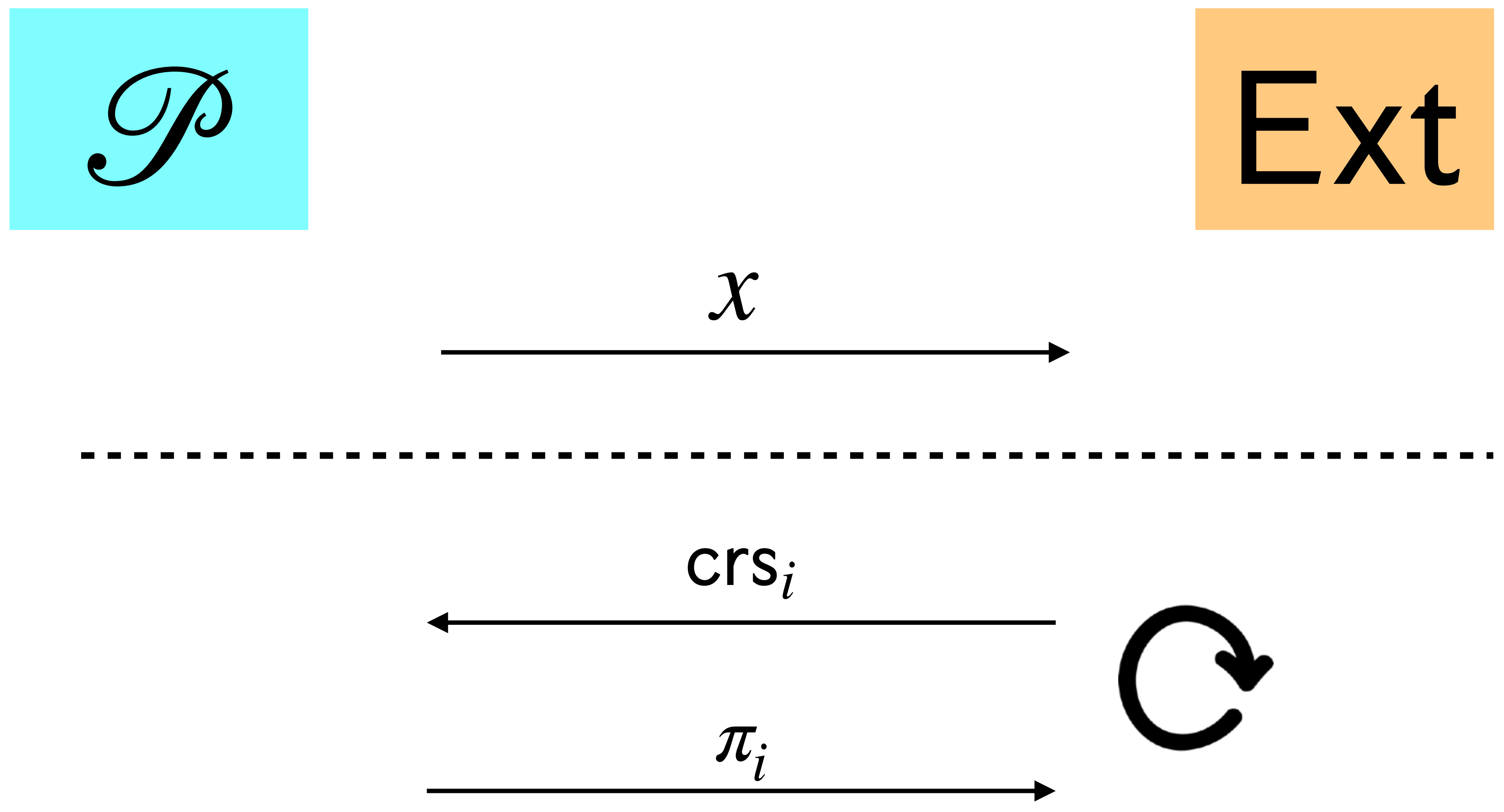

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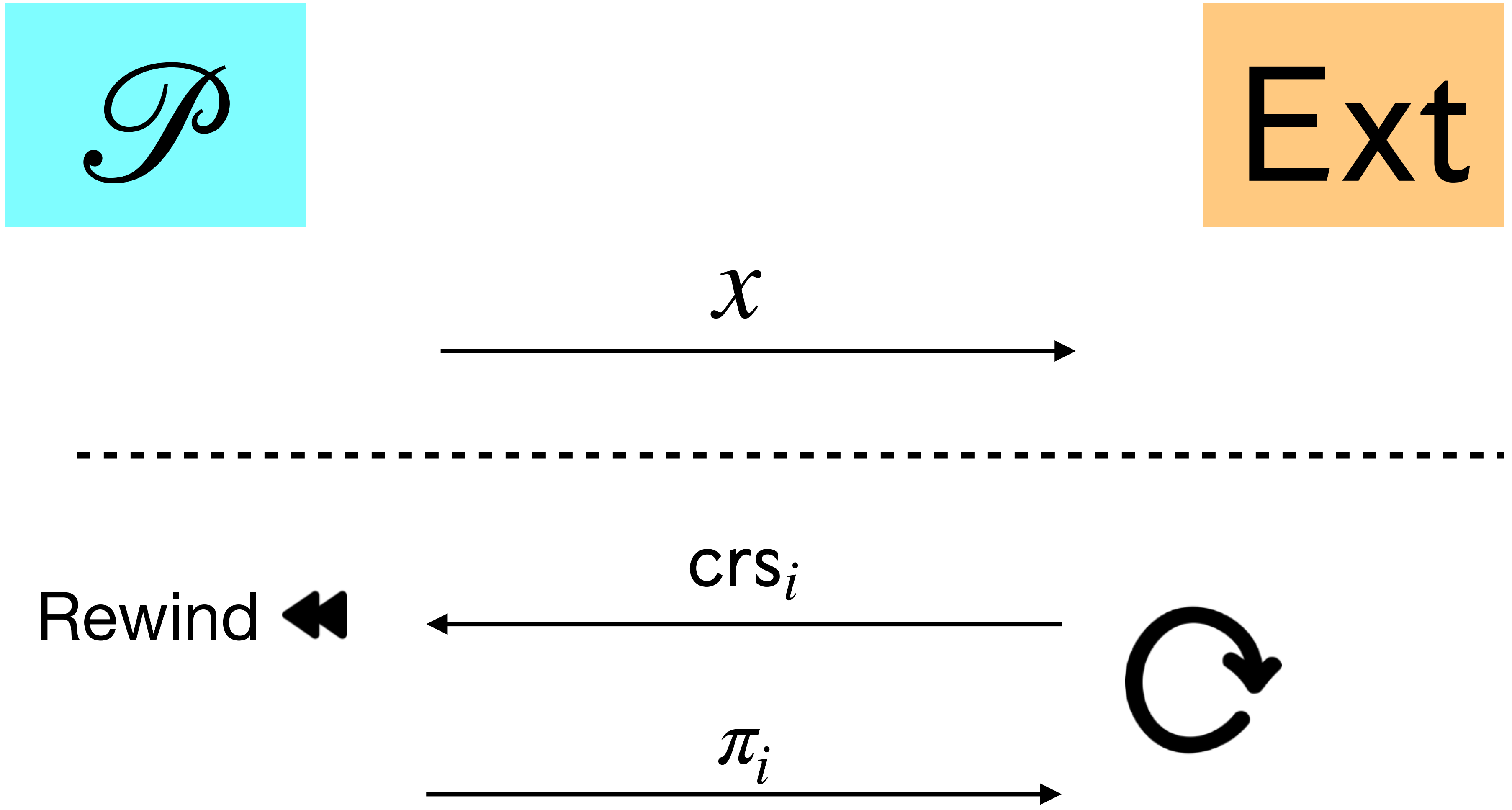

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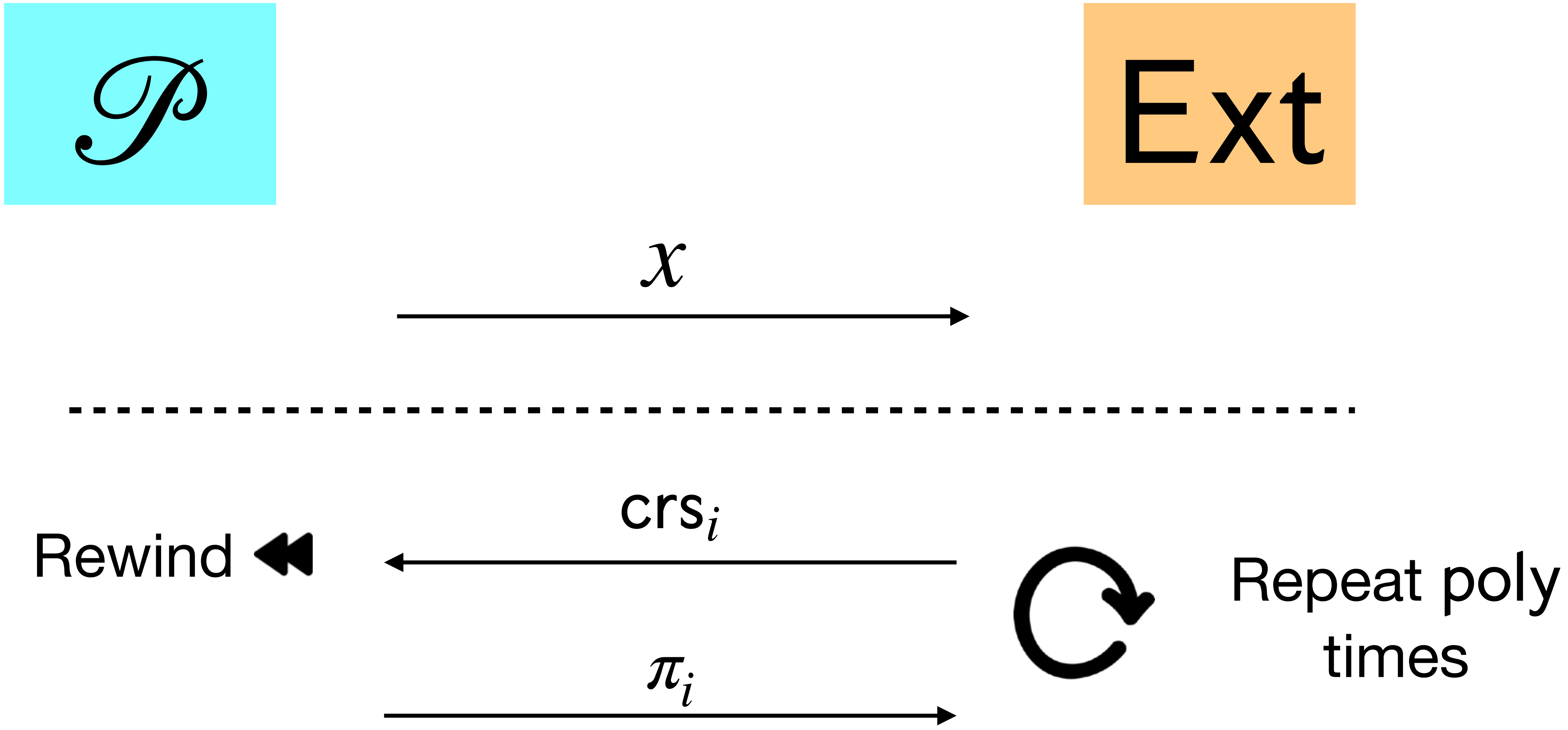

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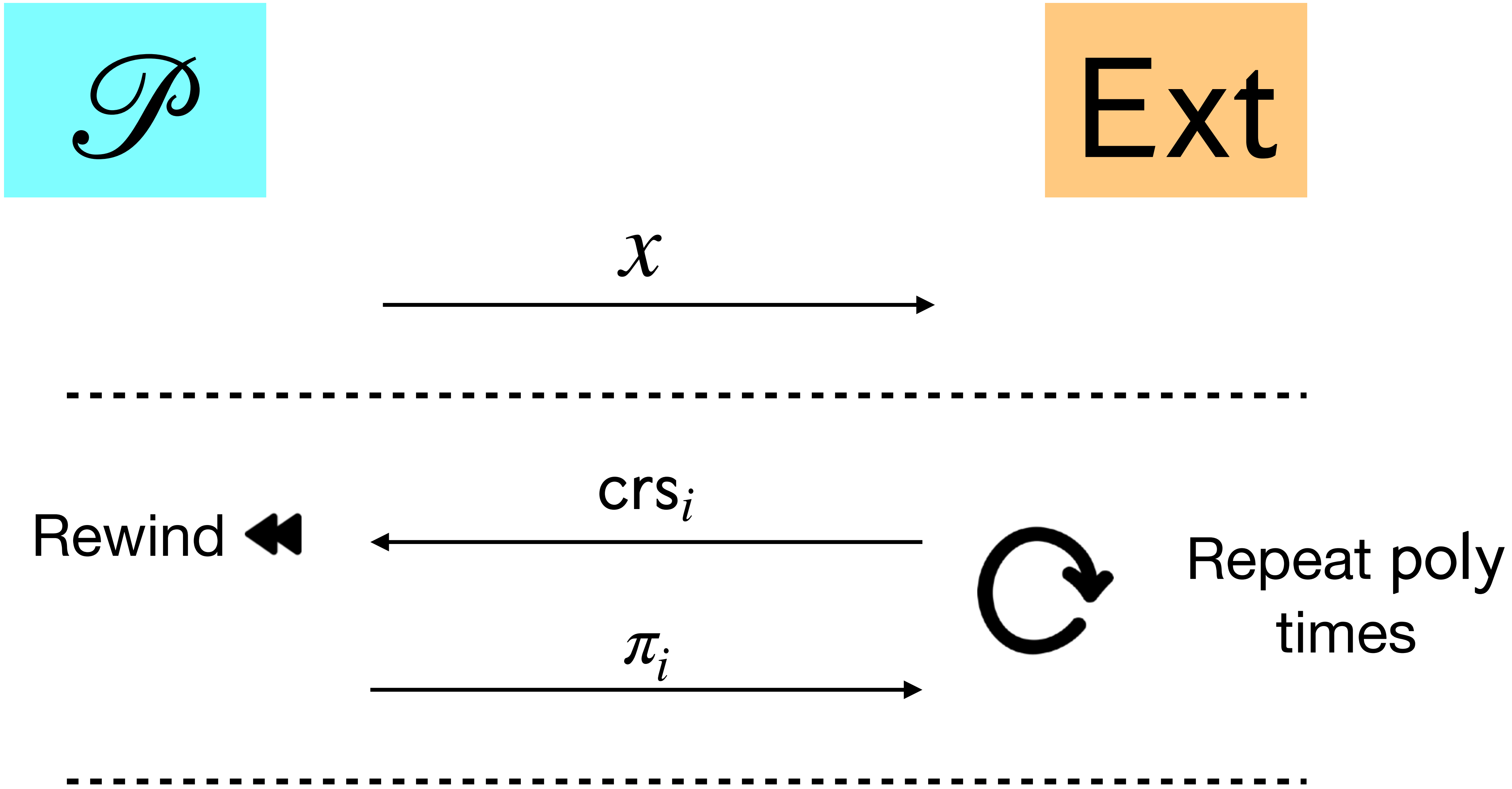

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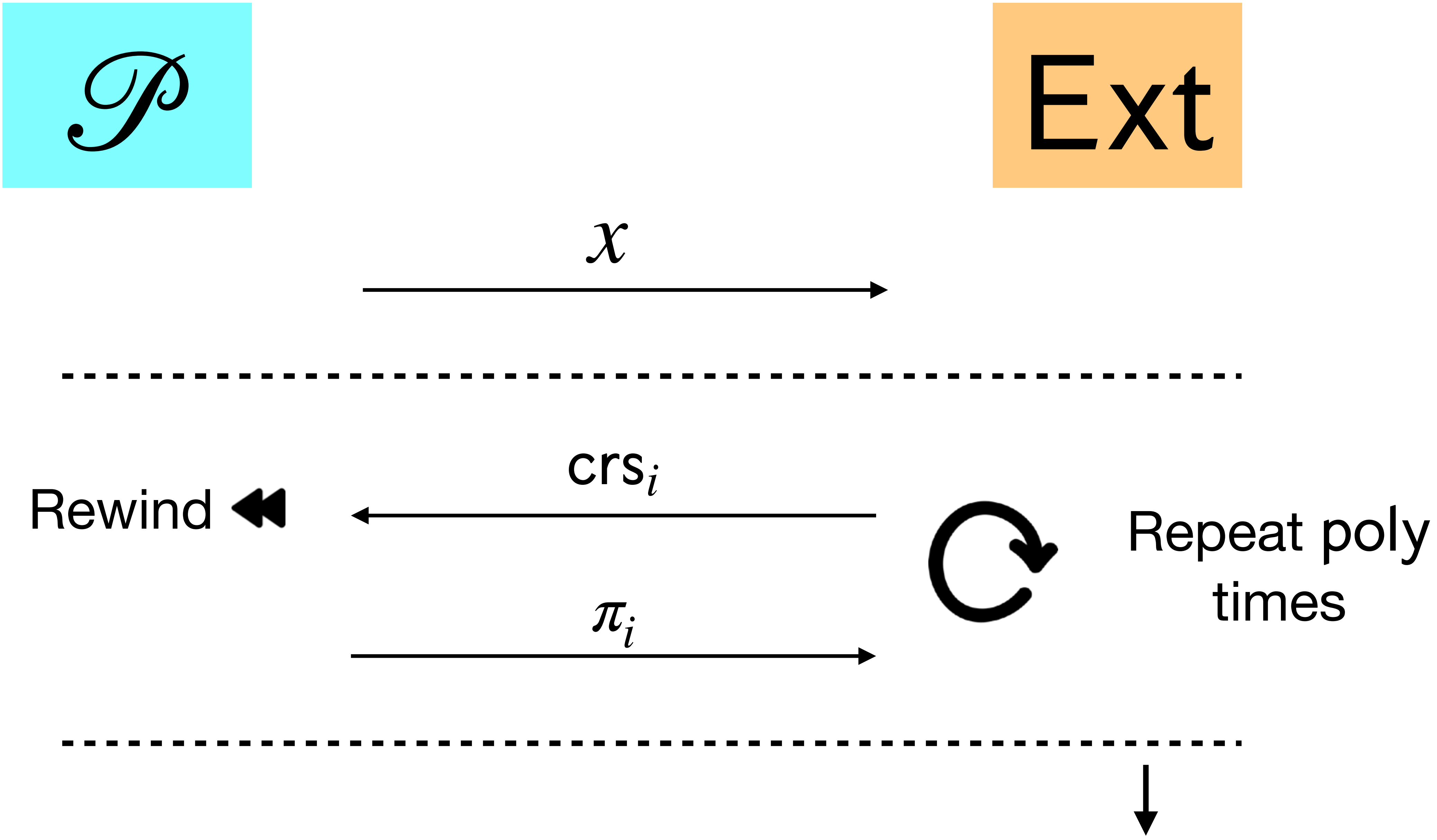

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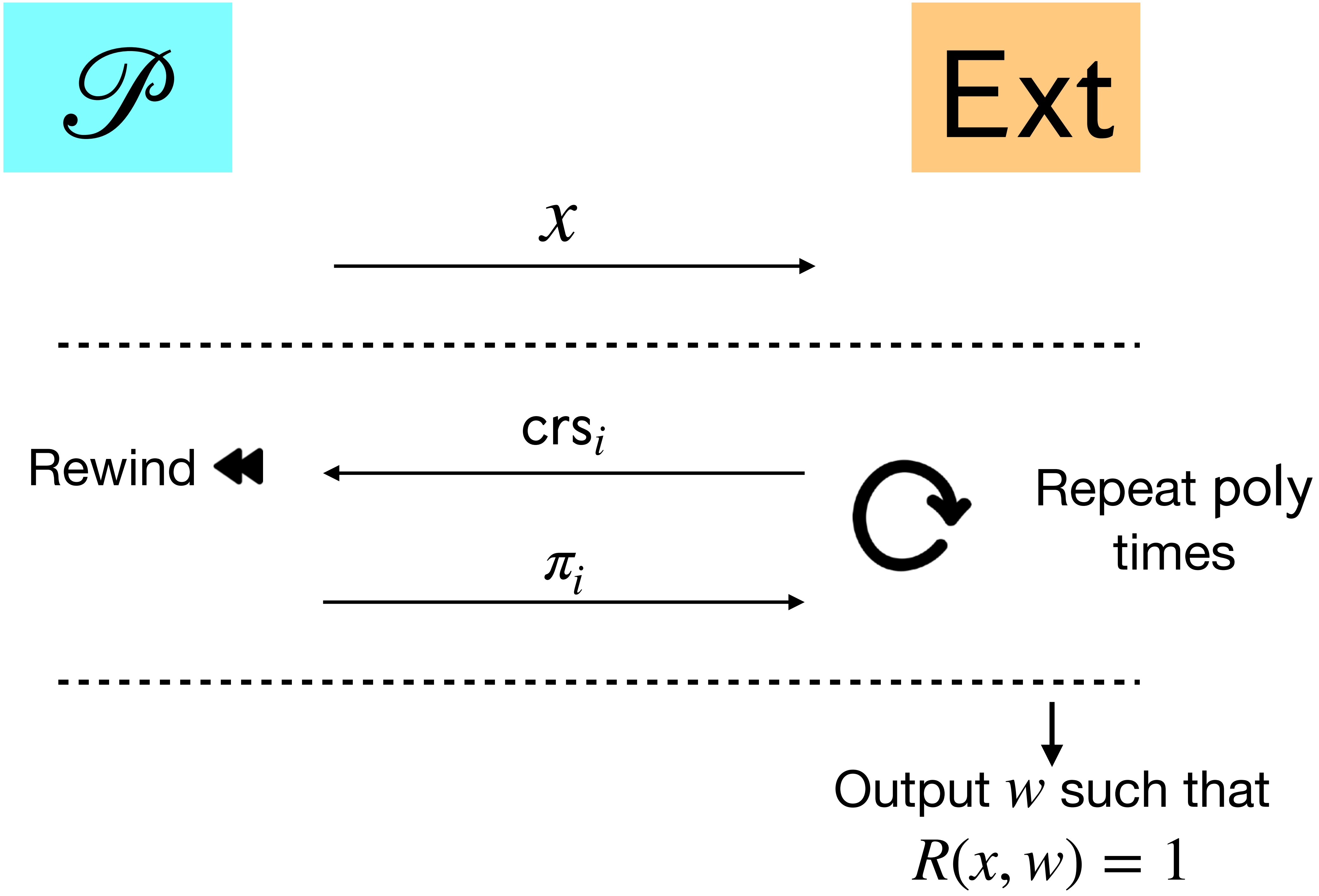

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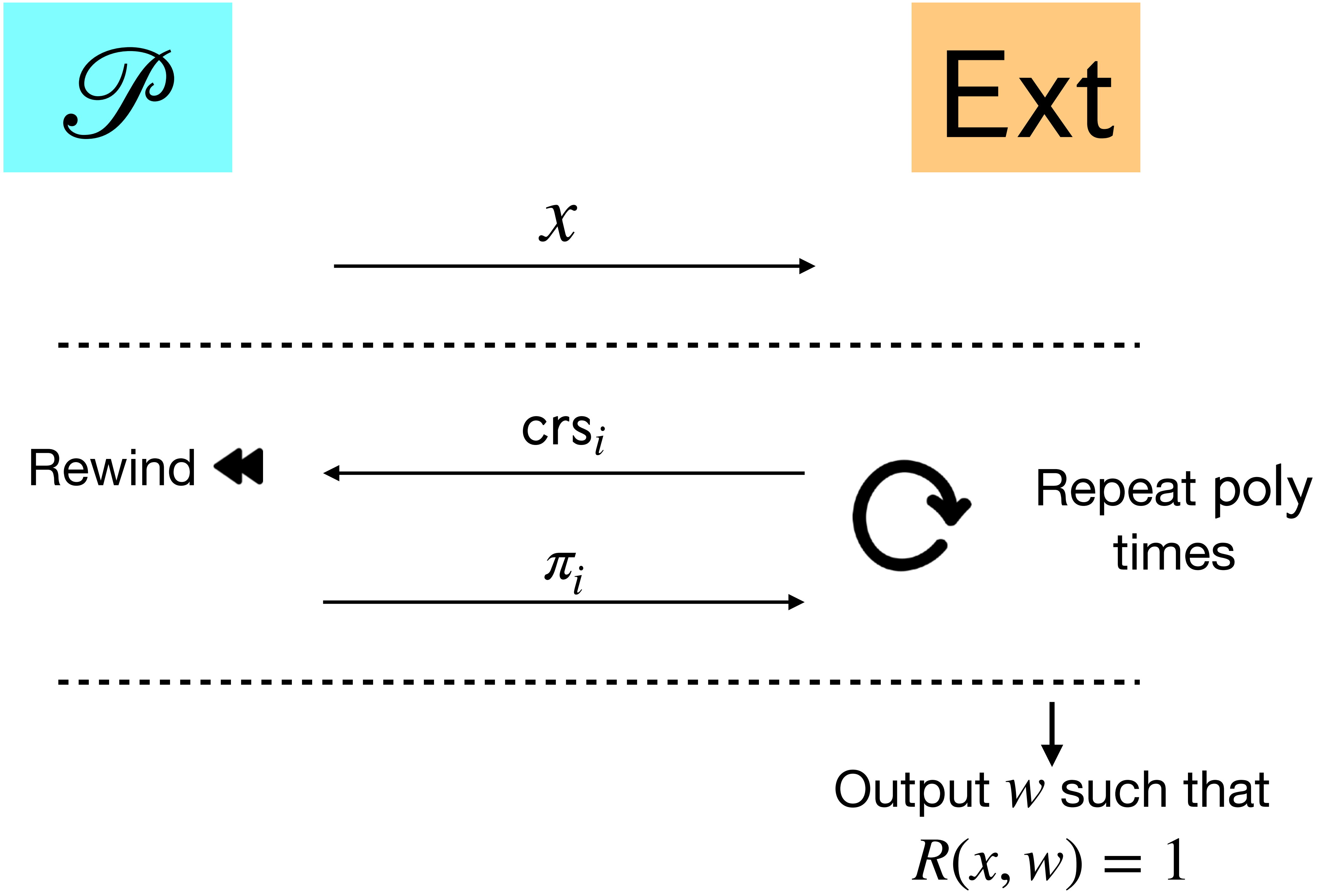

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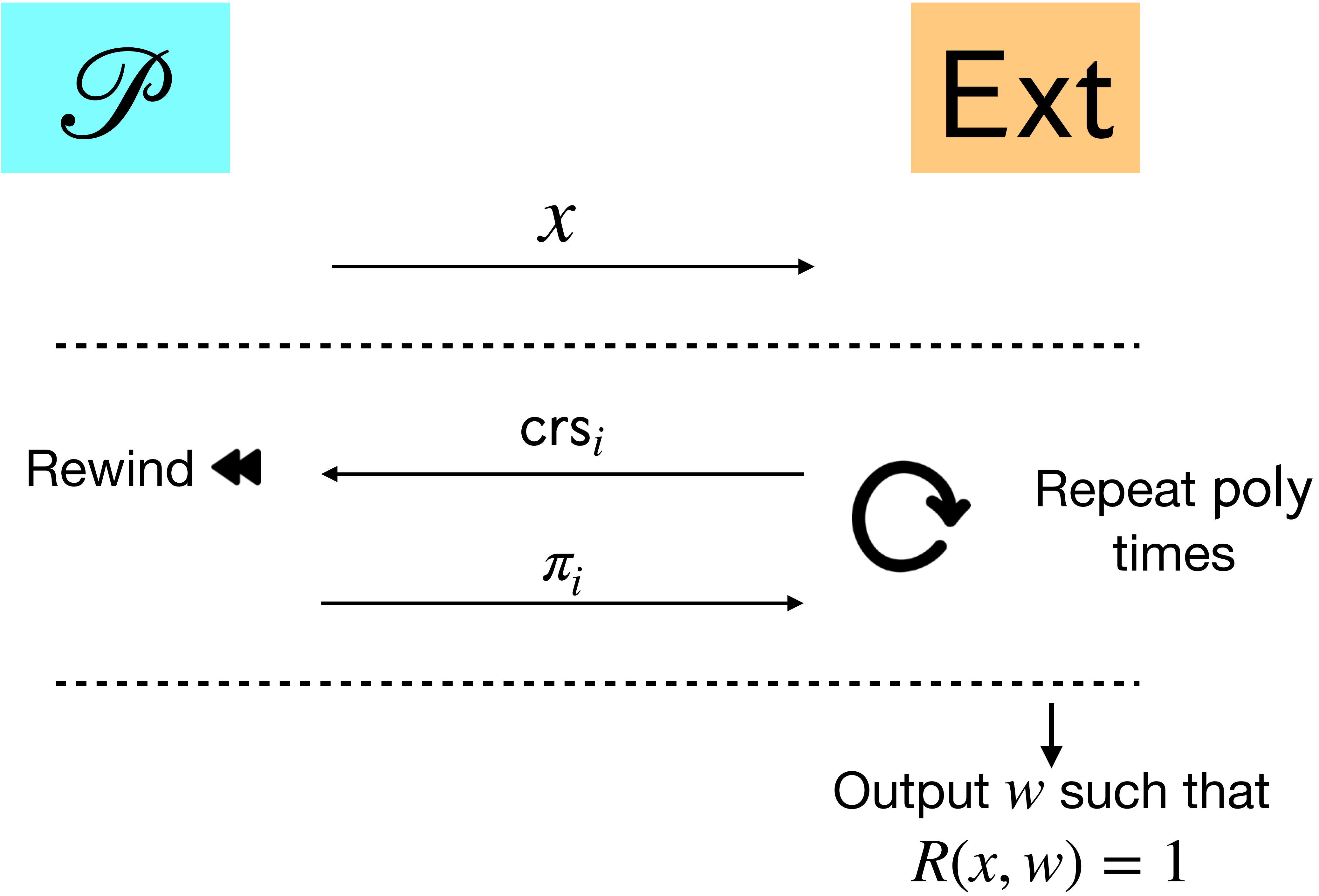

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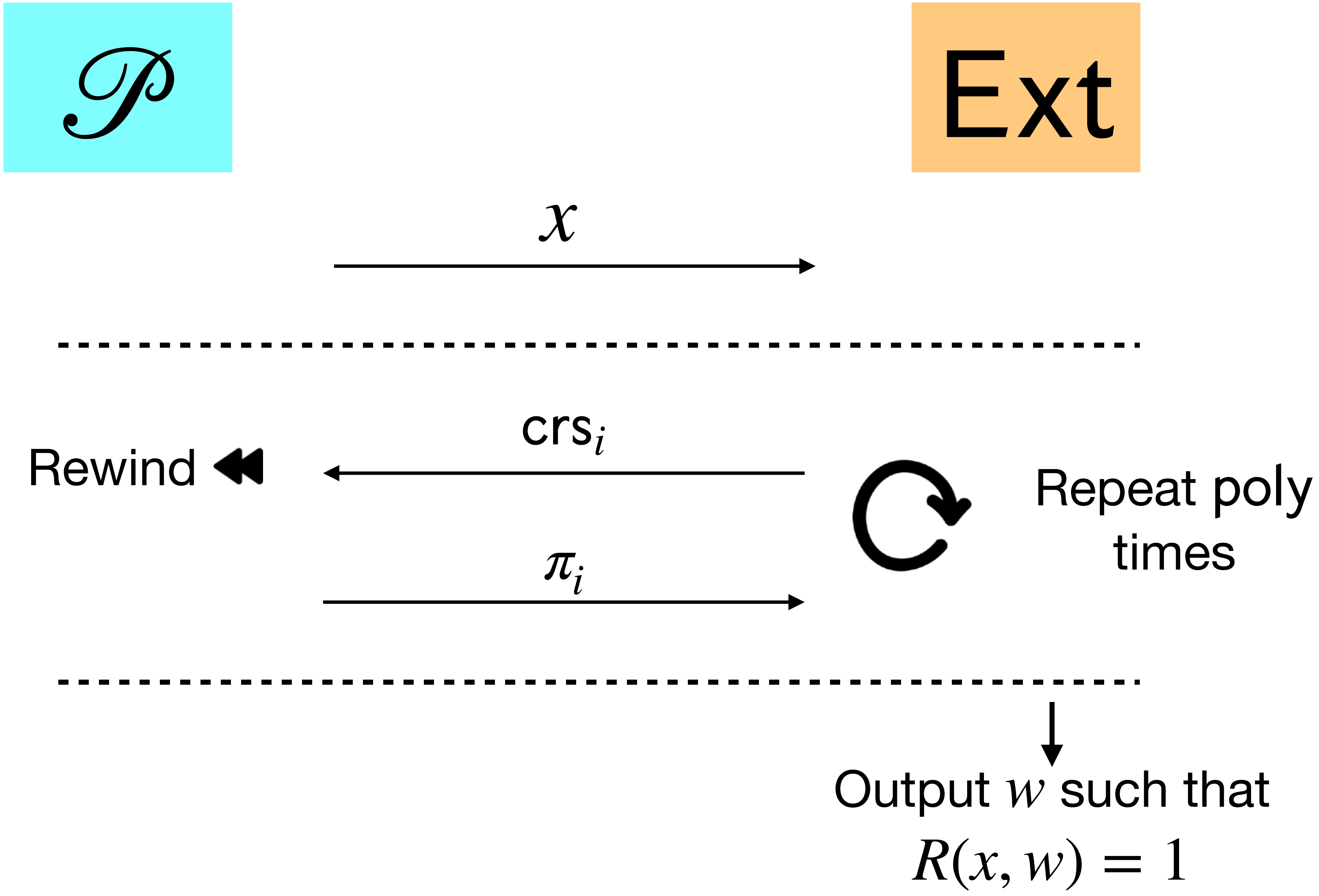

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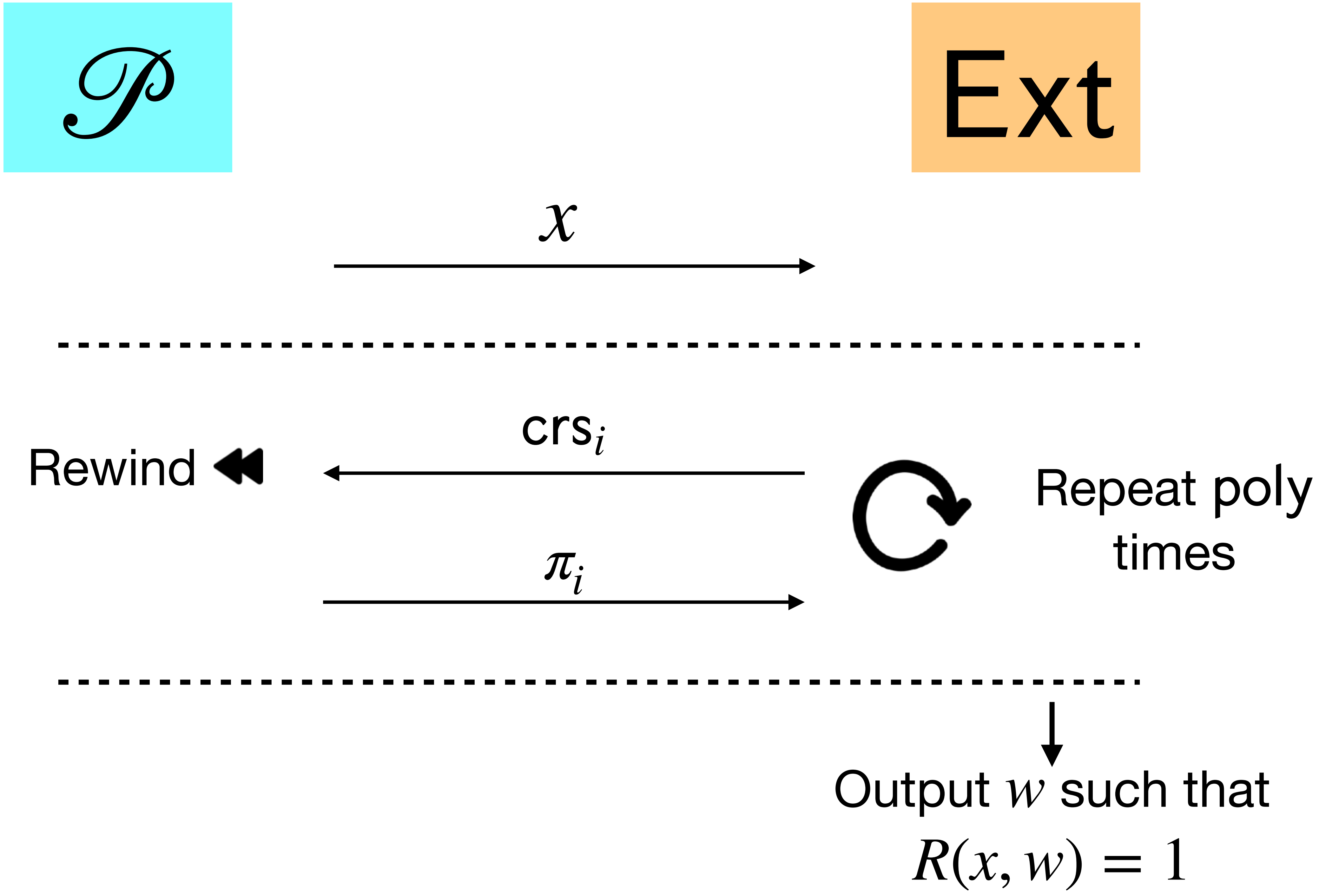

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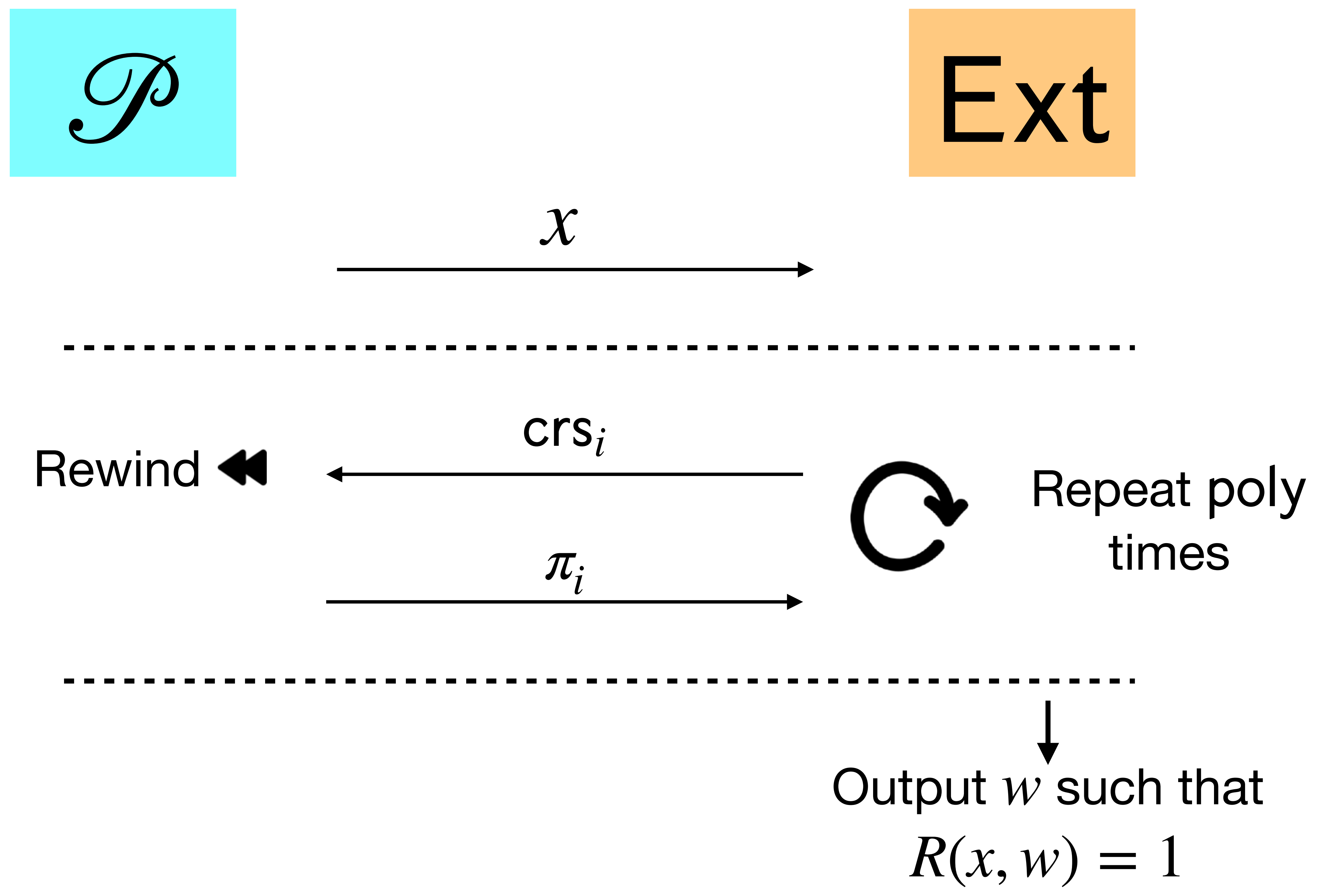

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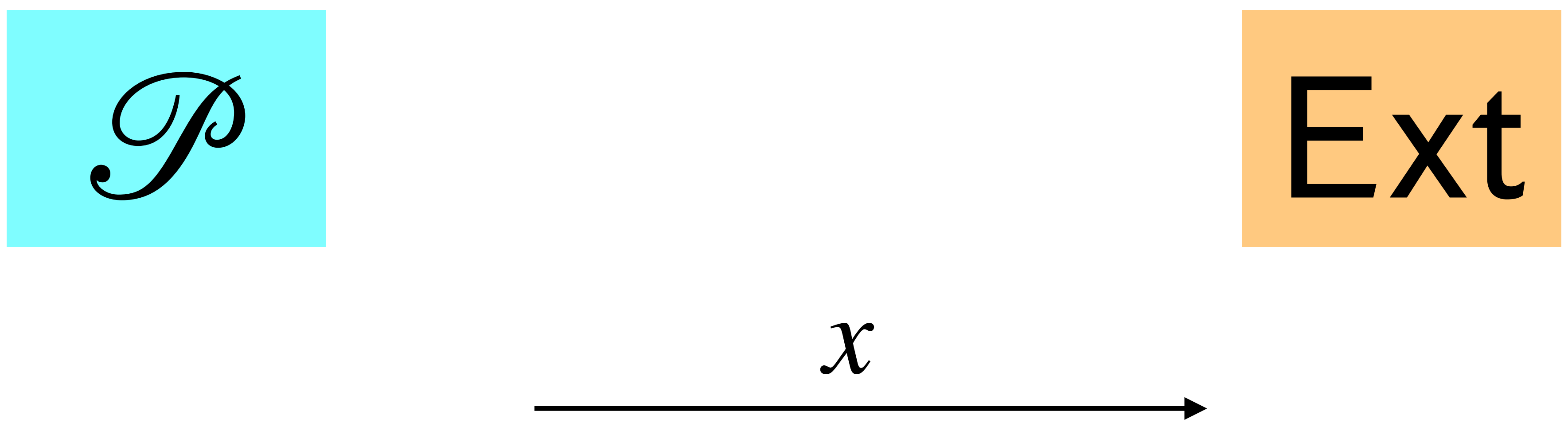

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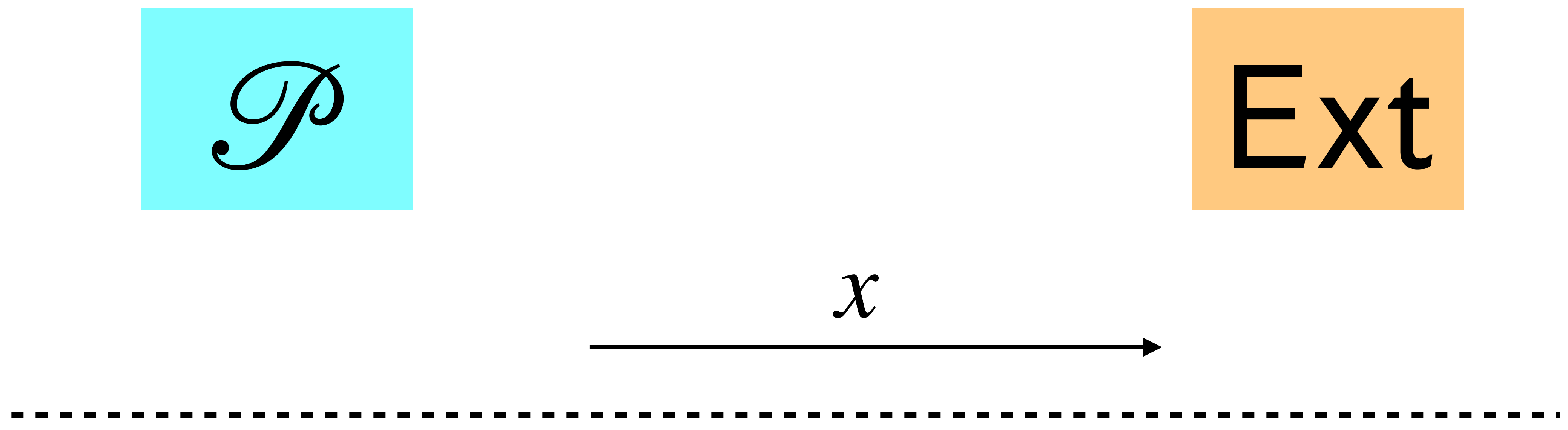

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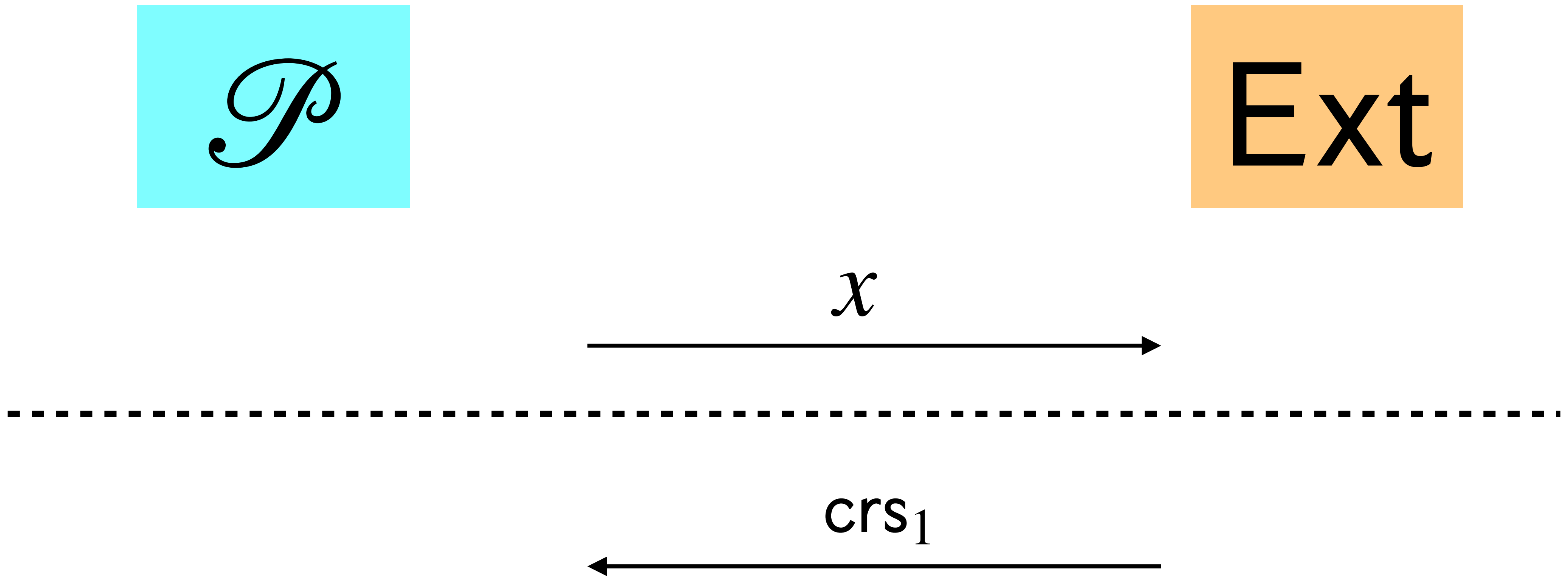

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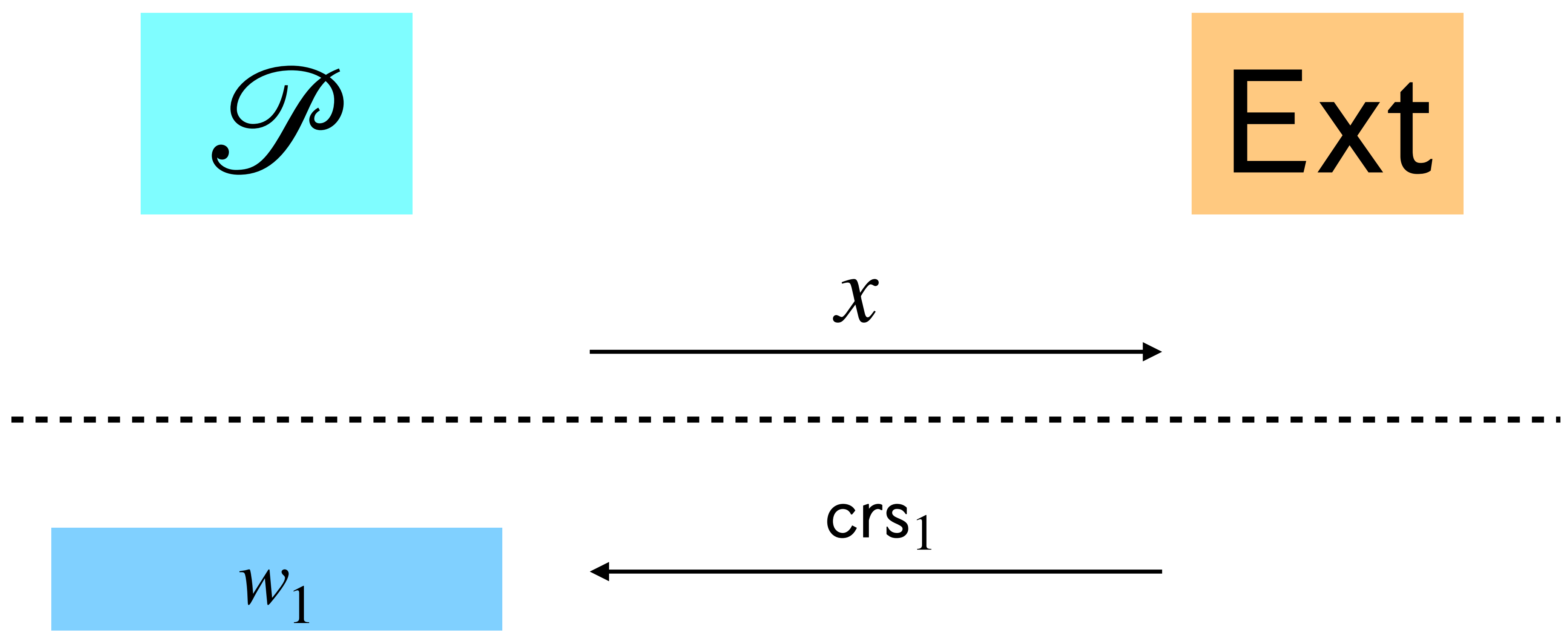

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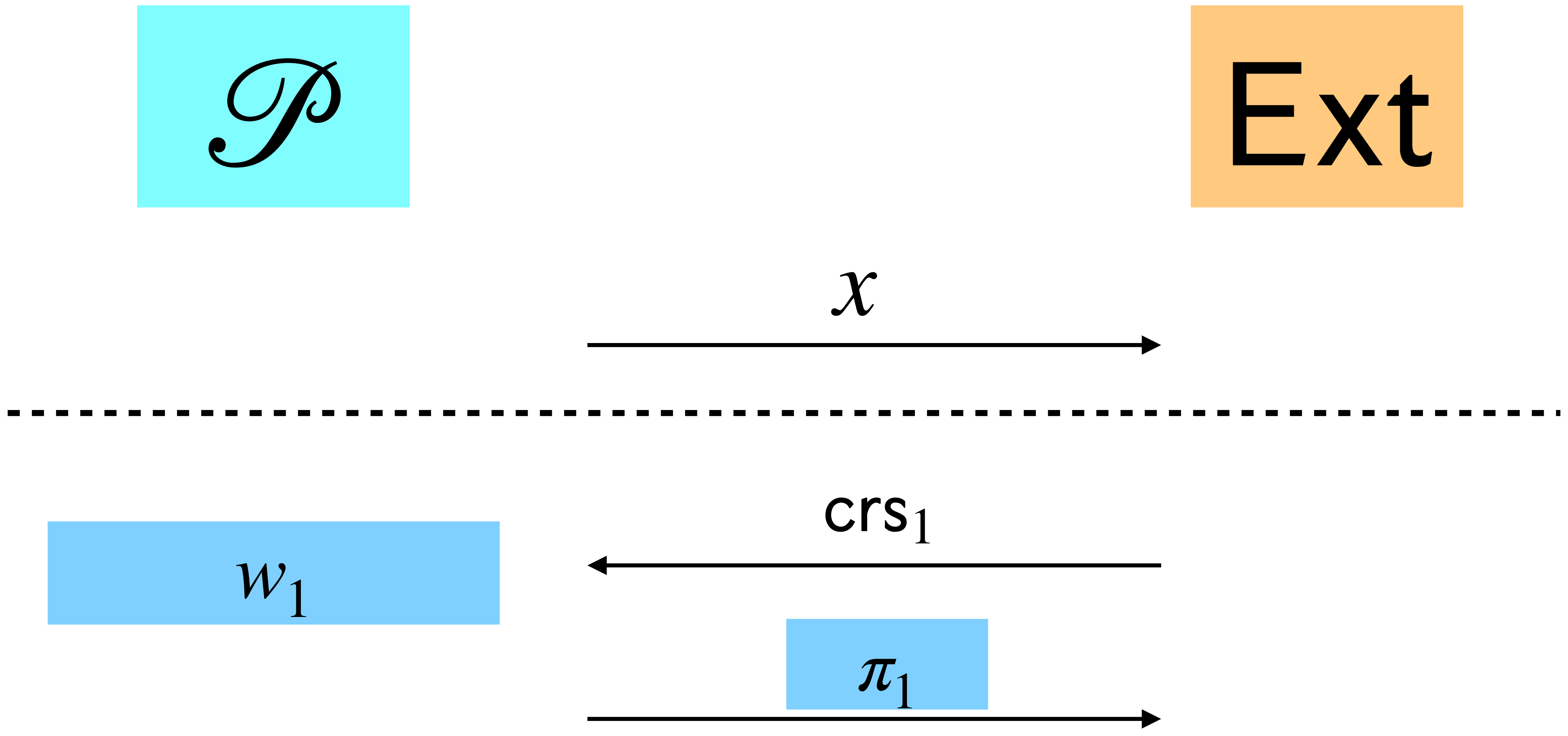

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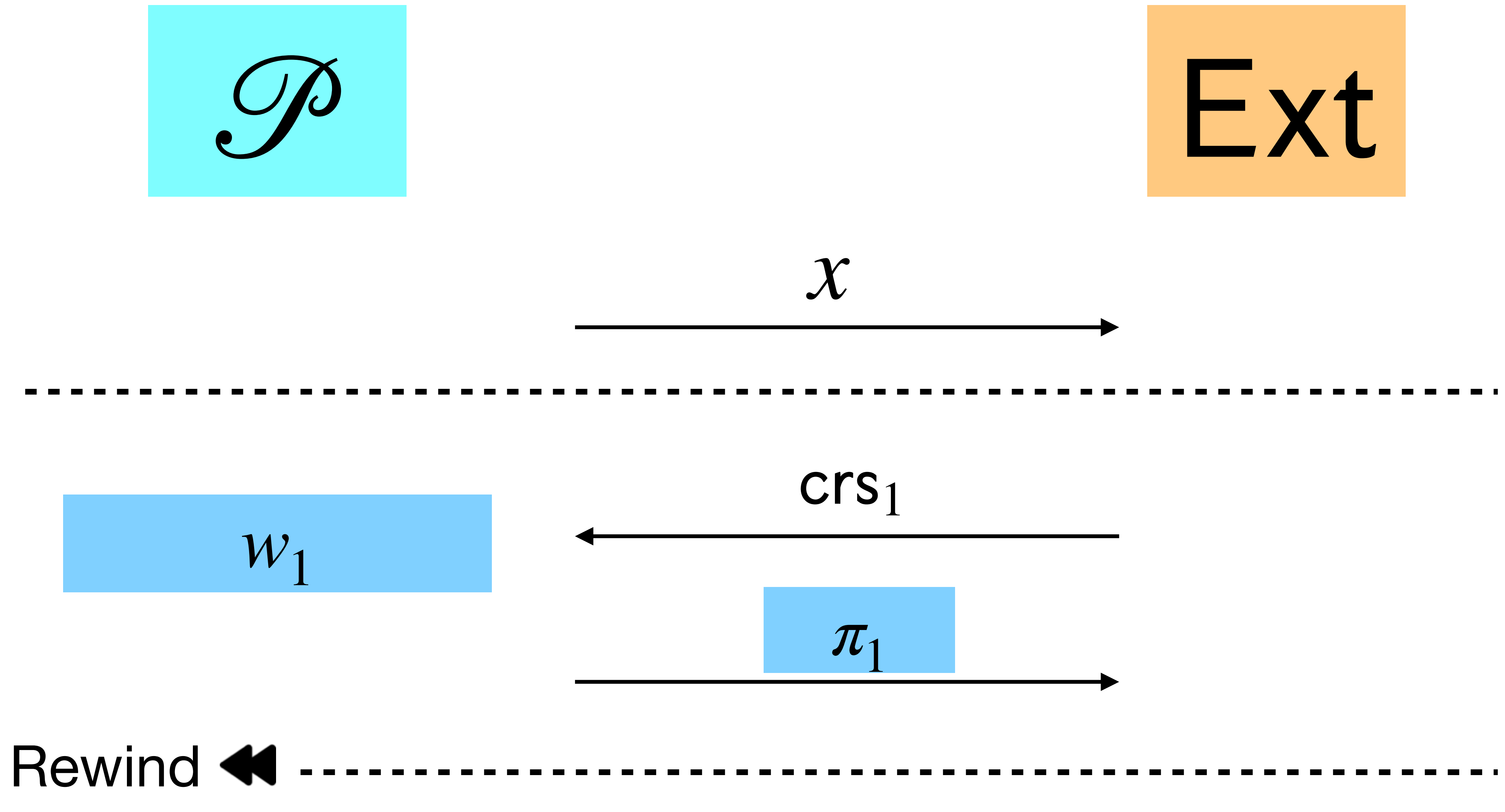

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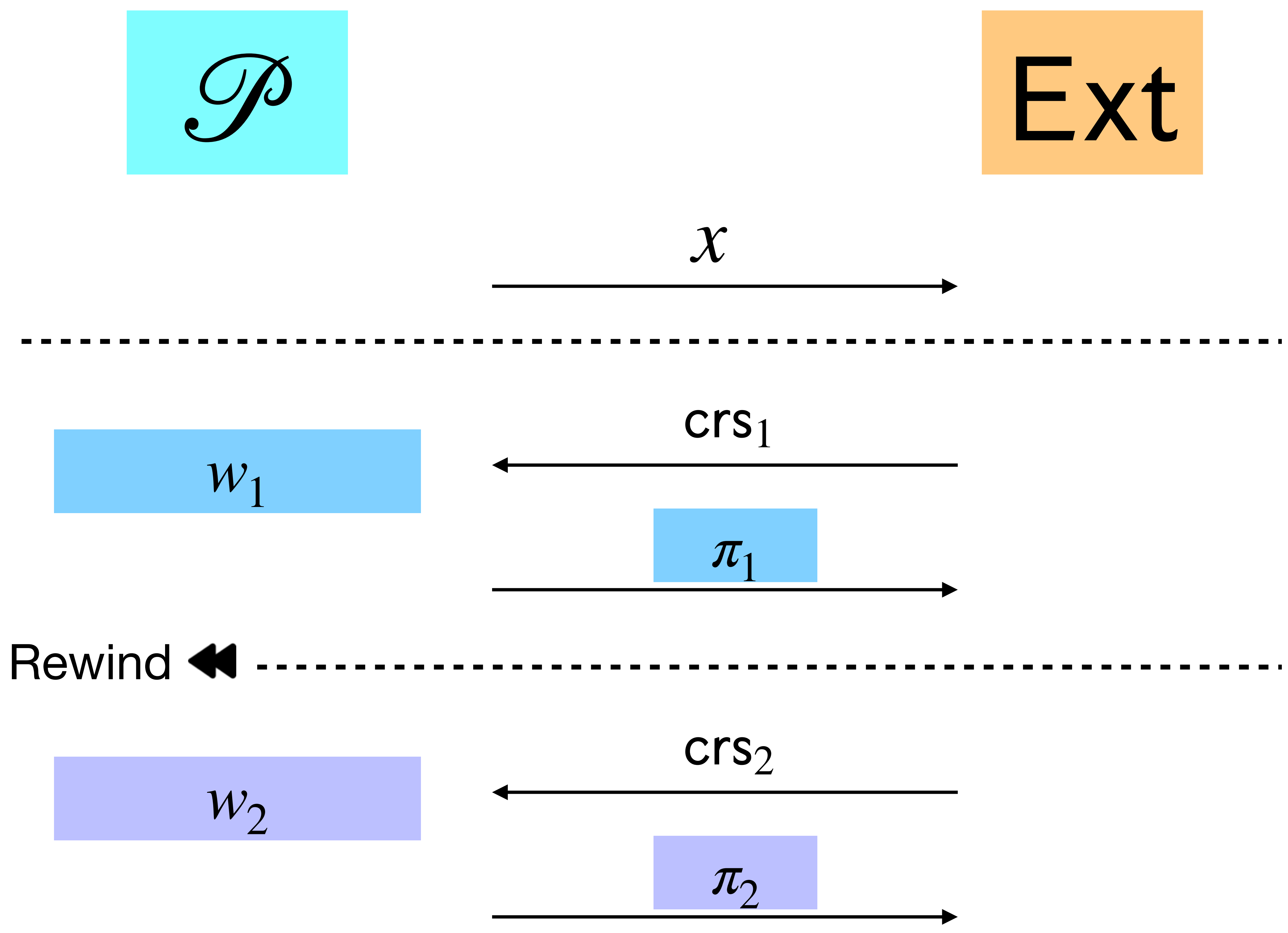

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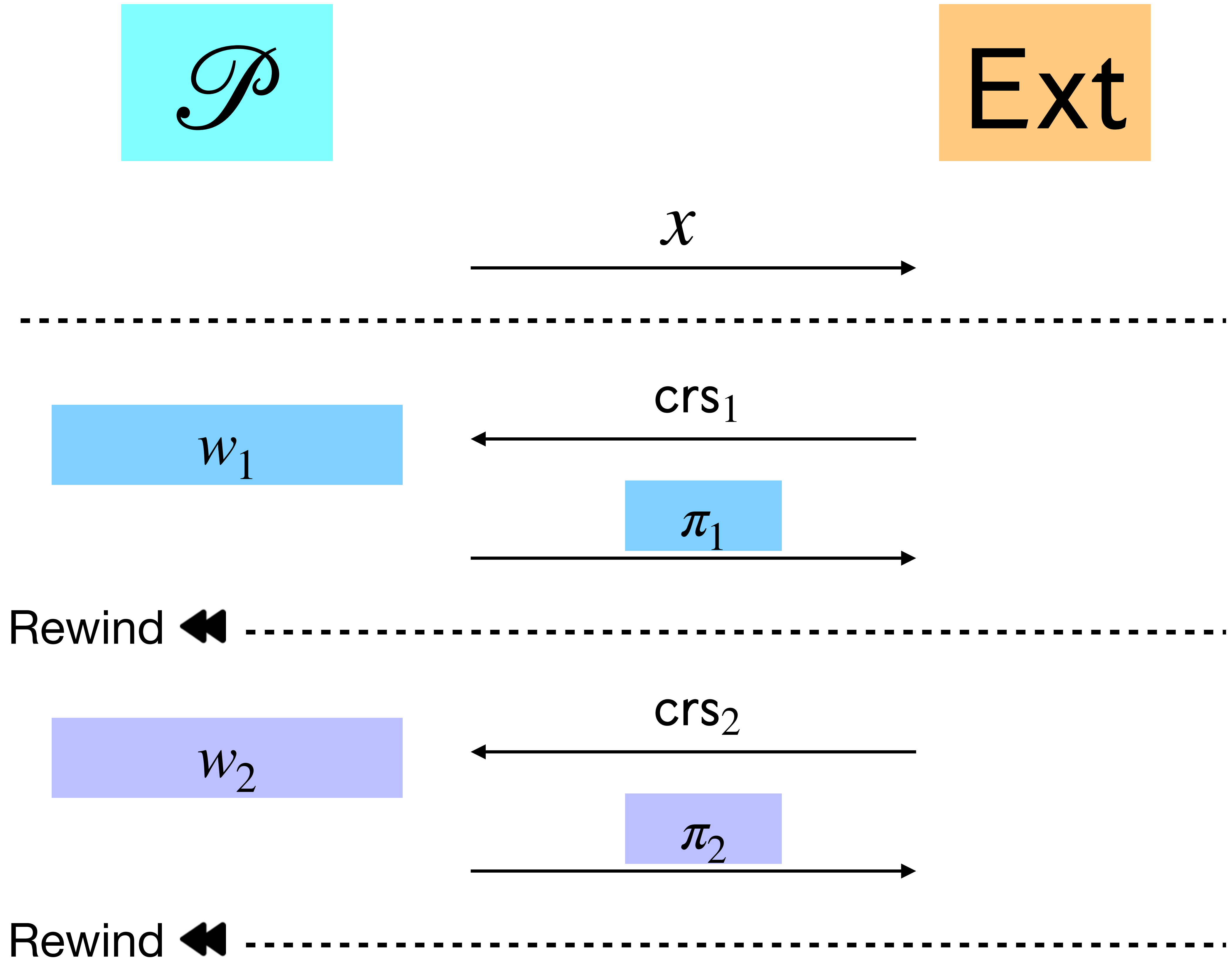

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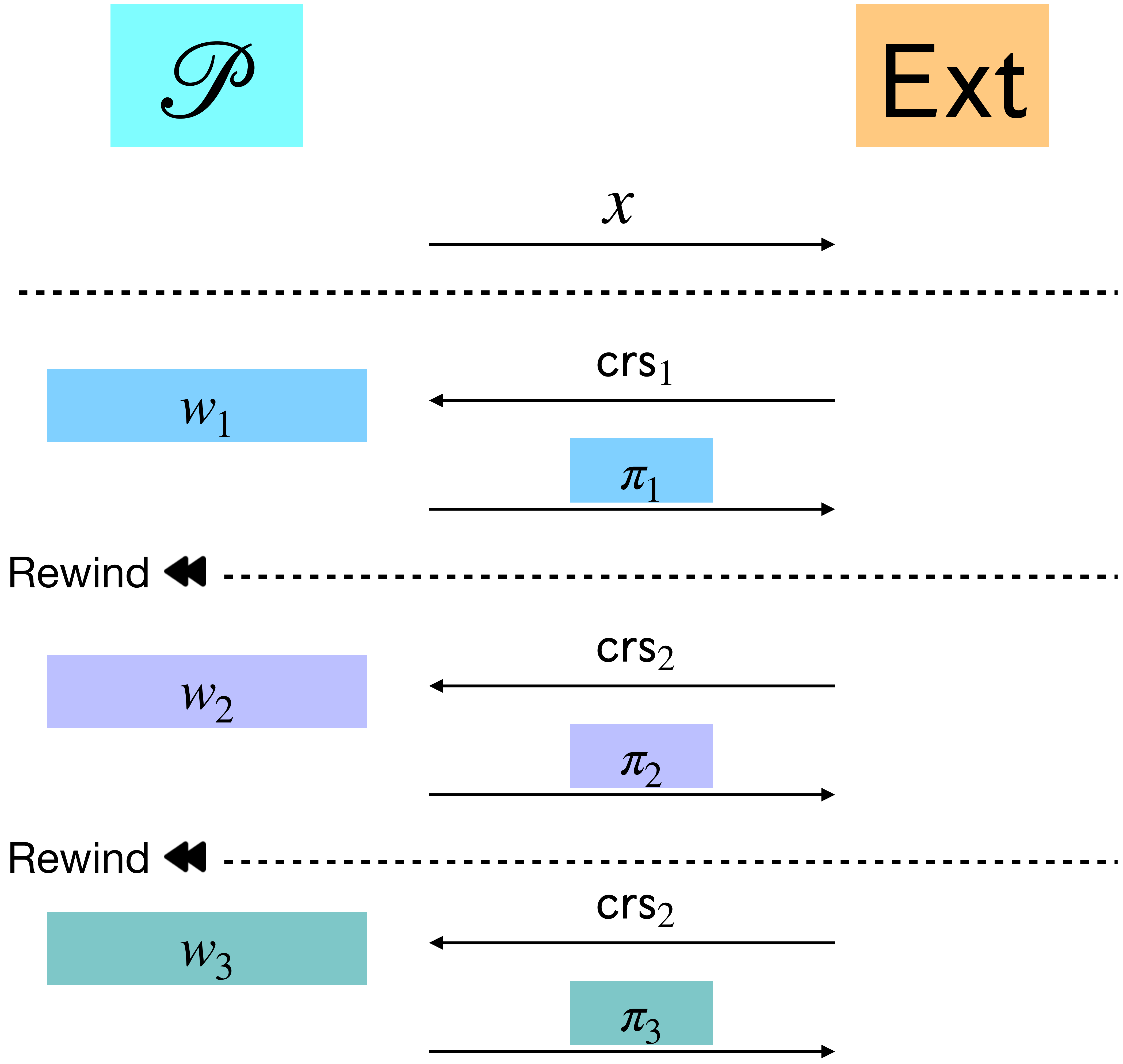

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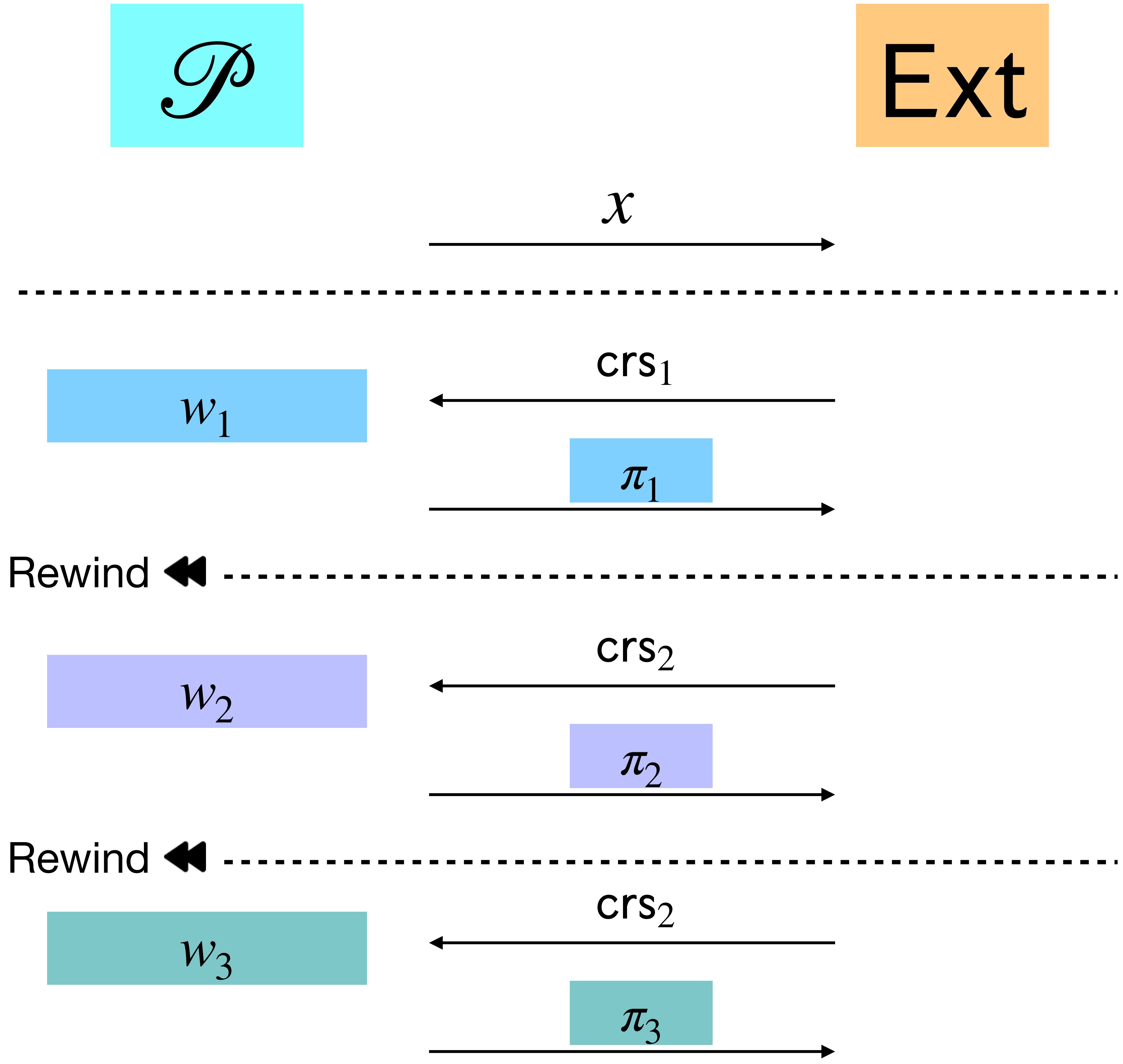

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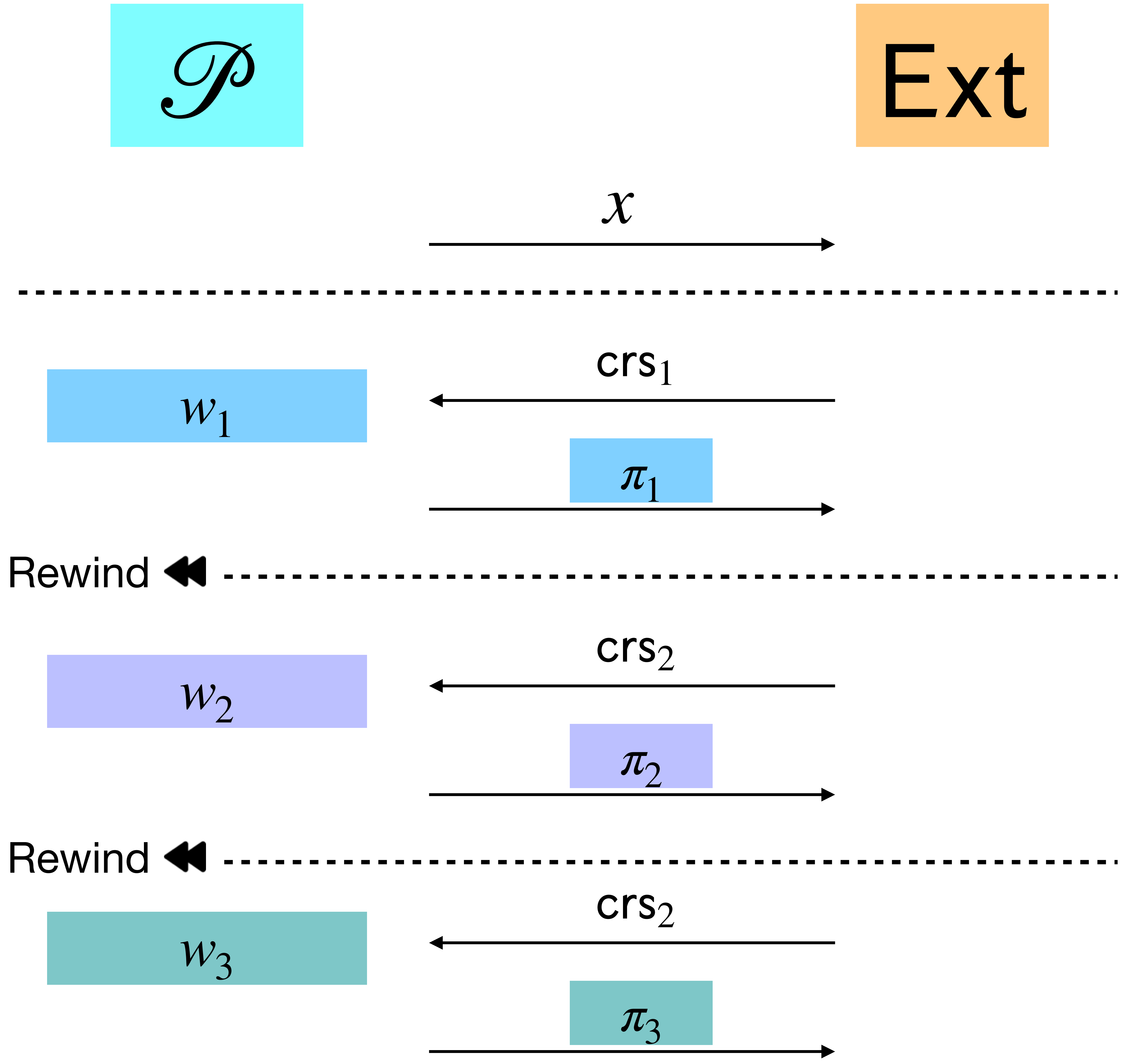

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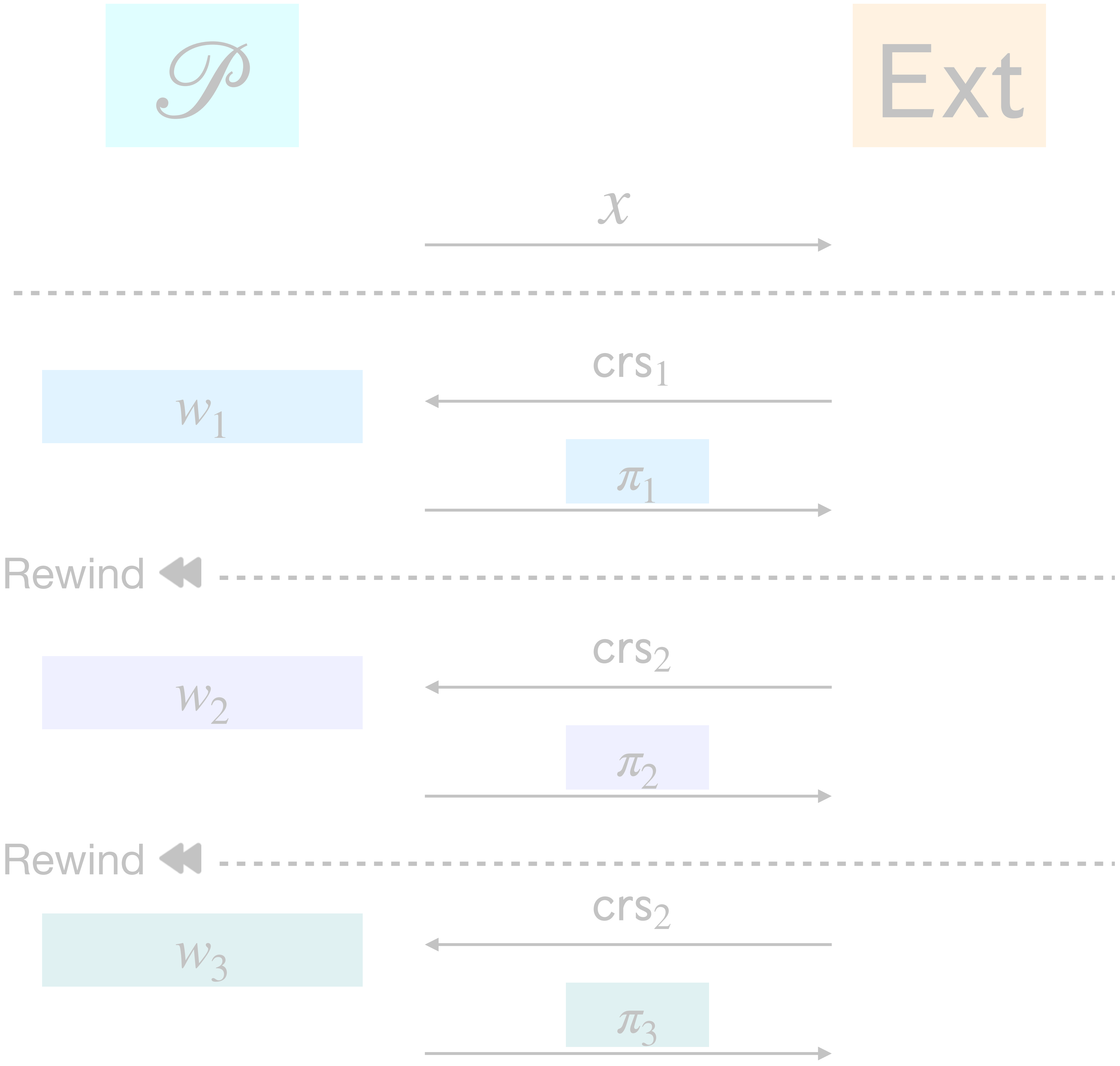

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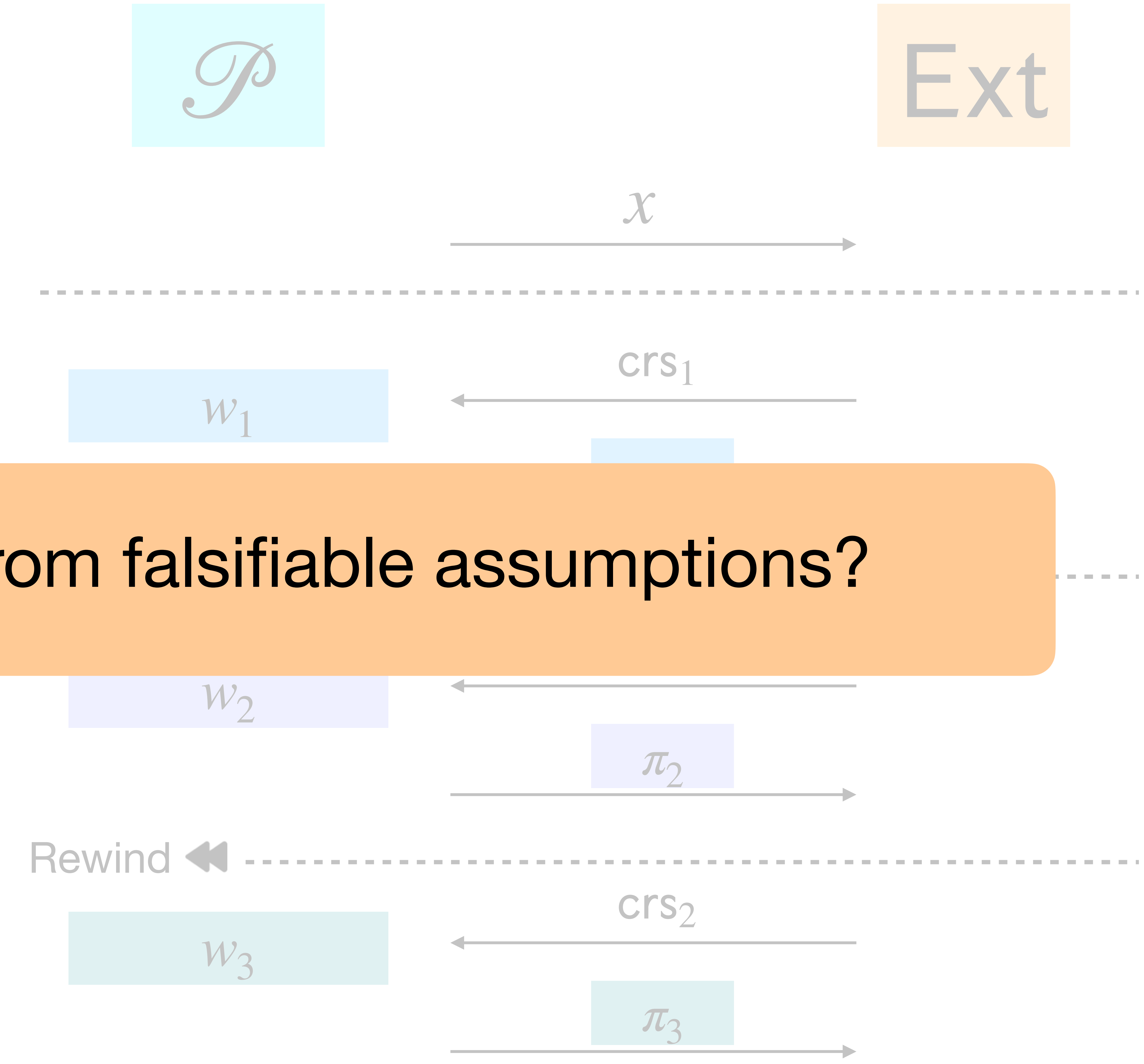

Qn: Can we build SNAR**K**s for UP from falsifiable assumptions?

SNARG to SNARK for UP

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Adaptive vs. Non-Adaptive Soundness

Adaptive Soundness

$\mathcal{P}^*$

$\mathcal{V}$

Non-Adaptive Soundness

$\mathcal{P}^*$

$\mathcal{V}$

Adaptive vs. Non-Adaptive Soundness

Adaptive Soundness

$\mathcal{P}^*$

$\mathcal{V}$

Common Reference String (crs)

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$\mathcal{P}^*$

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Adaptive vs. Non-Adaptive Soundness

Adaptive Soundness

$\mathscr{P}^*$

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$\xrightarrow{x^*, \pi^*}$

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Adaptive vs. Non-Adaptive Soundness

Adaptive Soundness

$\mathscr{P}^*$ $\mathscr{V}$

Common Reference String (crs)

$\xrightarrow{x^*, \pi^*}$

Non-Adaptive Soundness

$\mathscr{P}^*$ $\xrightarrow{x^* \notin L}$ $\mathscr{V}$

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(Informal) Gentry-Wichs Barrier: If a language takes time 2^{n^δ} to "decide", there is no 2^{n^ϵ} for $\epsilon < \delta$ black-box reduction to *falsifiable assumptions* that shows **adaptive soundness.**

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Adaptive Soundness

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Intuition: How can you tell if soundness was broken? Need to decide if $x^* \in L$!

Non-Adaptive Soundness

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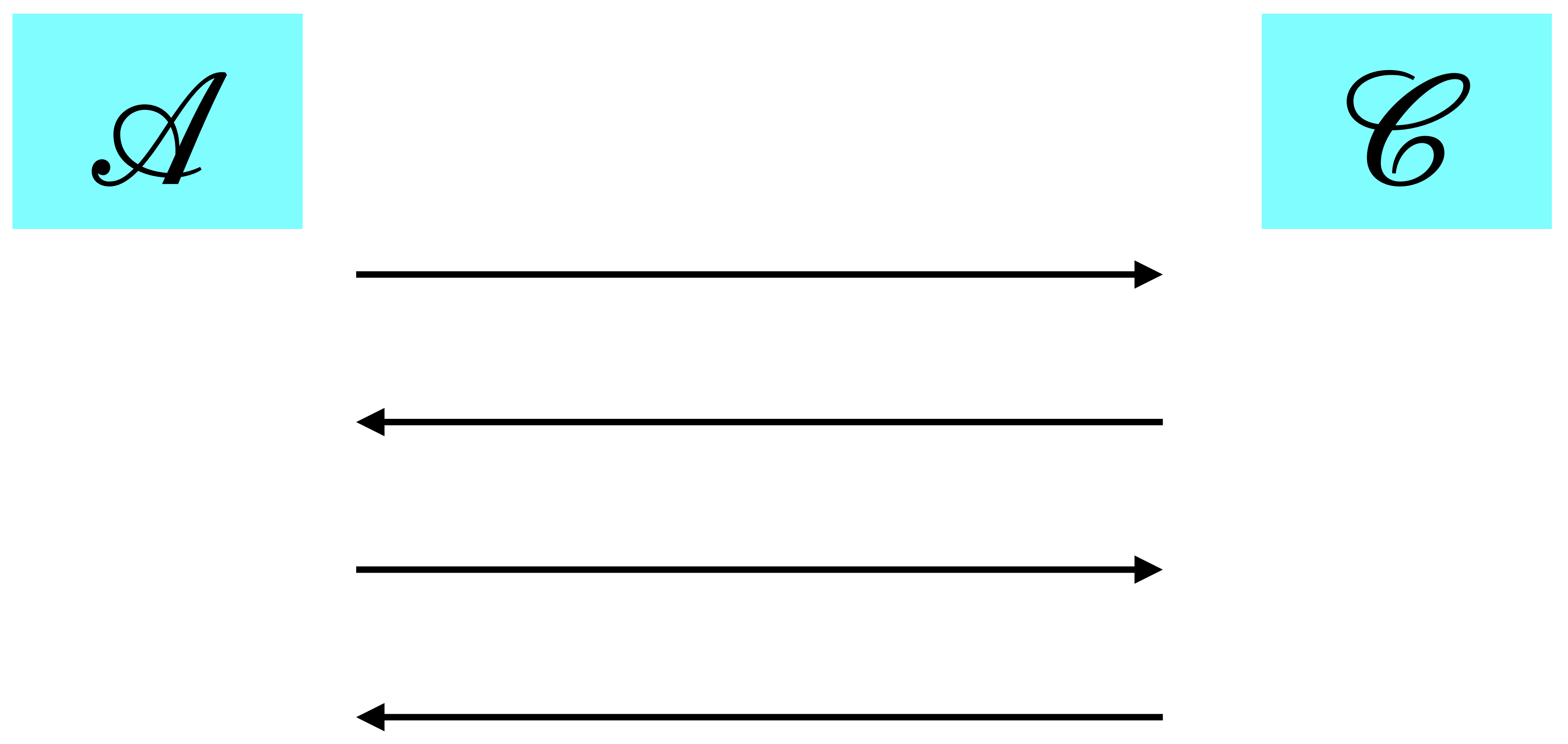

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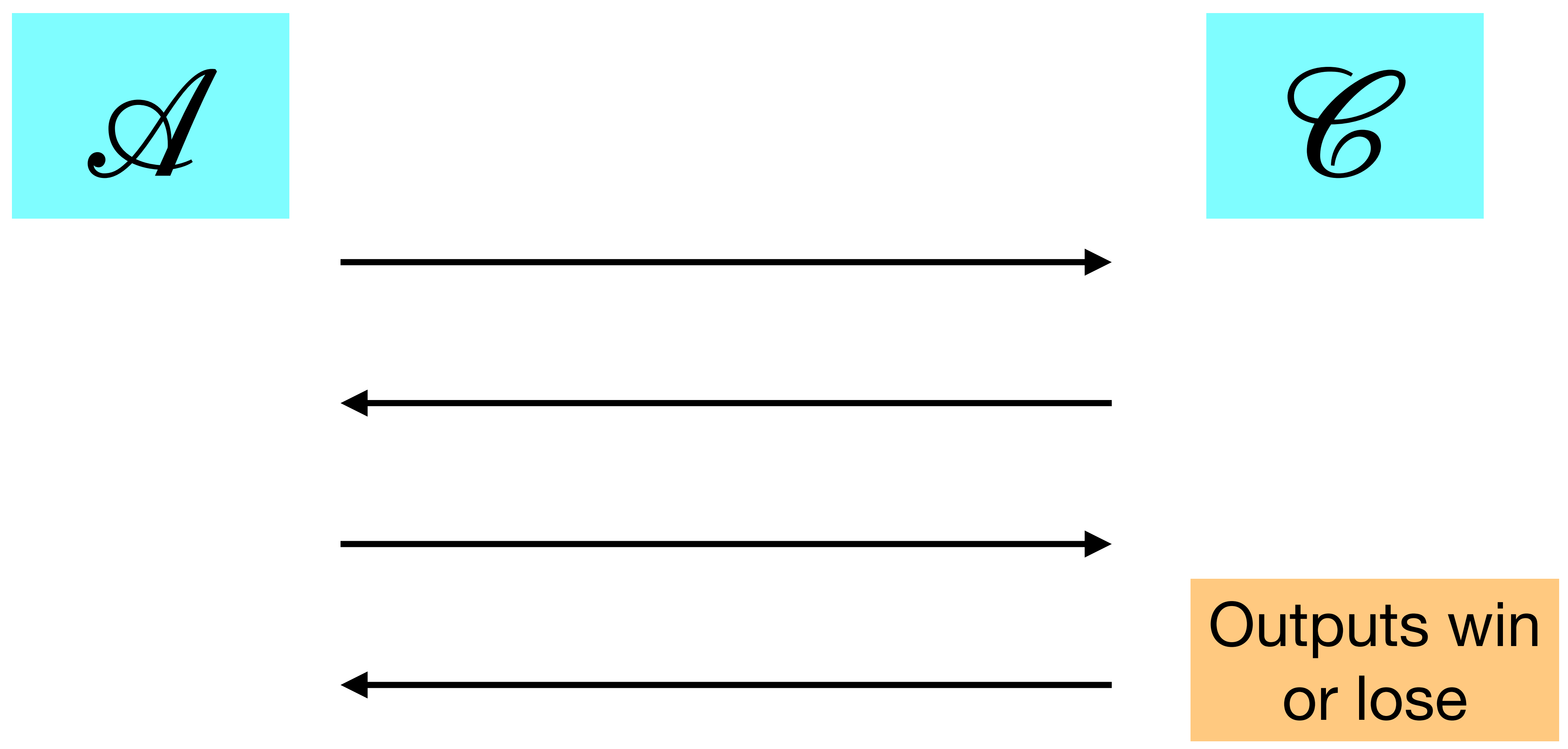

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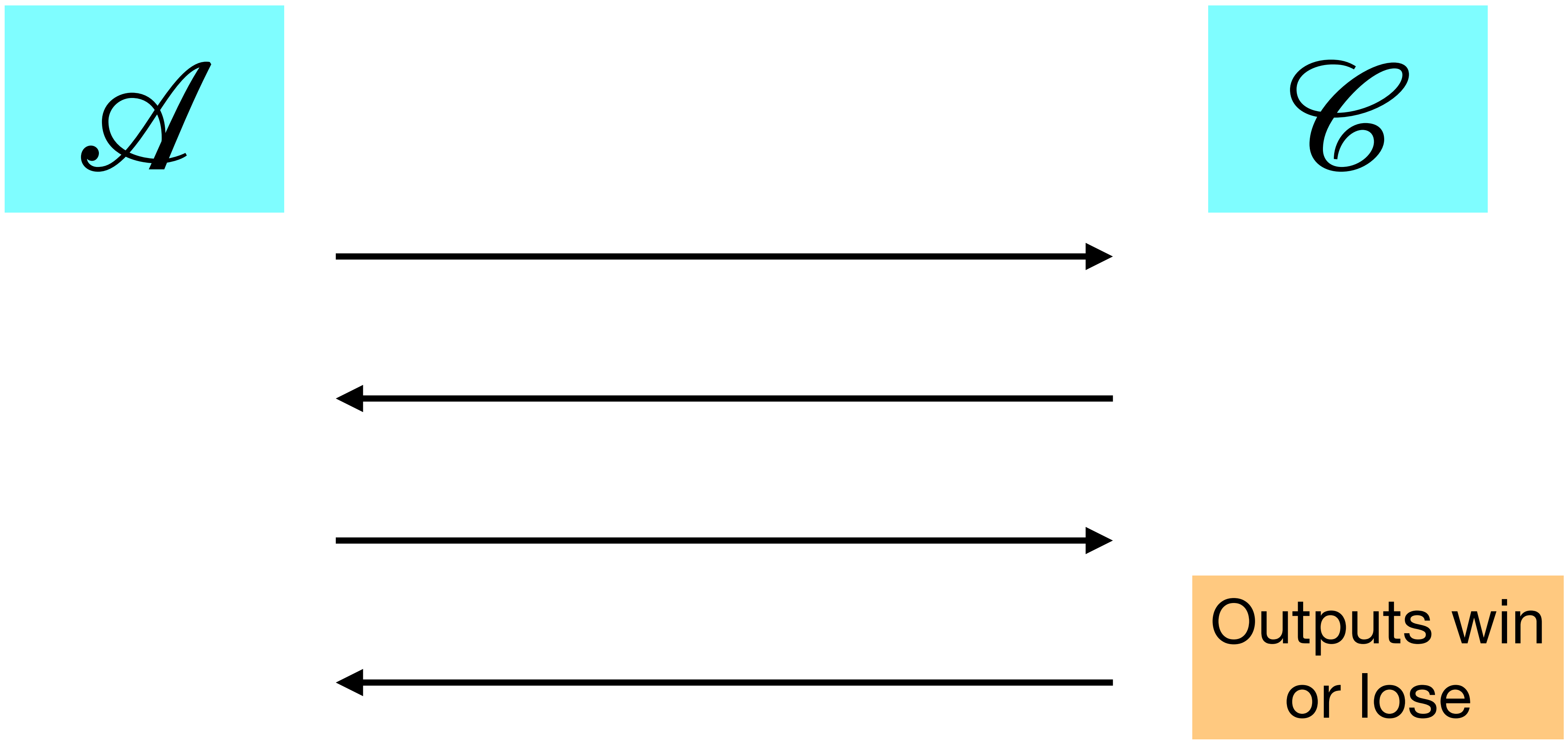

- E.g. Decision problems like DDH and LWE have parameter $c = 1/2$
- E.g. Search problems like OWF, DLOG have parameter $c = 0$

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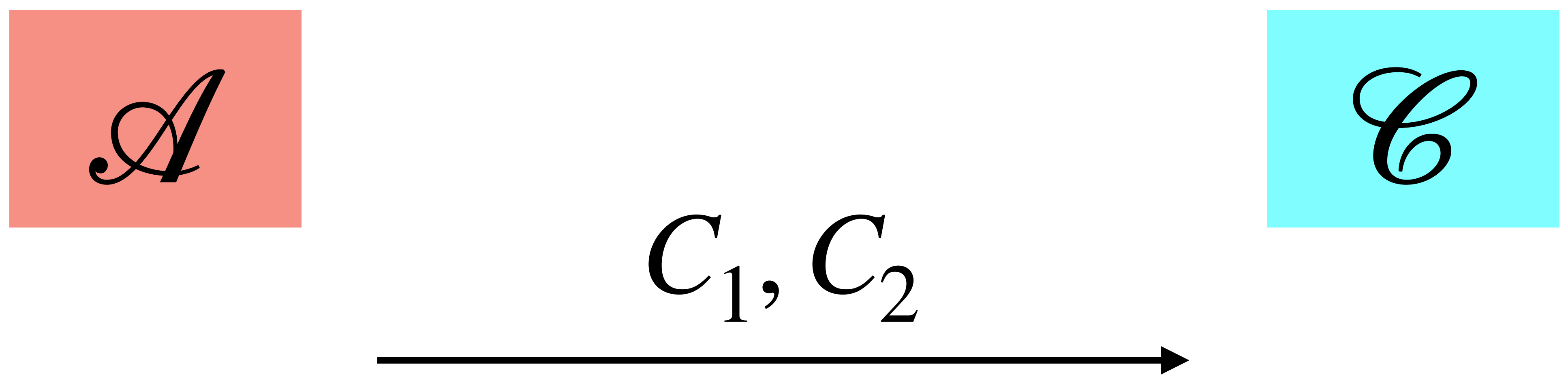

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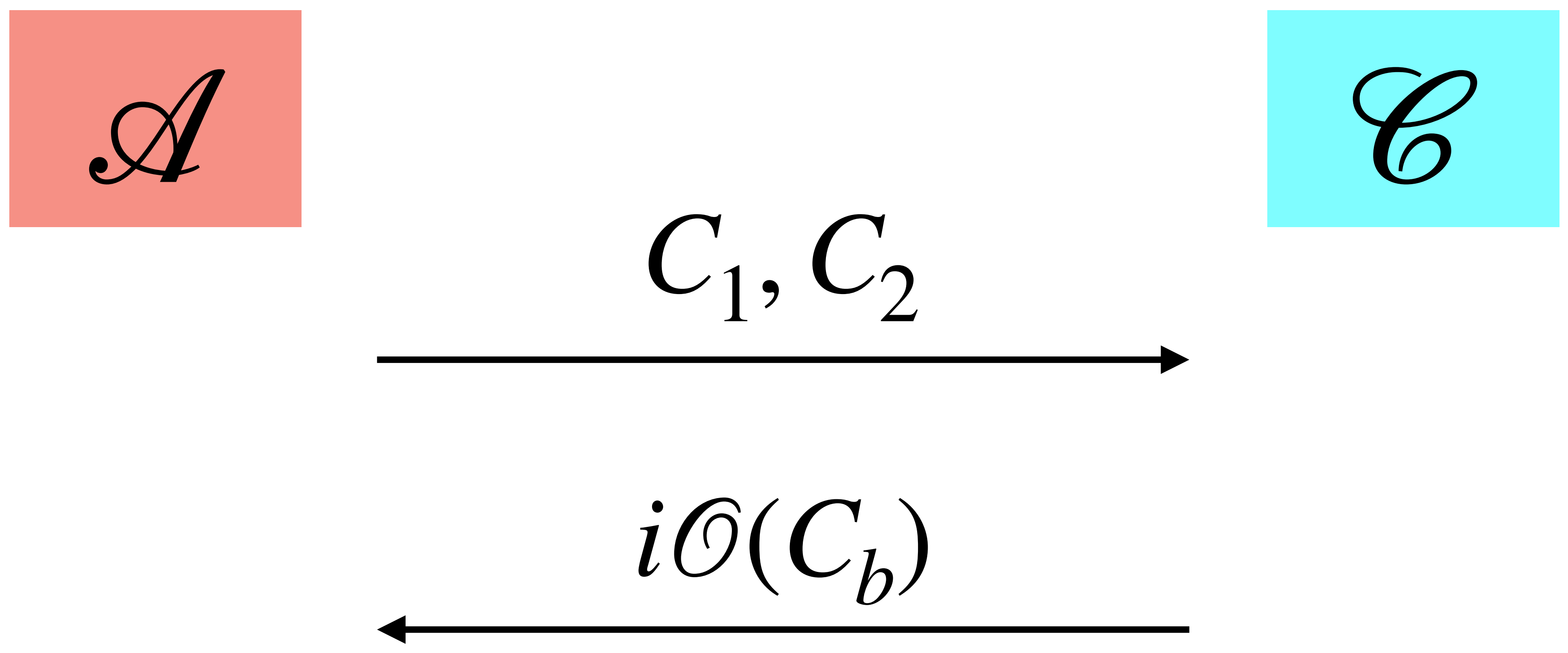

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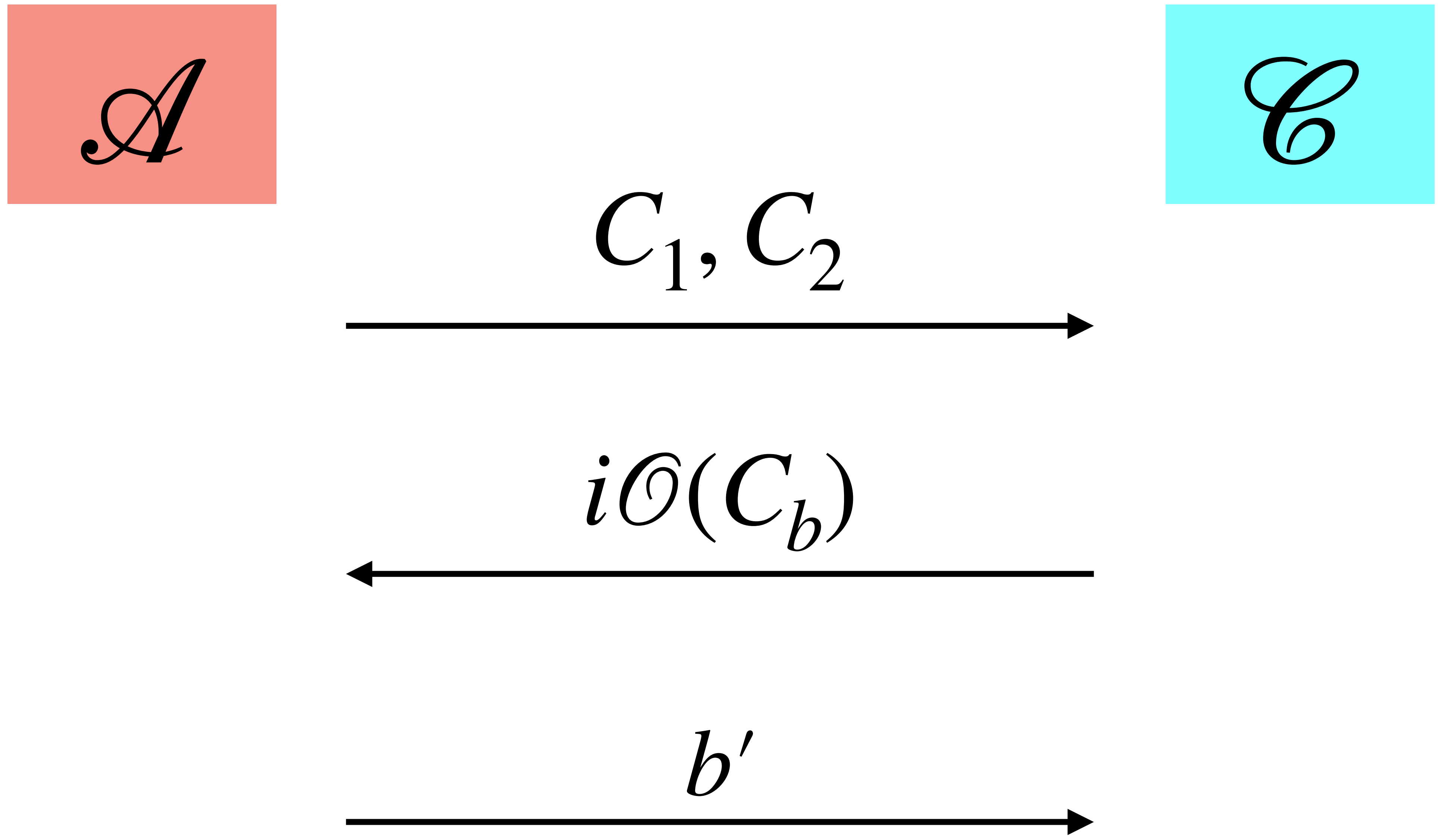

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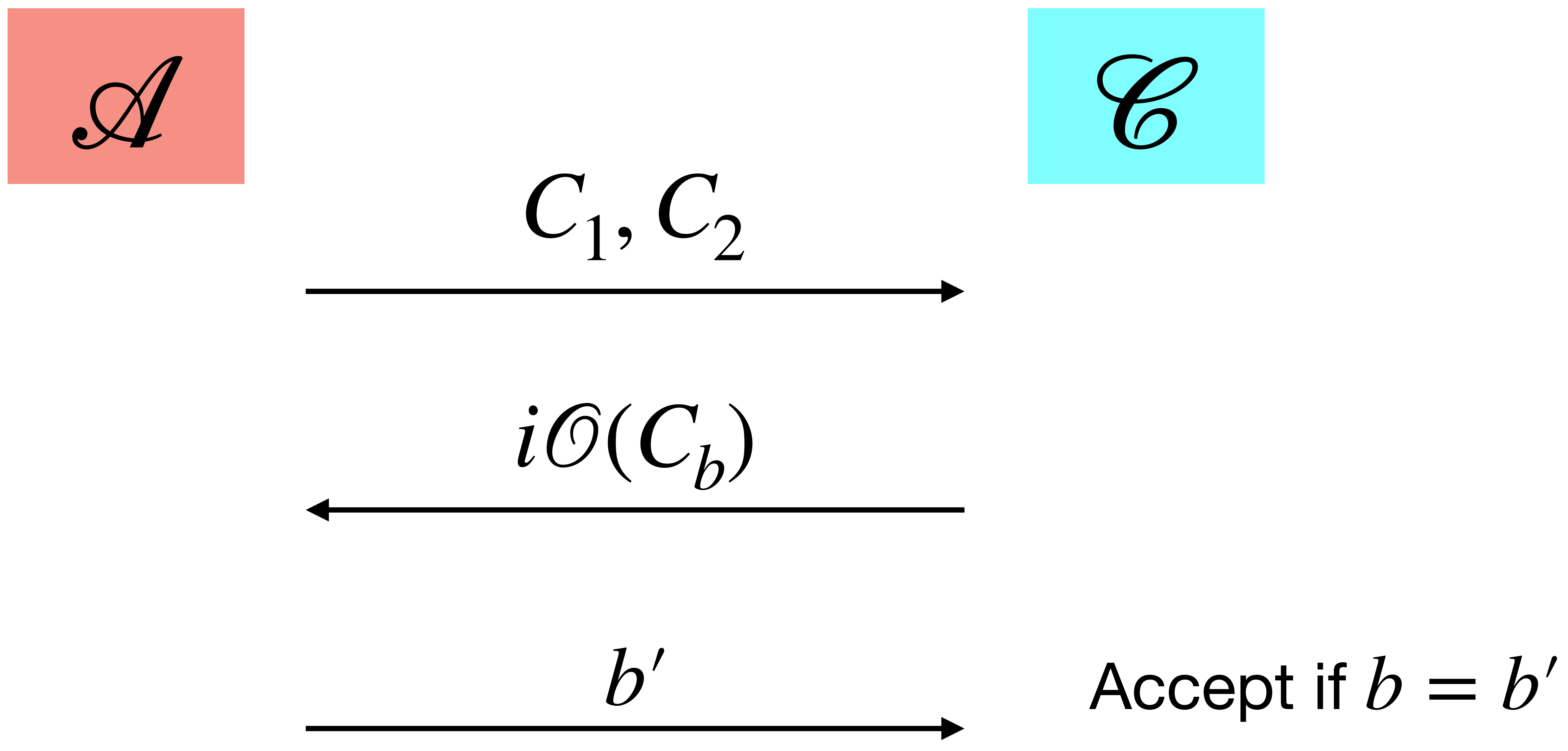

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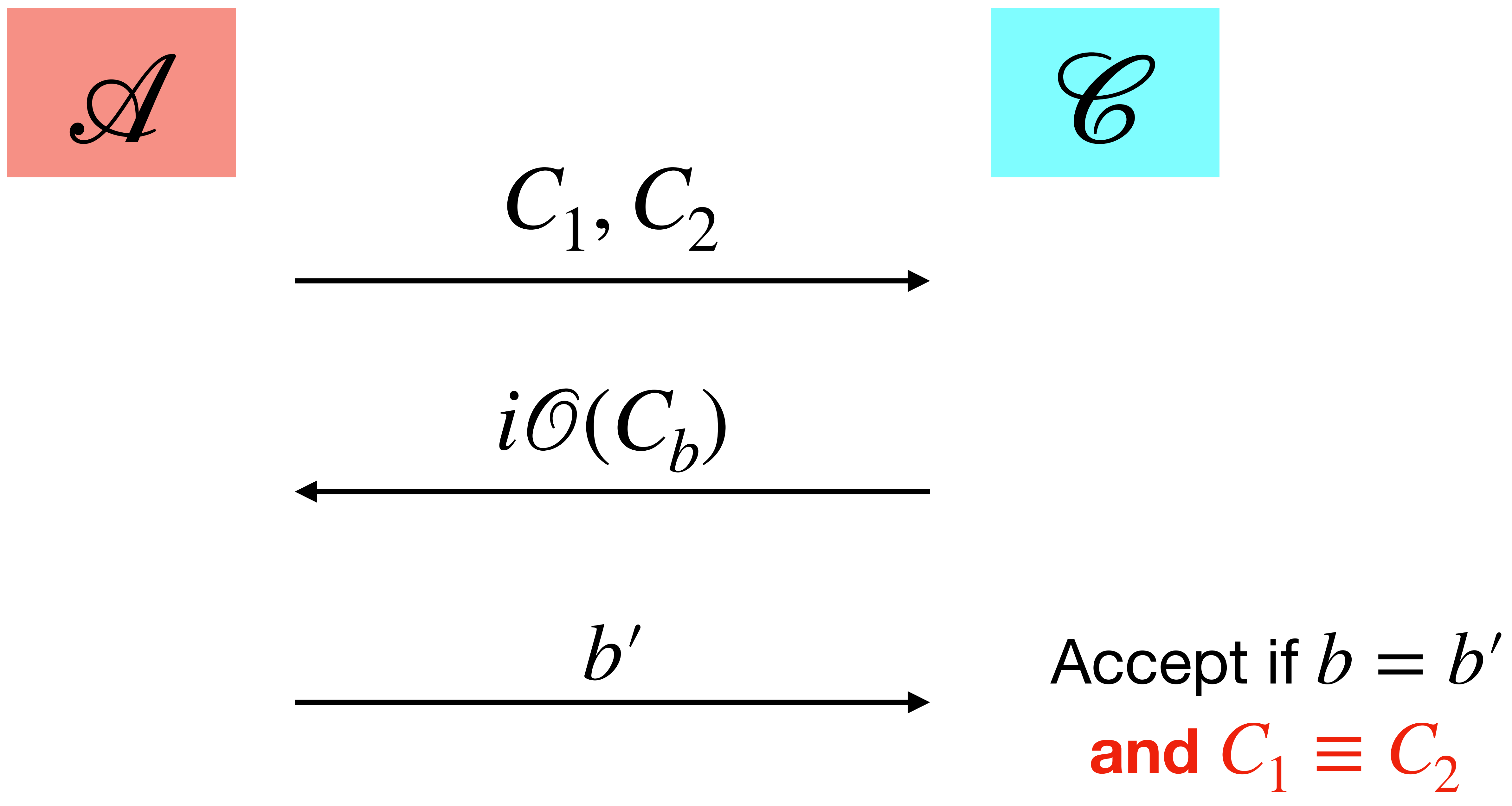

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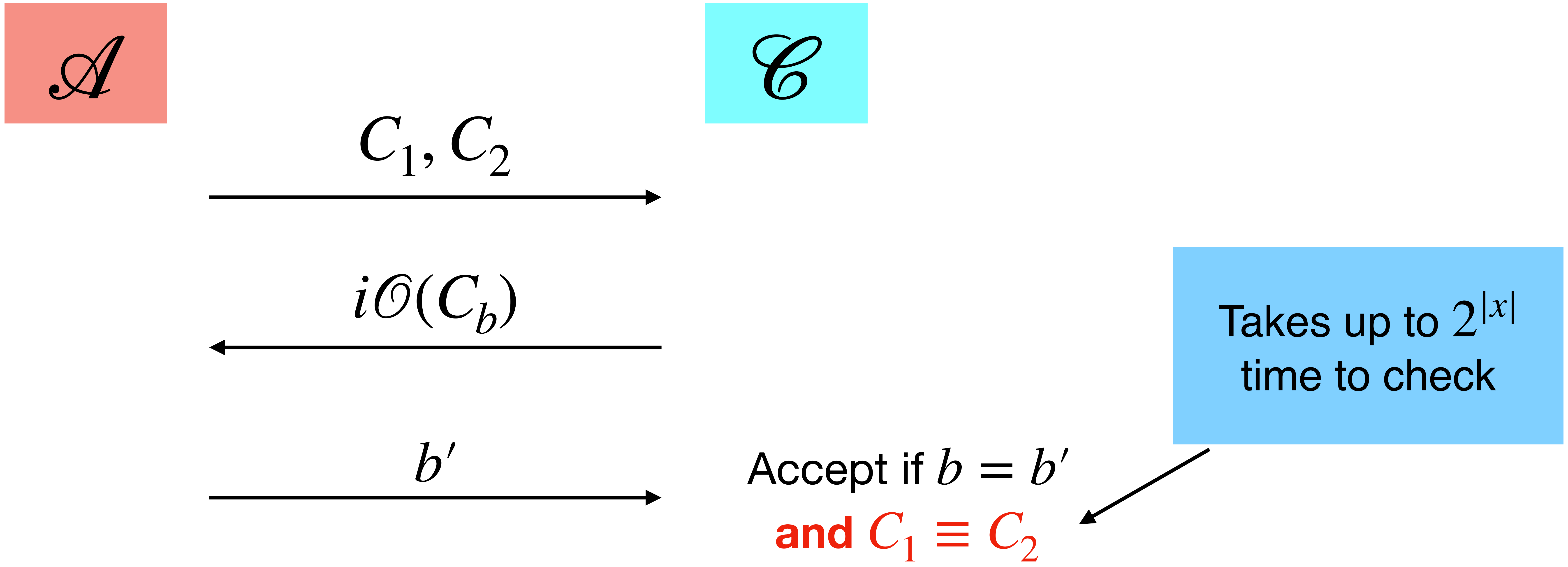
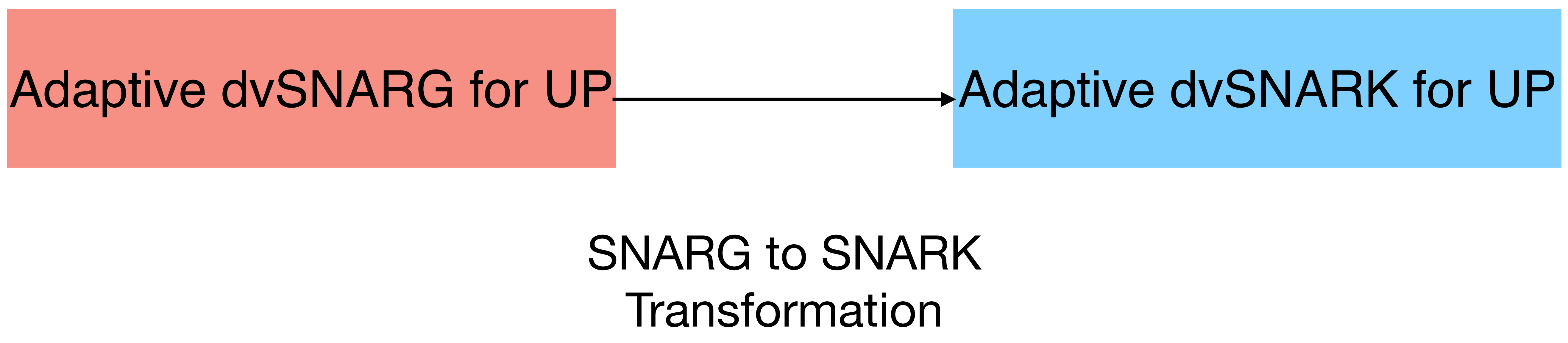


Adaptive dvSNARK for UP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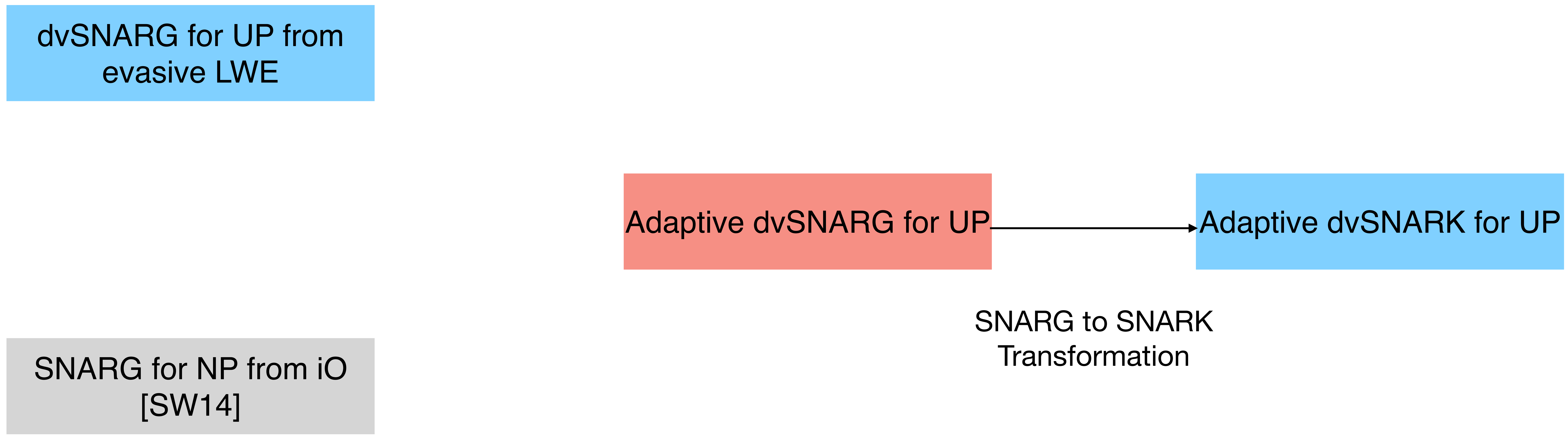

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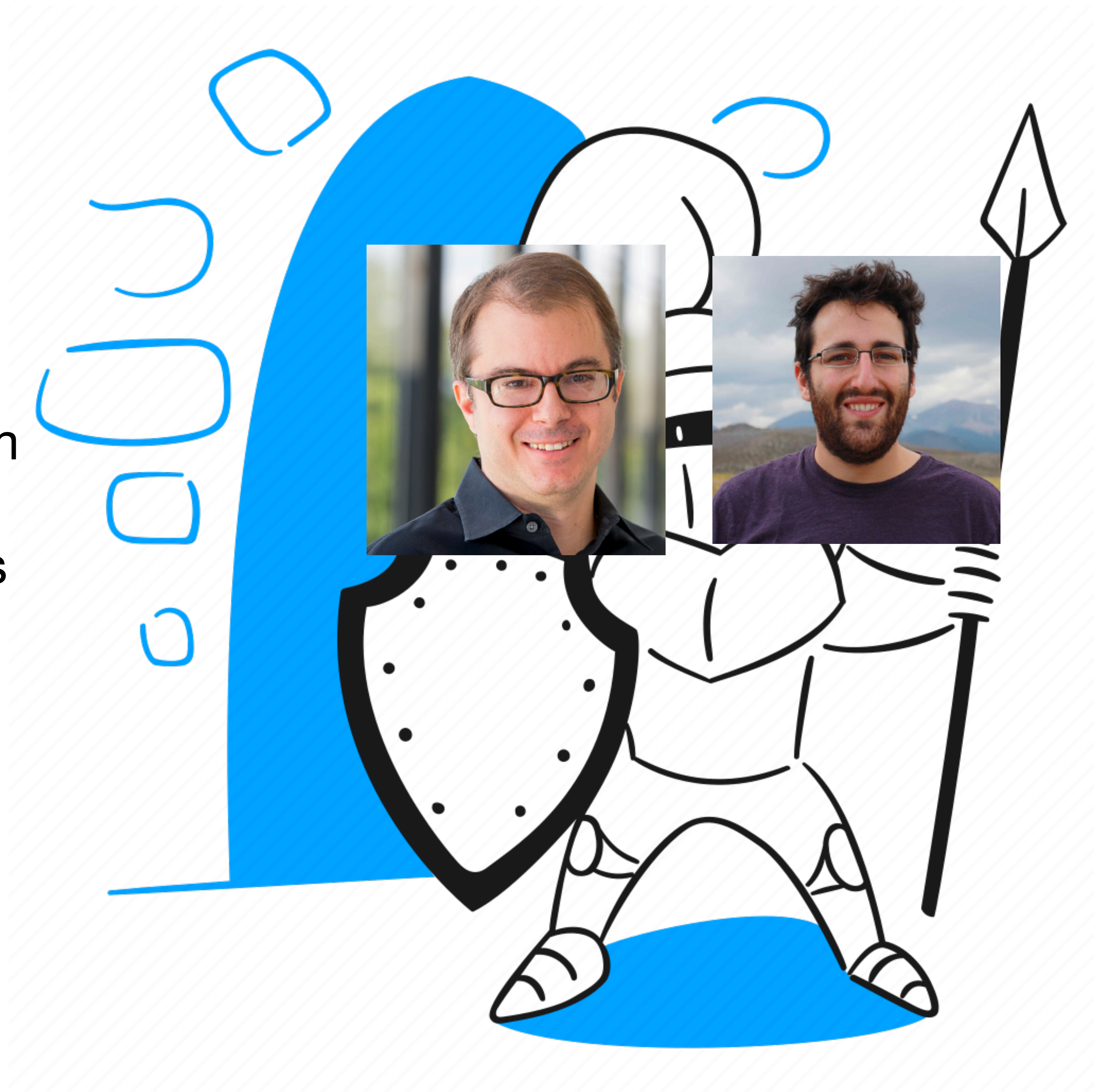

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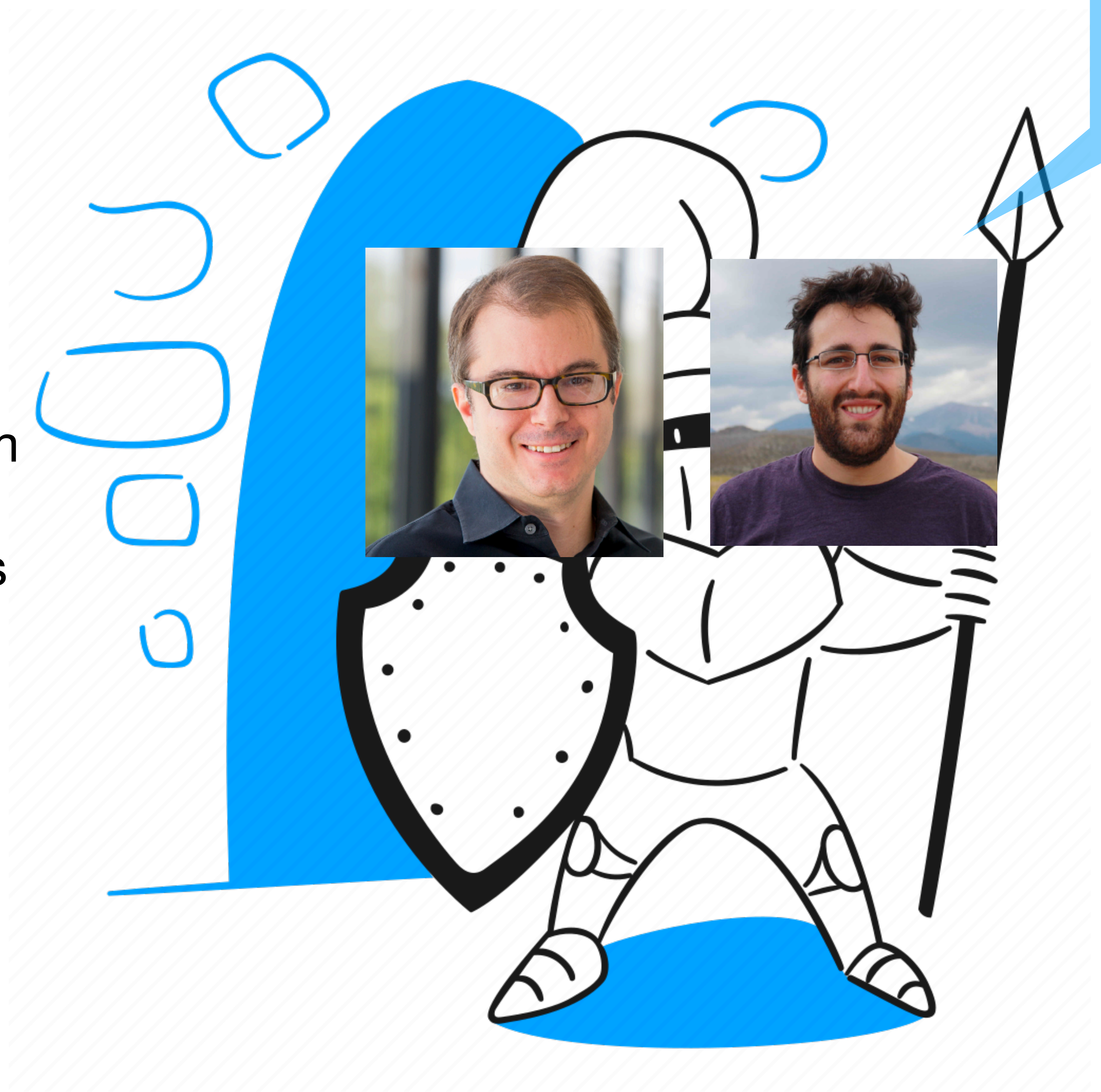

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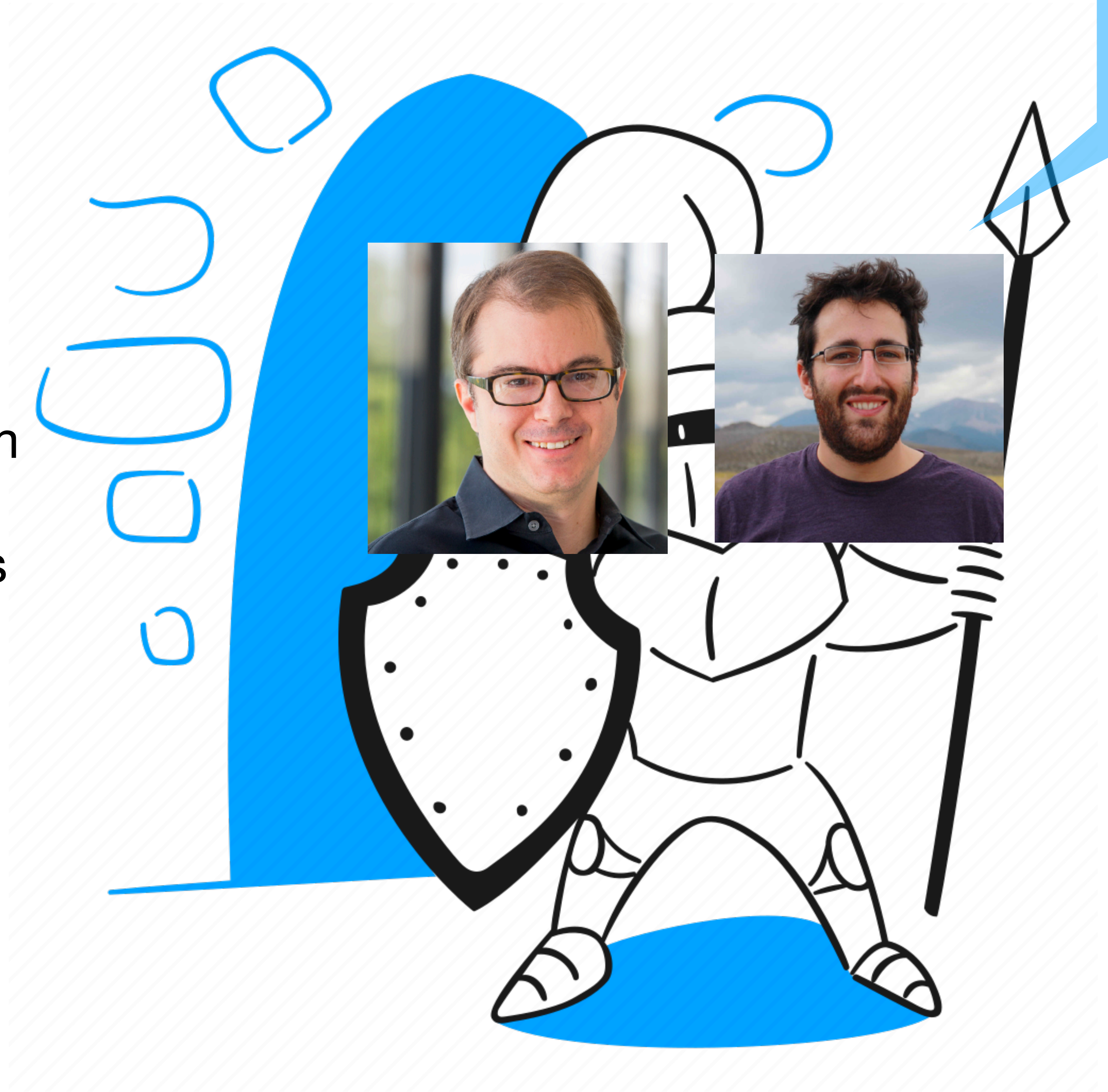

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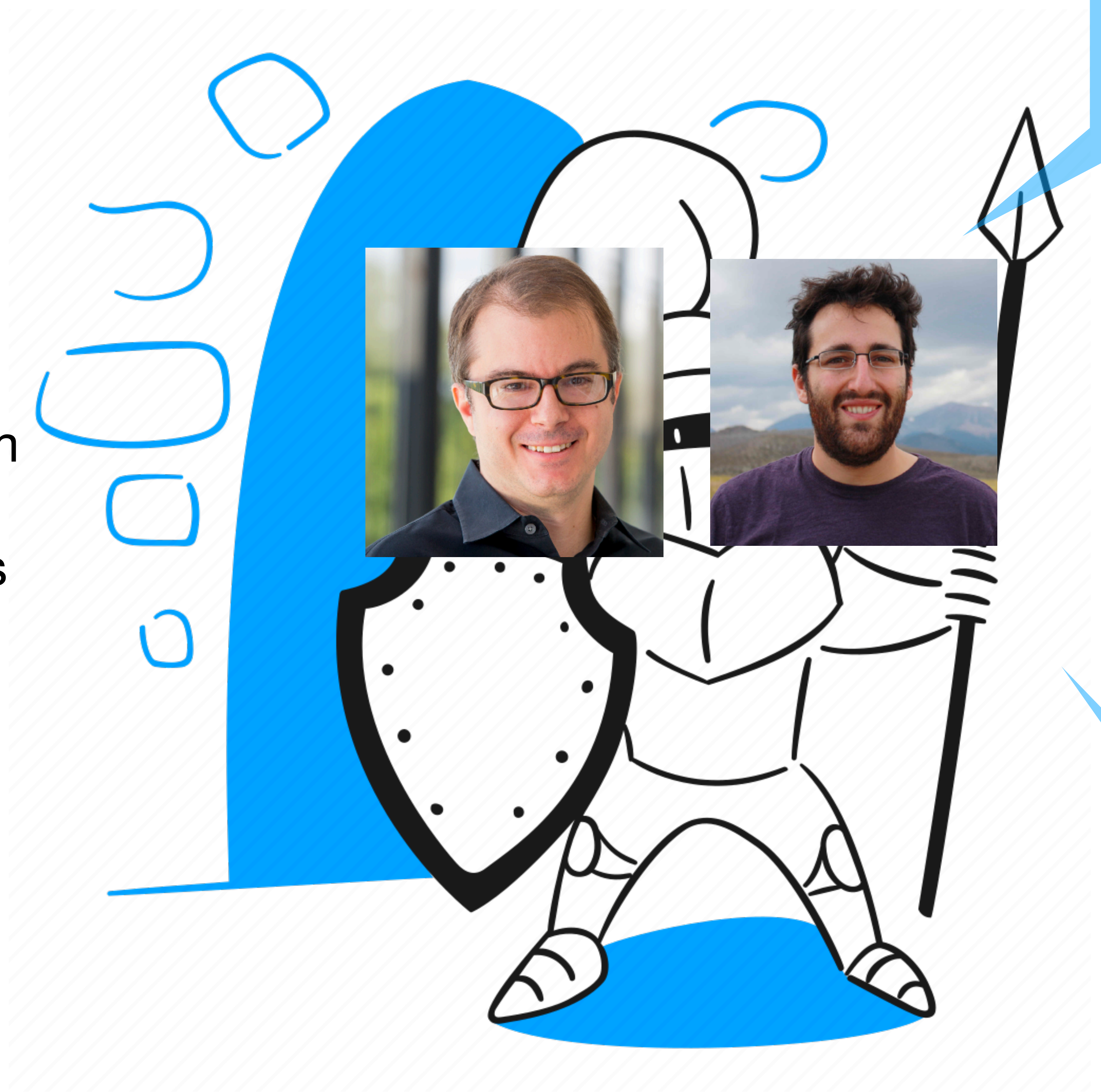

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You may proceed…
with caution

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- No known constructions of adaptively sound SNARGs from **falsifiable assumptions** (prior to Feb 2024*).

*Feb 2024: [WW24], [**M**PV24], [WZ24]

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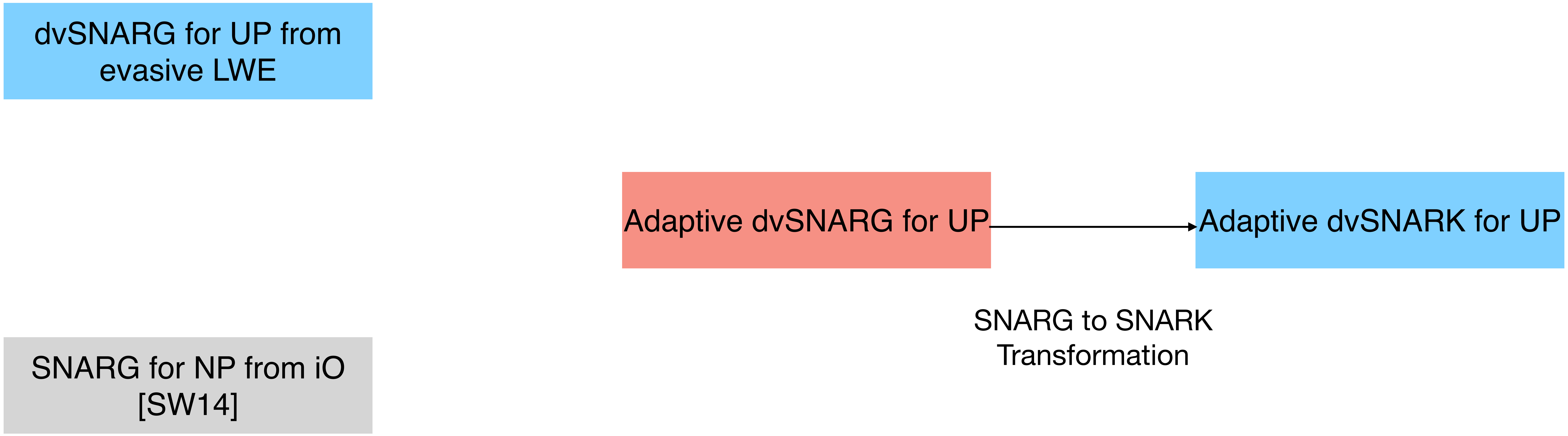
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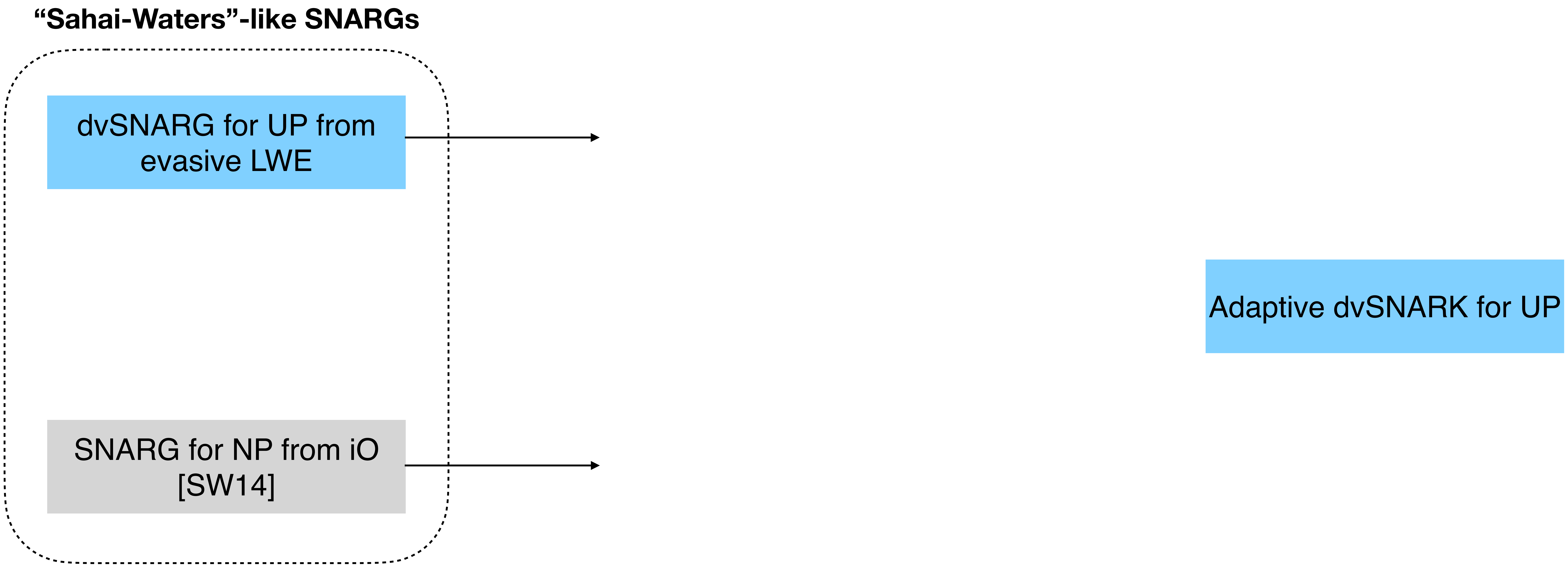
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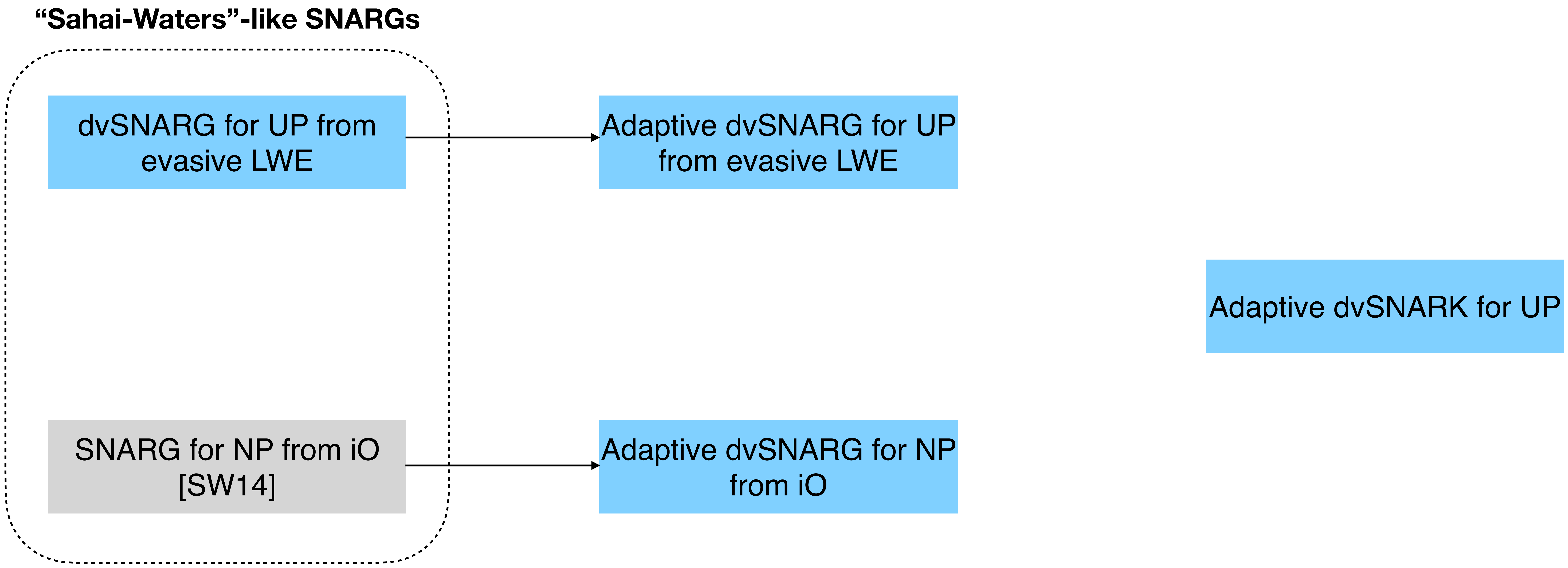

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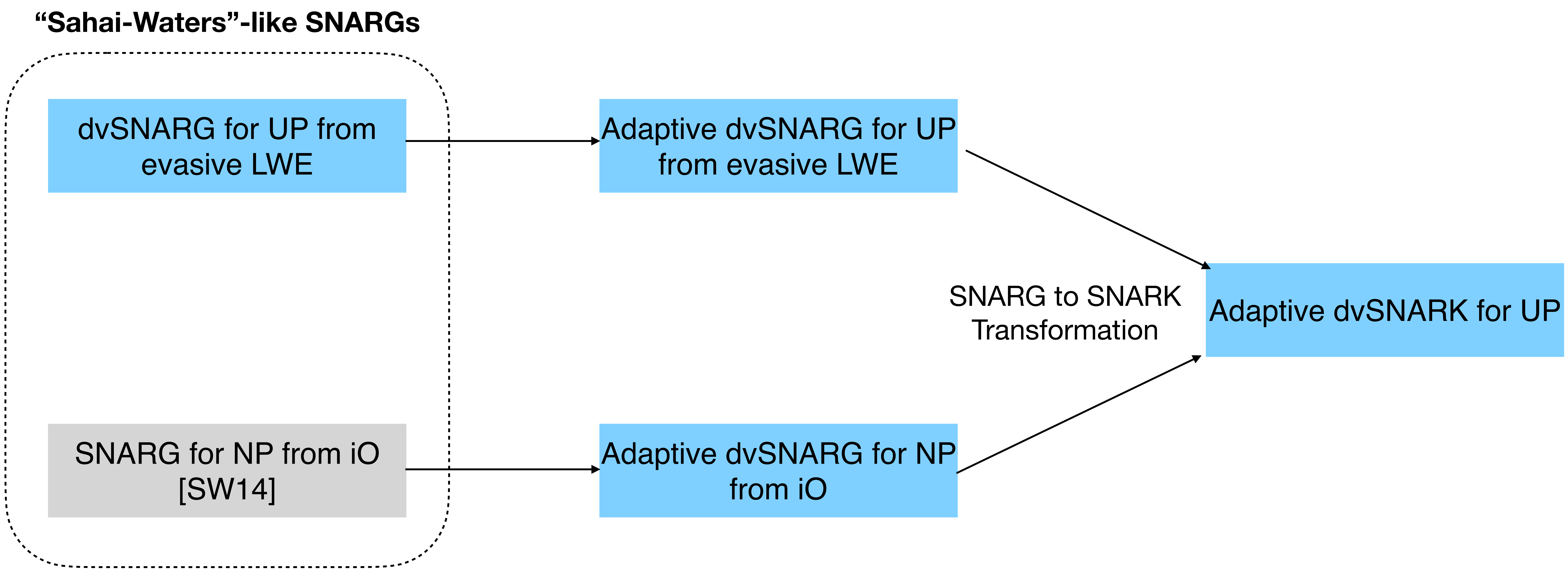

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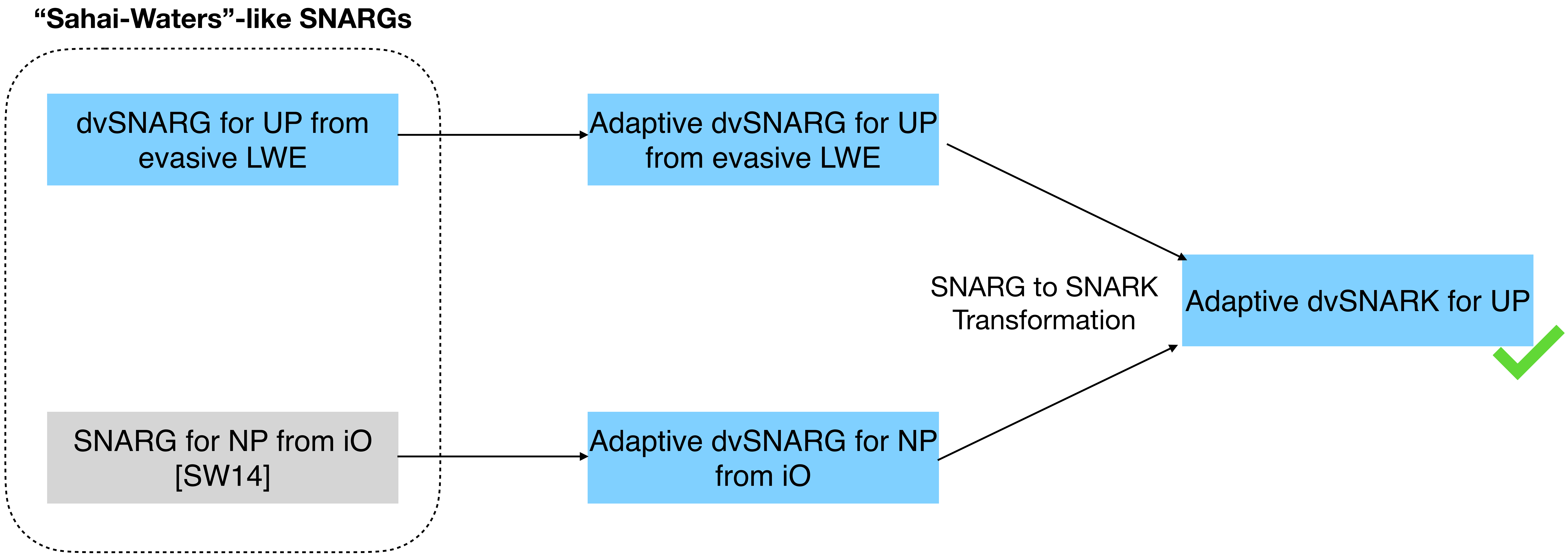
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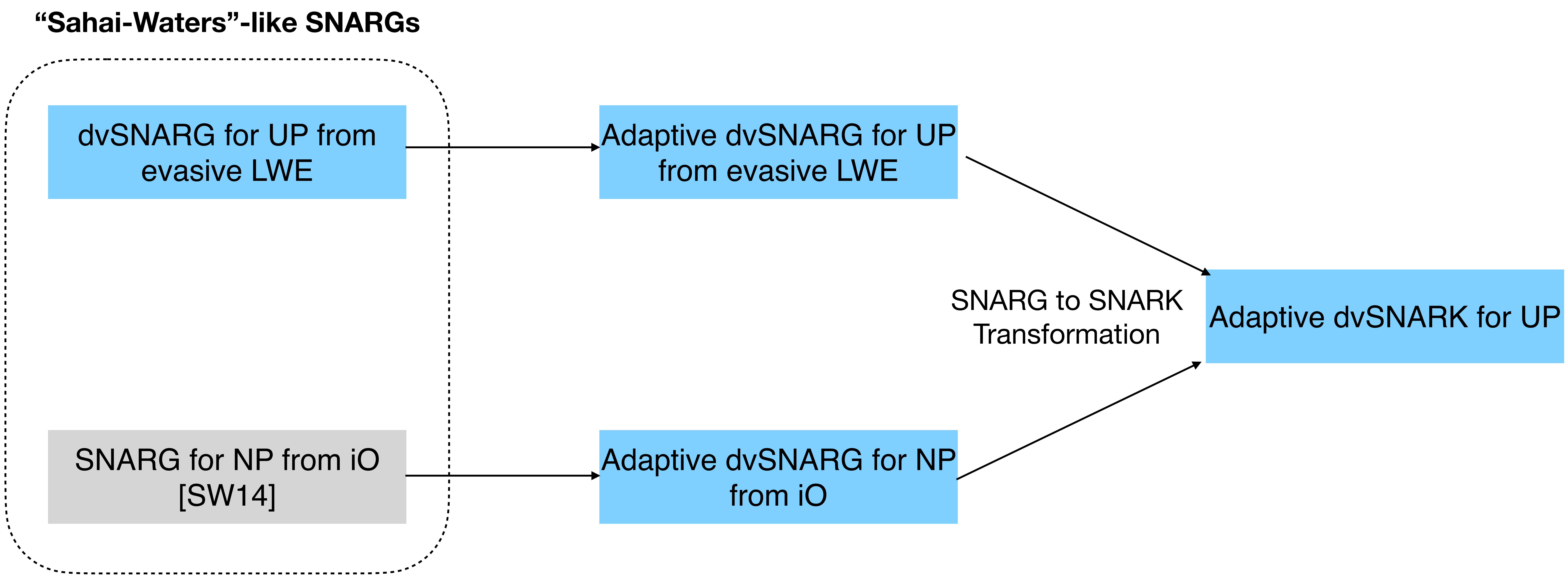
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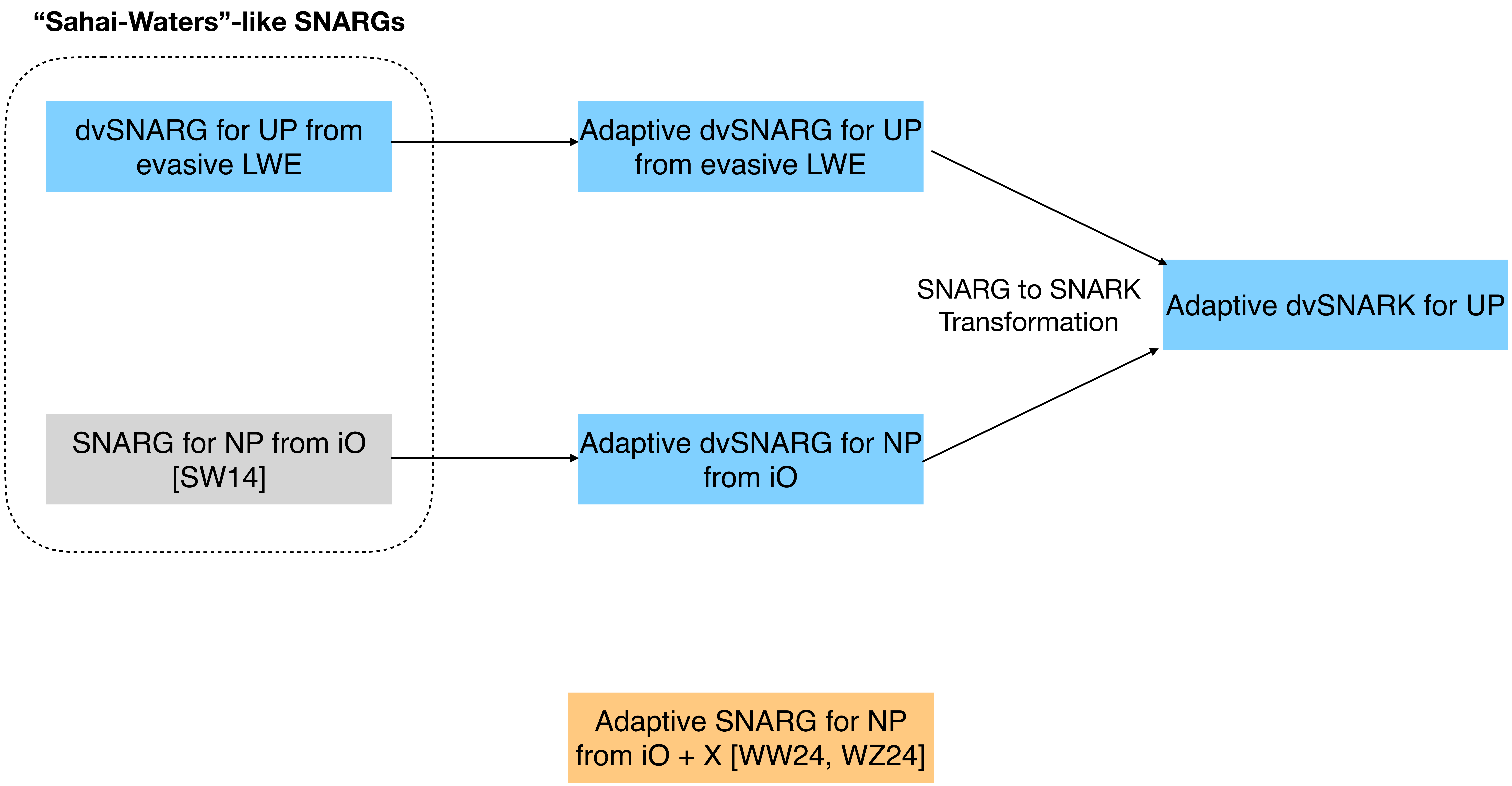
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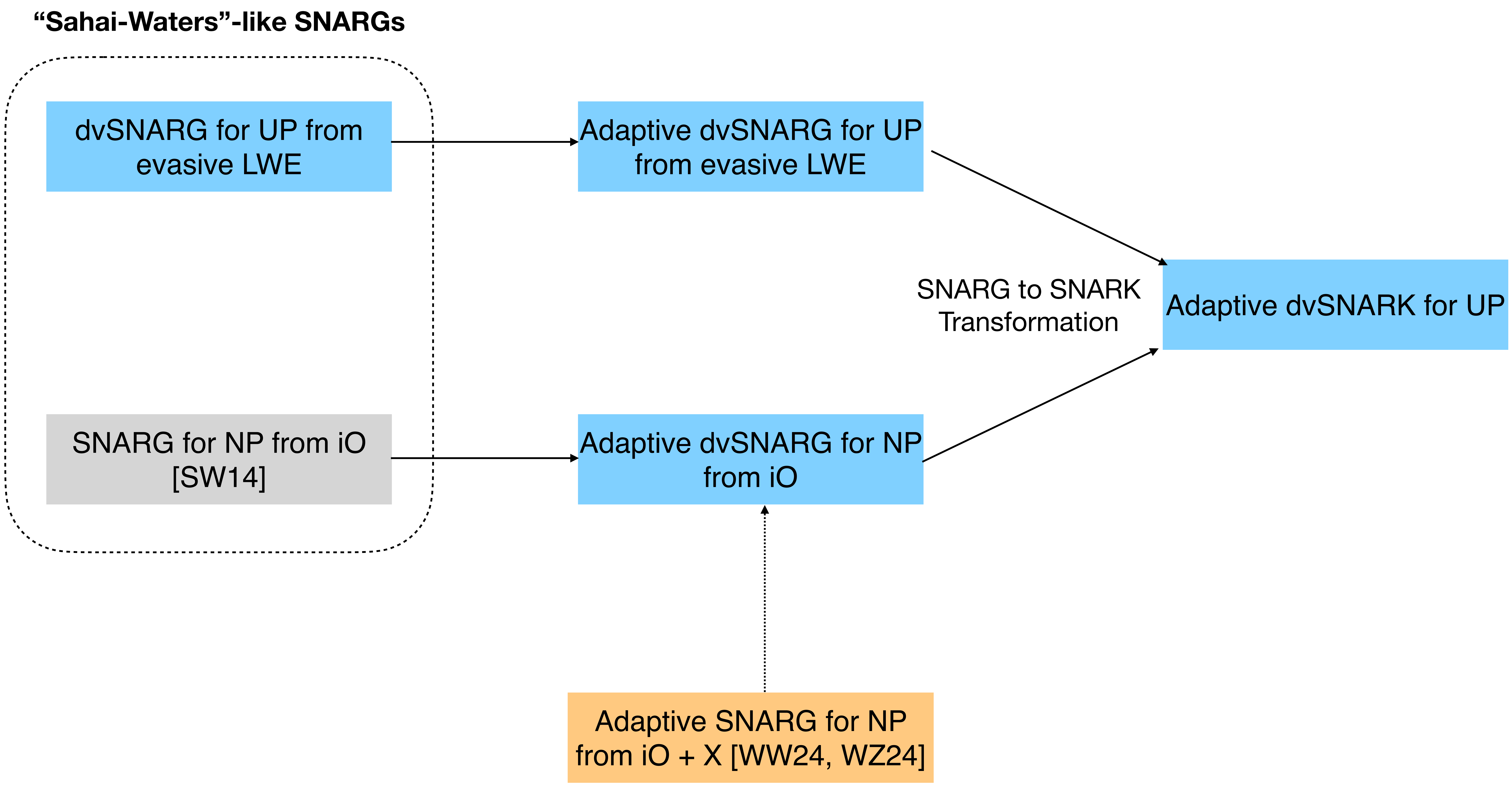
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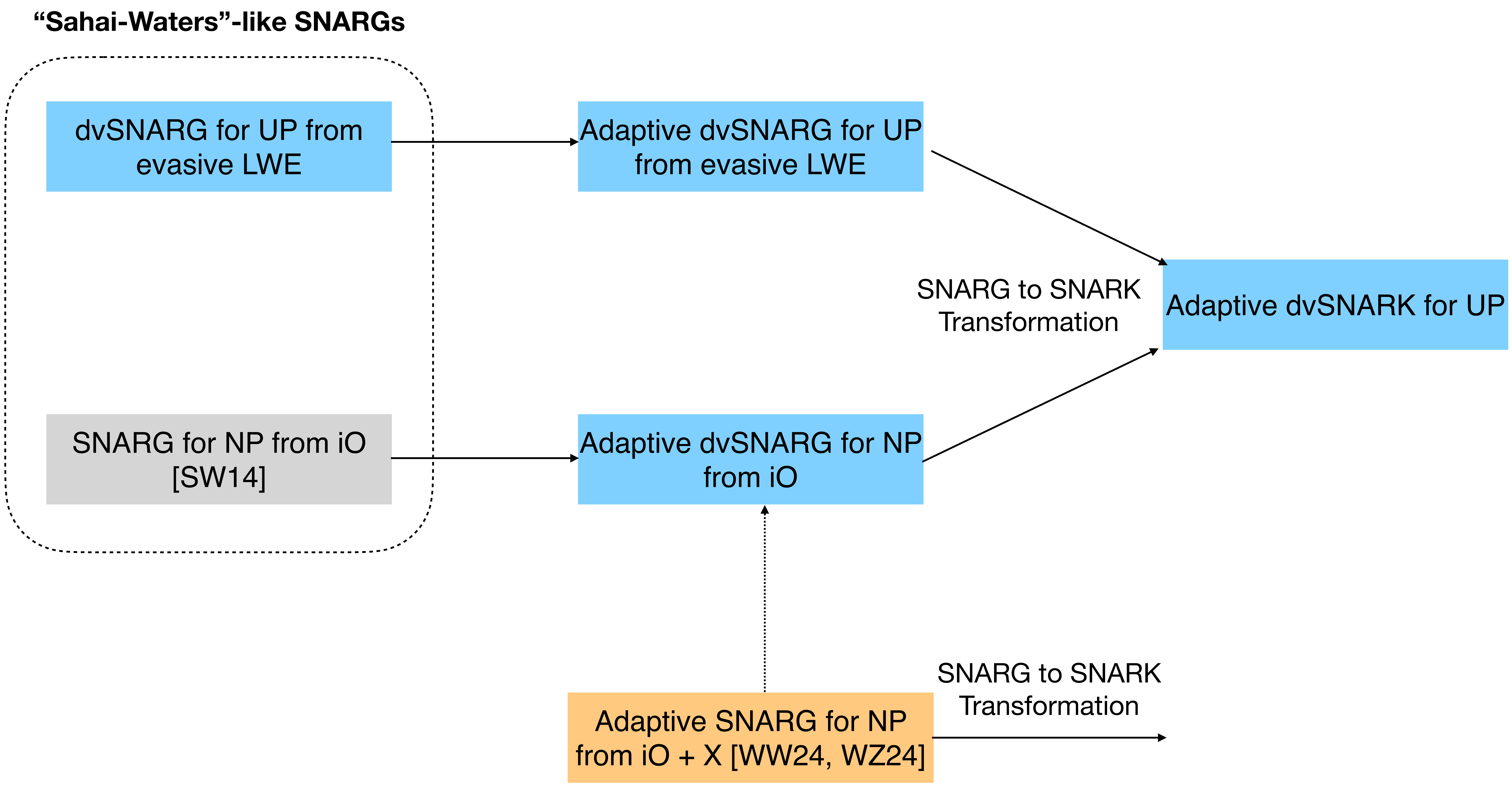

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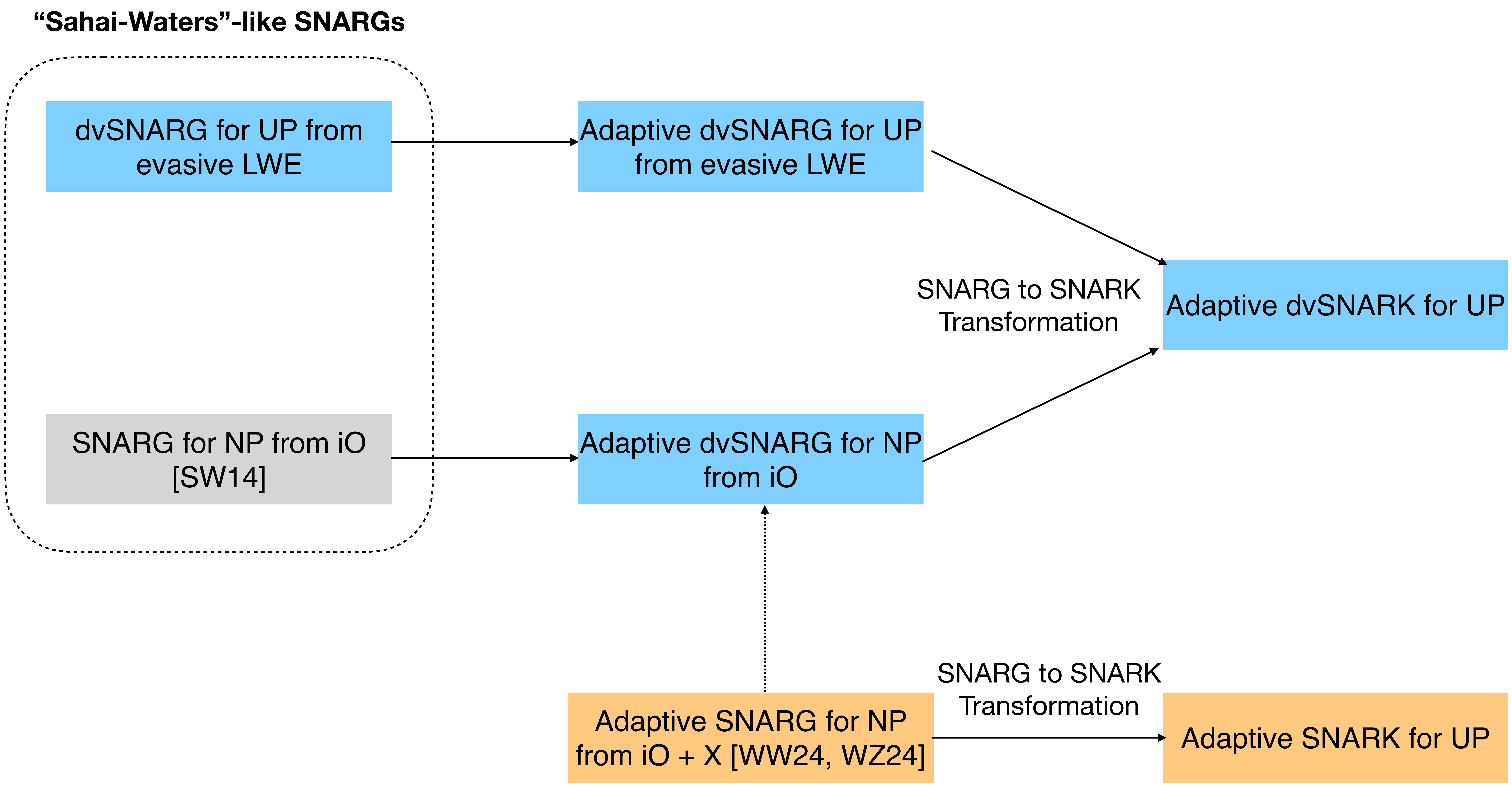
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TL;DR

In this work, we

1. Build a designated-verifier **SNARG for UP** from LWE and evasive LWE
2. Show our dvSNARG, and any “Sahai-Waters”-like dvSNARG can be made **adaptively sound**.
   - **Adaptively sound** SNARGs from **falsifiable assumptions** ([JLS20] iO + OWF)!
3. Transformation from **SNAR<u>G</u> for UP** to **SNAR<u>K</u>** for **UP**.
   - Corollary: **Adaptively sound** dv-SNAR<u>K</u> for UP from **falsifiable assumptions.**

All of the above constructions/transformations also **satisfy/preserve zero-knowledge!**

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Hybrid between witness encryption and constrained PRFs

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Our UP SNARG: Adaptive witness PRF for UP from evasive LWE.

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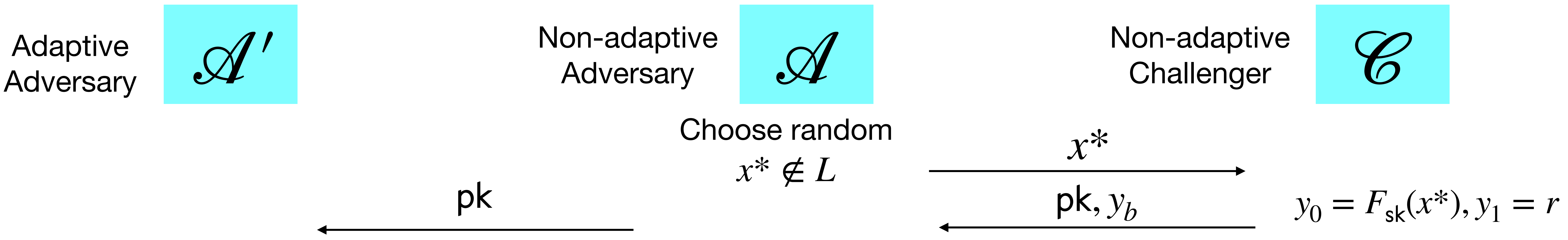

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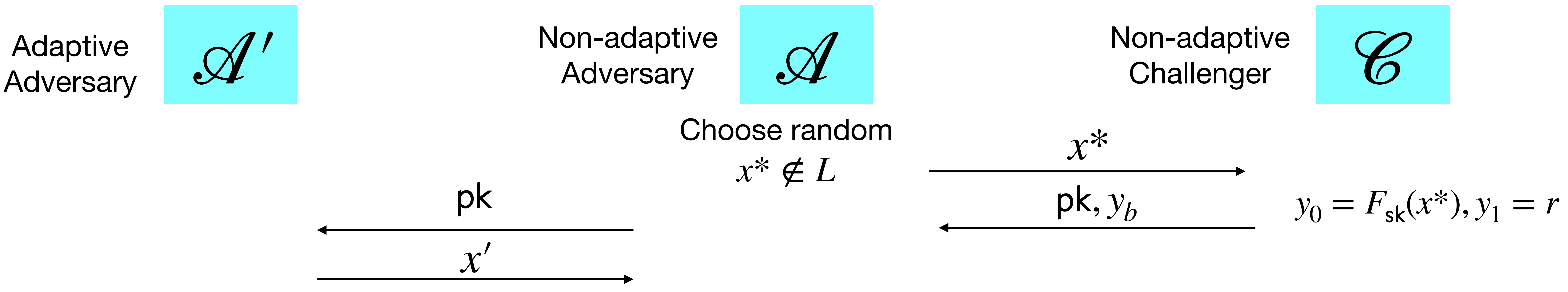

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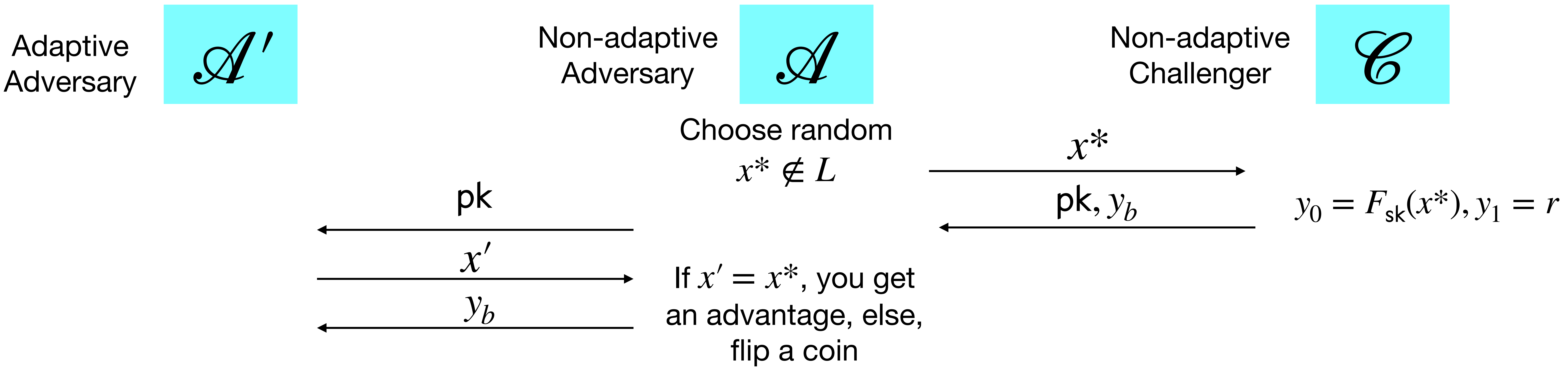

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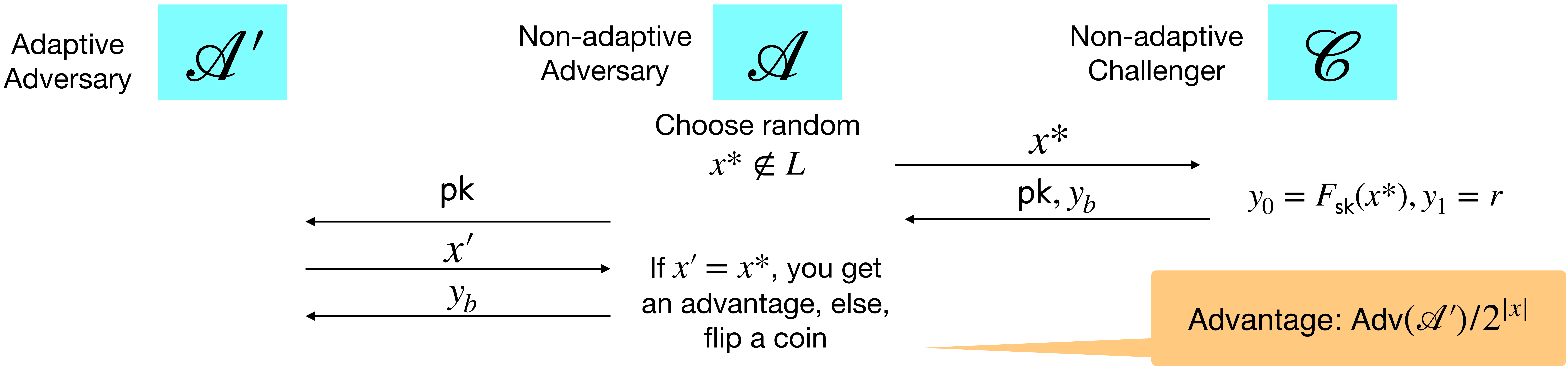

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- We can choose proof size $\sim \lambda$ for $2^{-\lambda}$ soundness!

TL;DR

In this work, we

1. Build a designated-verifier **SNARG for UP** from LWE and evasive LWE

2. Show our dvSNARG, and any “Sahai-Waters”-like dvSNARG can be made **adaptively sound**.

   - **Adaptively sound** SNARGs from **falsifiable assumptions** ([JLS20] iO + OWF)!

3. Transformation from **SNAR<u>G</u> for UP** to **SNAR<u>K</u>** for **UP**.

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All of the above constructions/transformations also **satisfy/preserve zero-knowledge!**

TL;DR

Build a witness PRF for UP from evasive LWE

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  - NP from sub-exponential iO + OWF
- We show that adaptively sound SNARGs for UP can be transformed into SNAR**K**s for UP assuming polynomially secure LWE.
  - We can build SNARKs from **falsifiable assumptions**!

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- What else can we **prove from evasive LWE** that we can build from obfuscation?
- Can we transform our SNARG into a **publicly verifiable SNARG**?
  - Have to be very careful about **zeroizing attacks!**
- Can we get a SNARG with a **smaller CRS**? Can we get a common *random/ transparent* string?

Thank you very much for your attention!

Bonus Slides

σ-PRF Obfuscation

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(i.e. the obfuscation leaks nothing more than the outputs)

Simplified Obfuscation Construction

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- **Step 1:** Consider a read-once branching program PRF $F_k : \{0,1\}^h \to \mathcal{Y}$ given by$\mathbf{u}$, $\{M_{i,b}\}_{i\in[h],b\in\{0,1\}}$, $\mathbf{v}$ satisfying:

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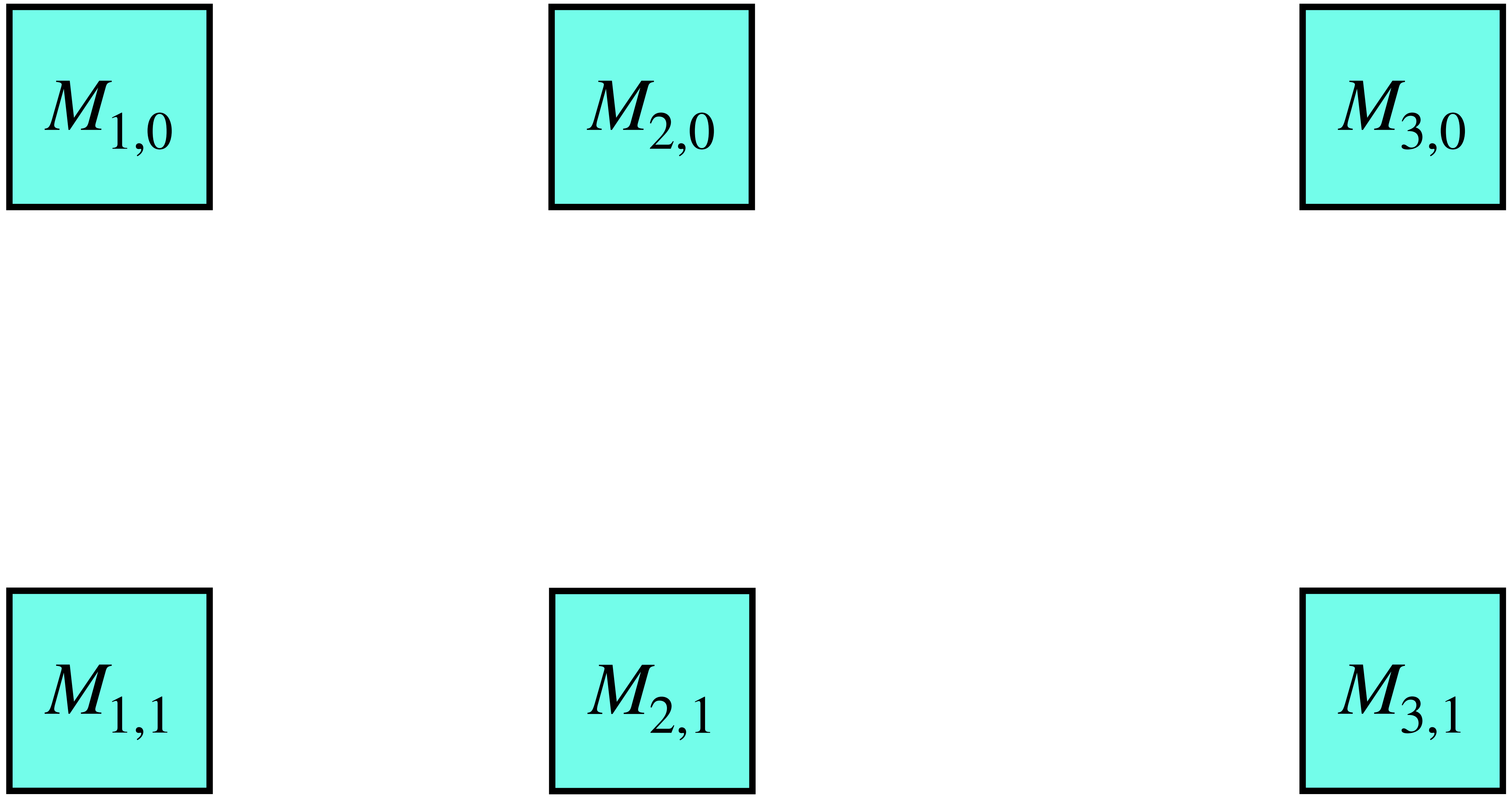
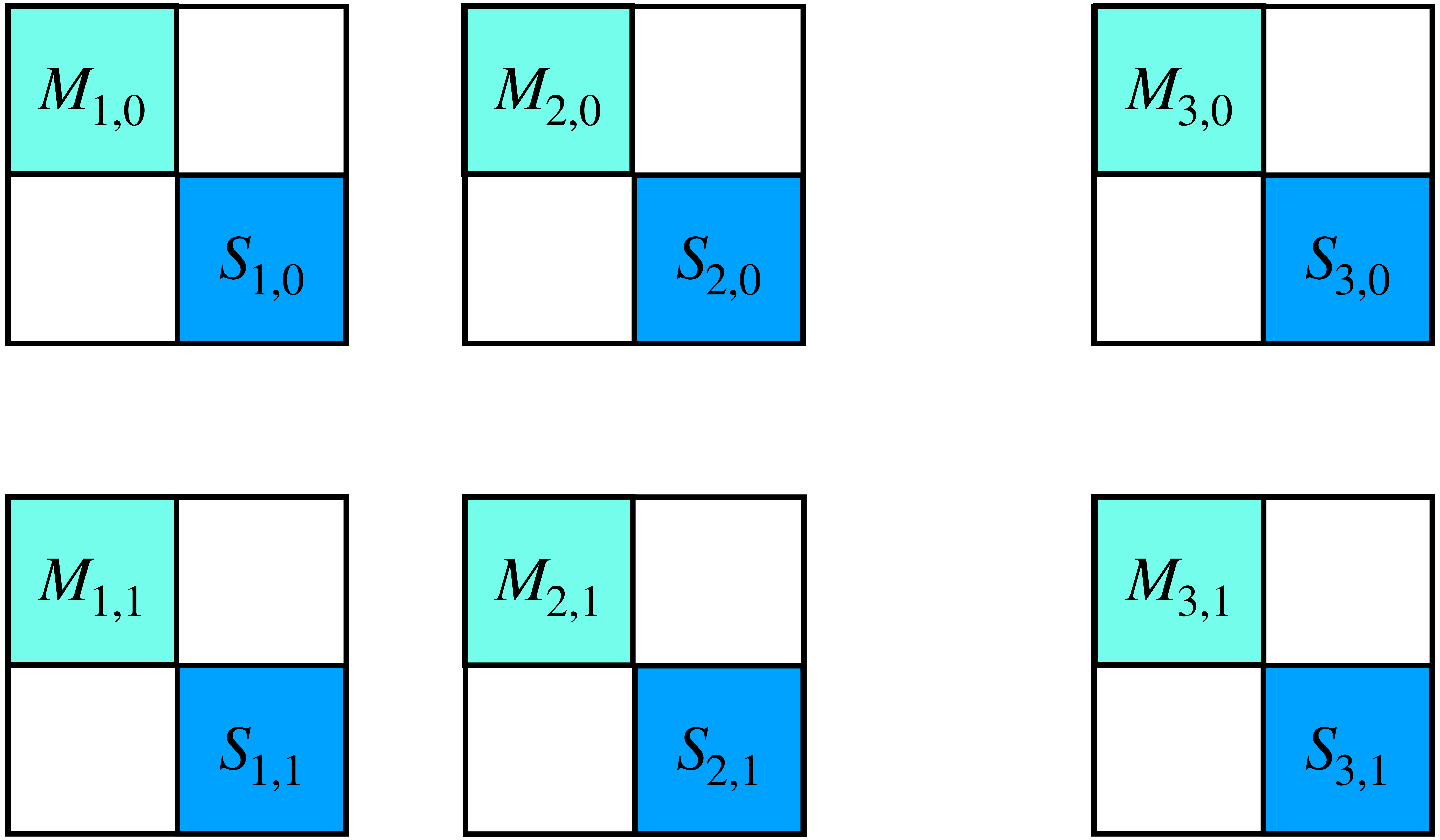
- **Step 2:** Perform GGH15 [Garg-Gentry-Halevi] encoding of the branching program.

GGH15 Encodings



GGH15 Encodings



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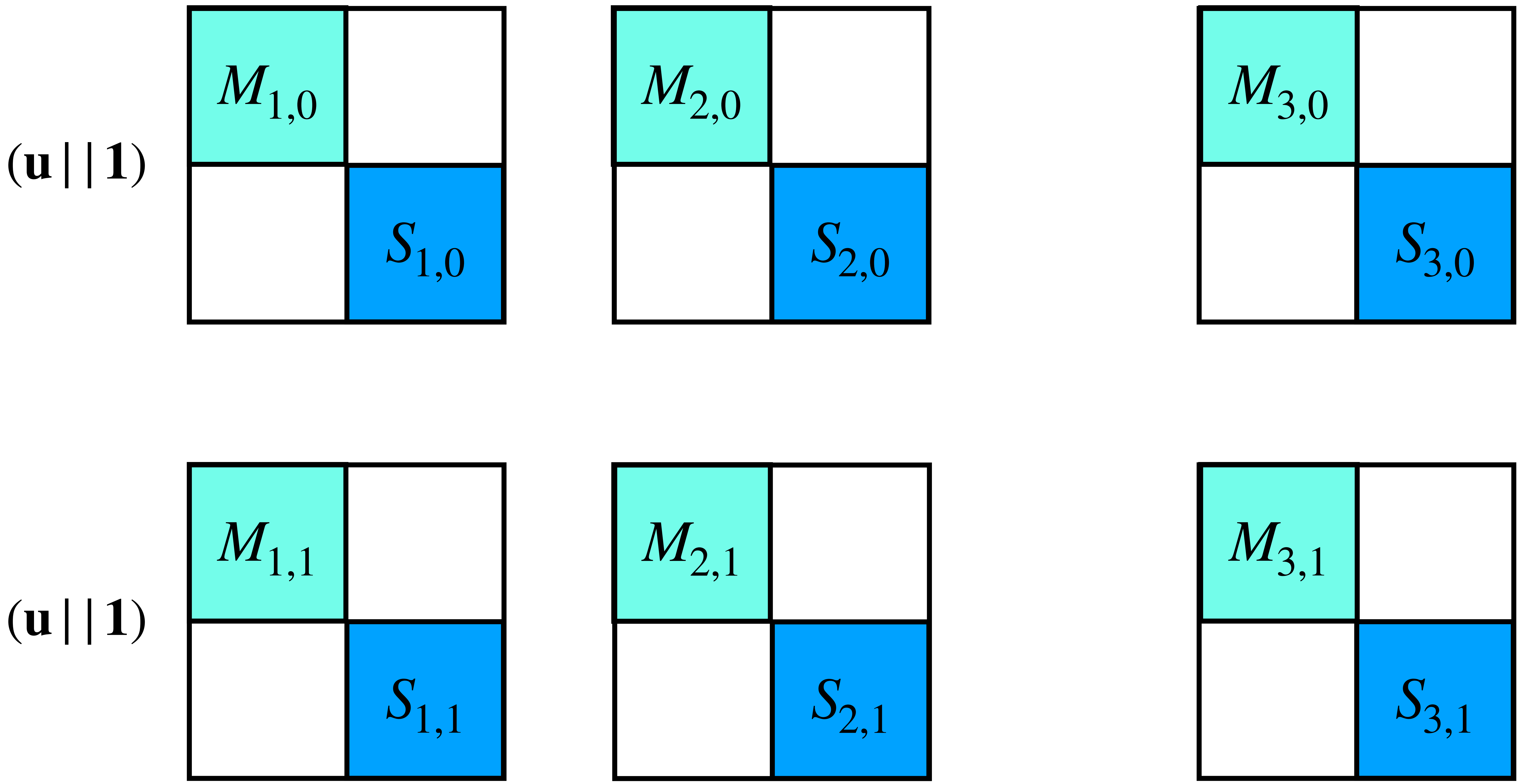

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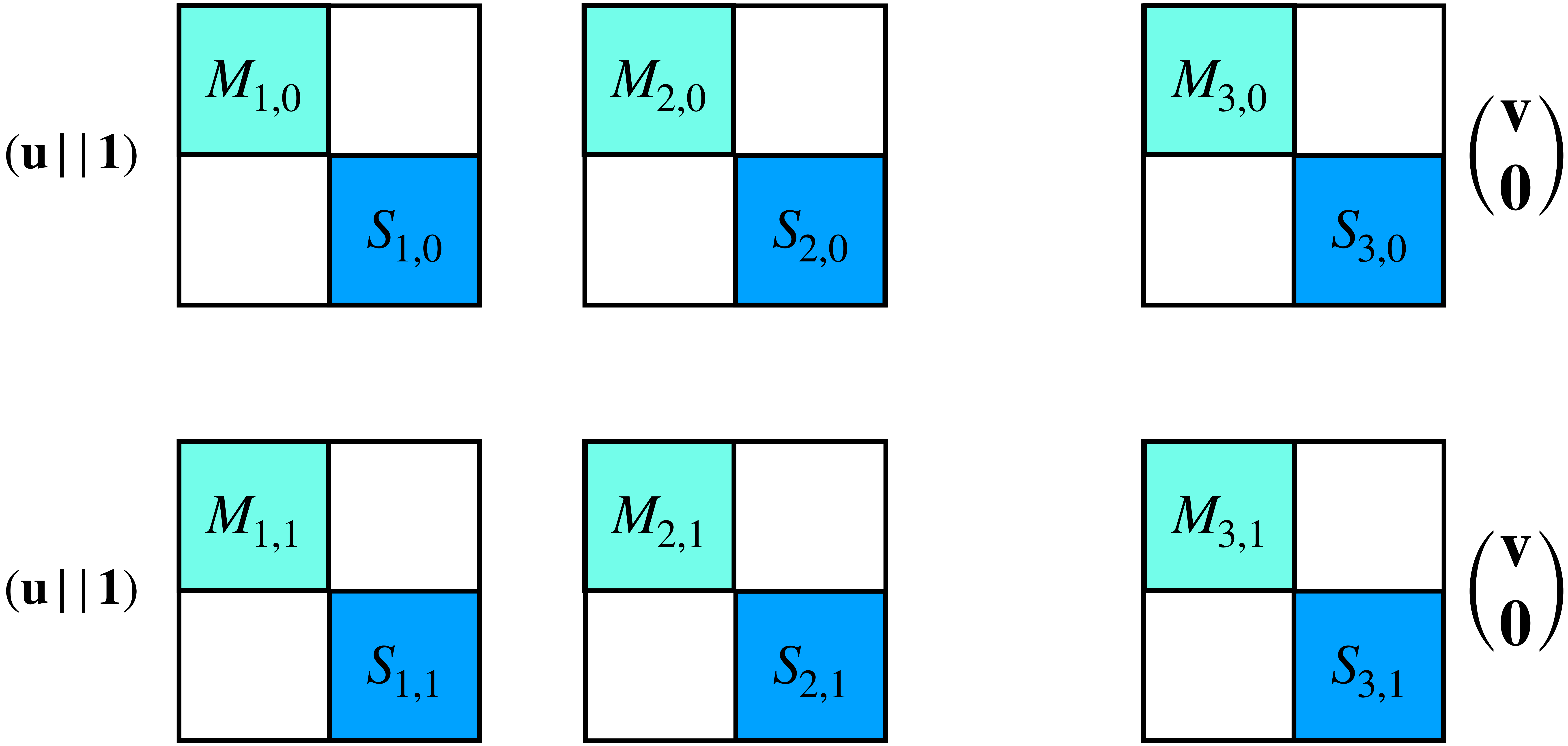

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$$(\mathbf{u}\,||\,\mathbf{1}) \begin{pmatrix} M_{1,0} & \\ & S_{1,0} \end{pmatrix} \begin{pmatrix} M_{2,0} & \\ & S_{2,0} \end{pmatrix} \begin{pmatrix} M_{3,0} & \\ & S_{3,0} \end{pmatrix} \begin{pmatrix} \mathbf{v} \\ \mathbf{0} \end{pmatrix}$$

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Taking subset product still gives:

$$\mathbf{u}\left(\prod_{i=1}^{3} M_{i,x_i}\right)\mathbf{v} = F_k(\mathbf{x})$$

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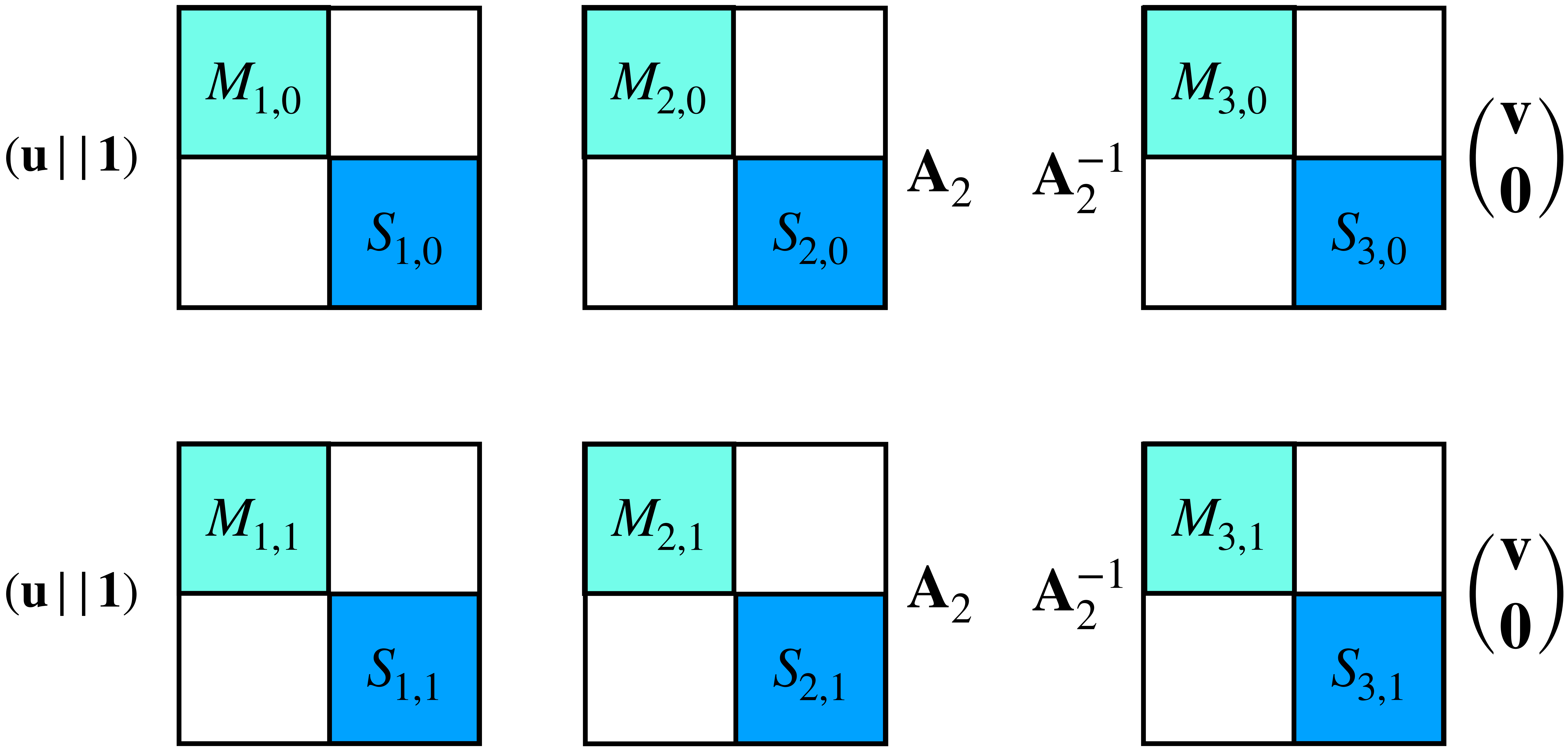

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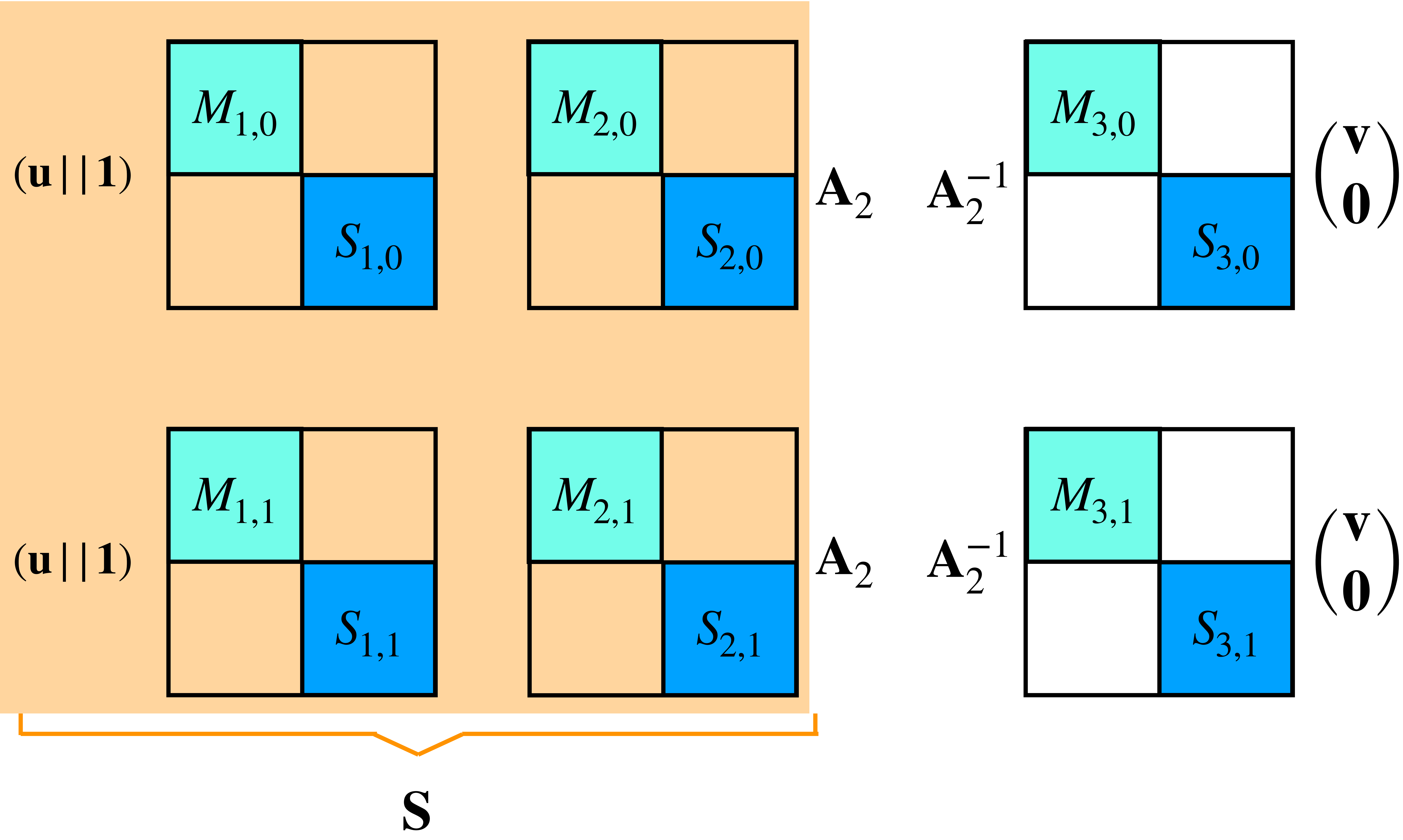

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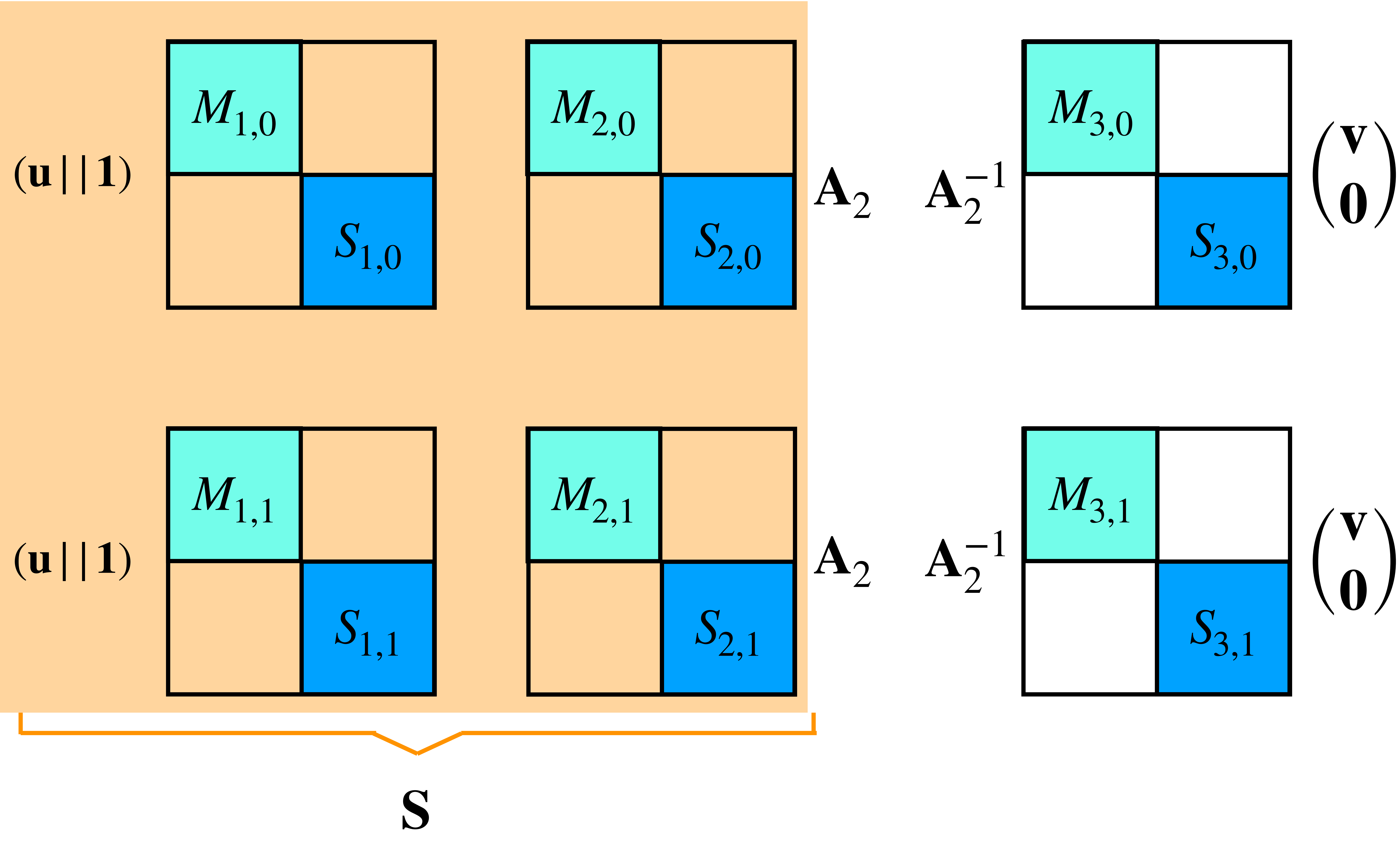

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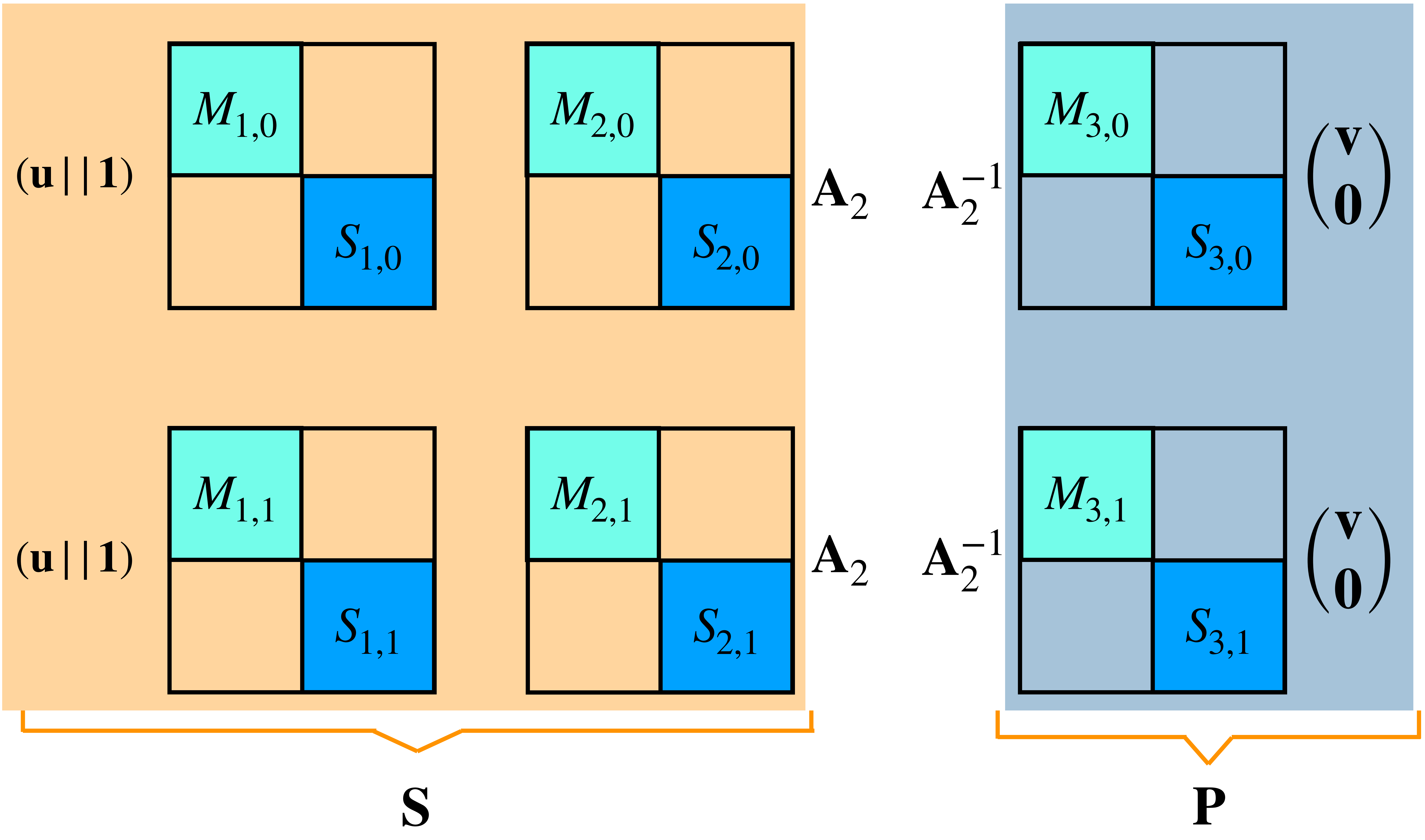

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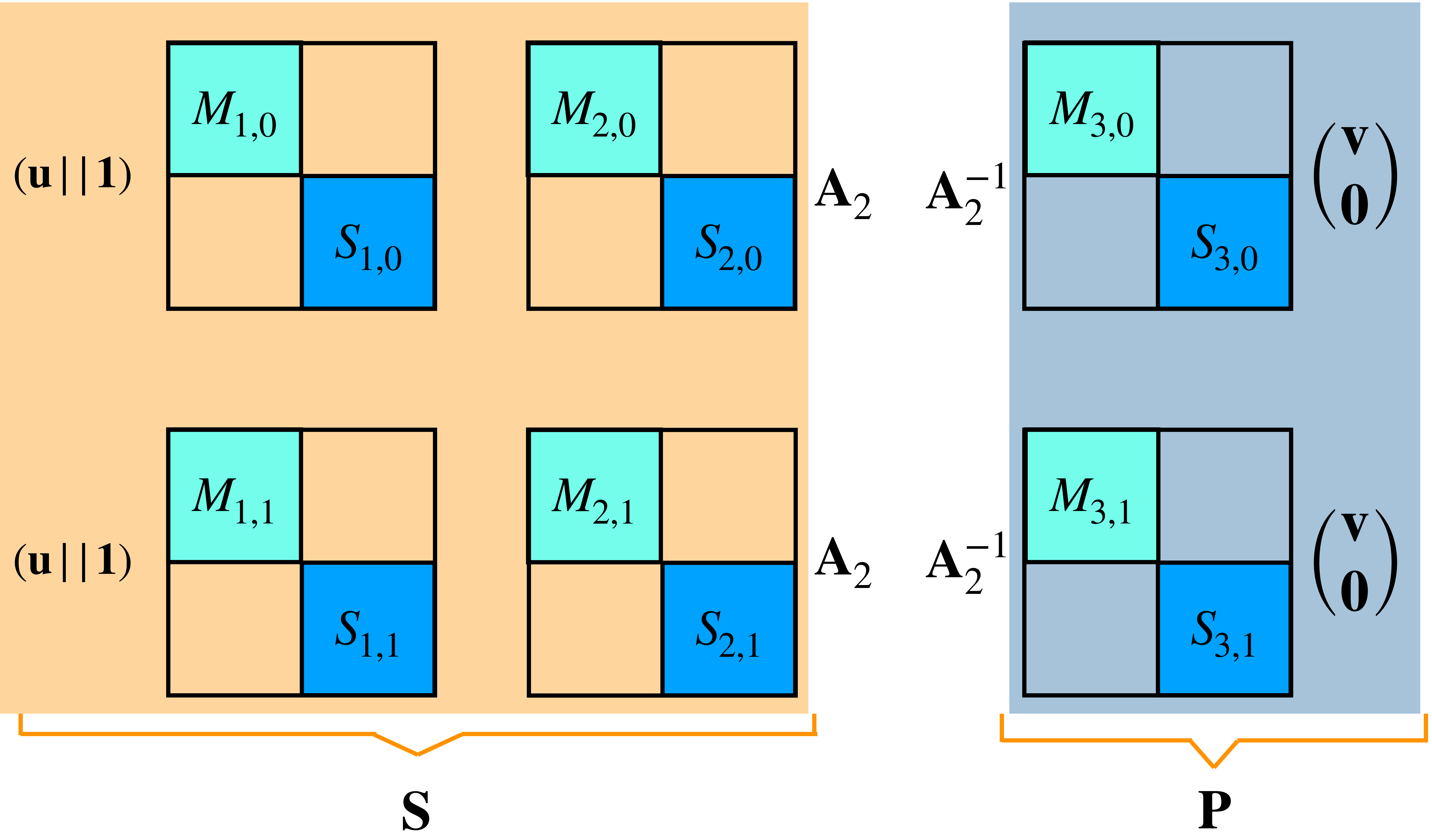

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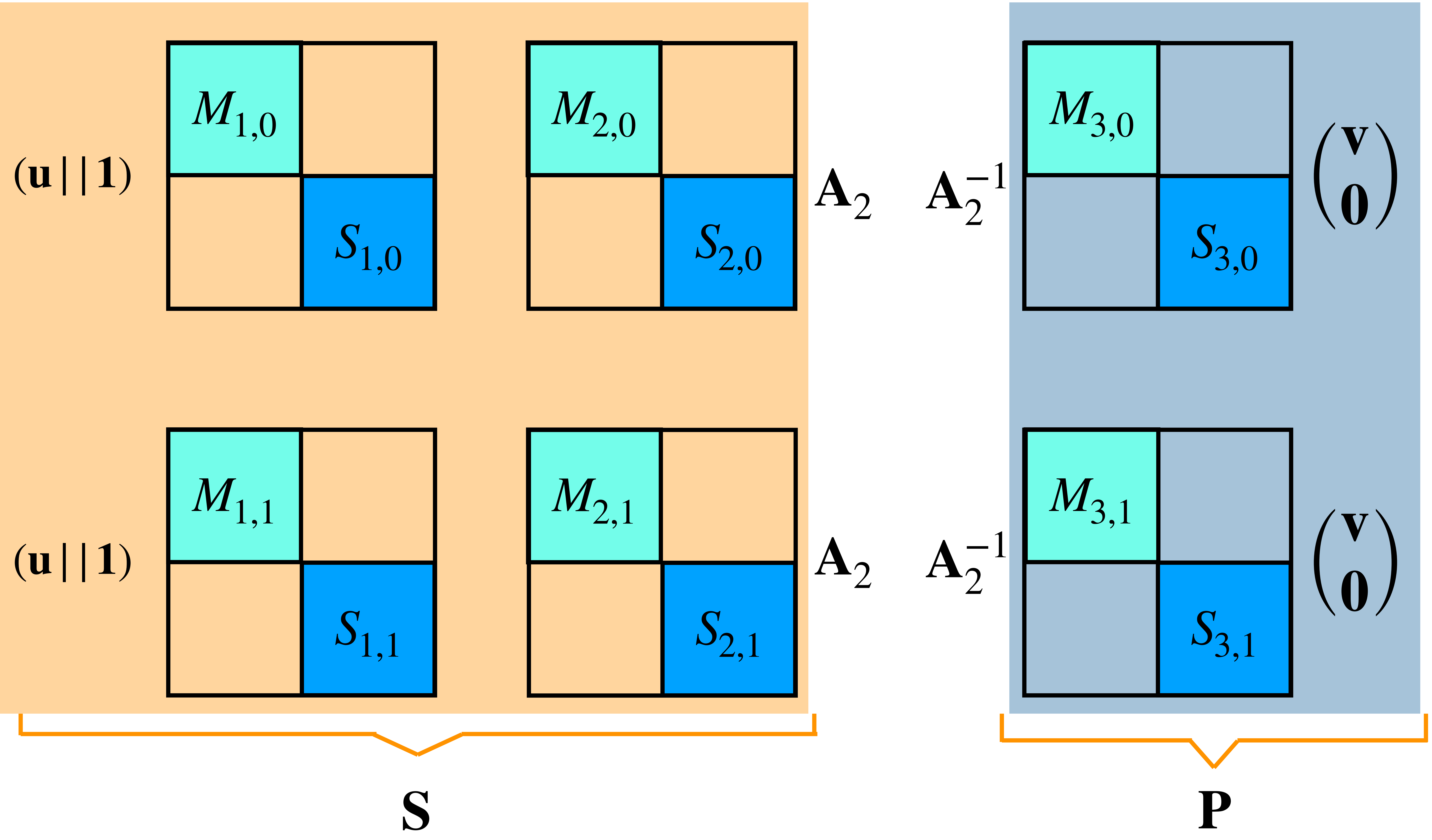

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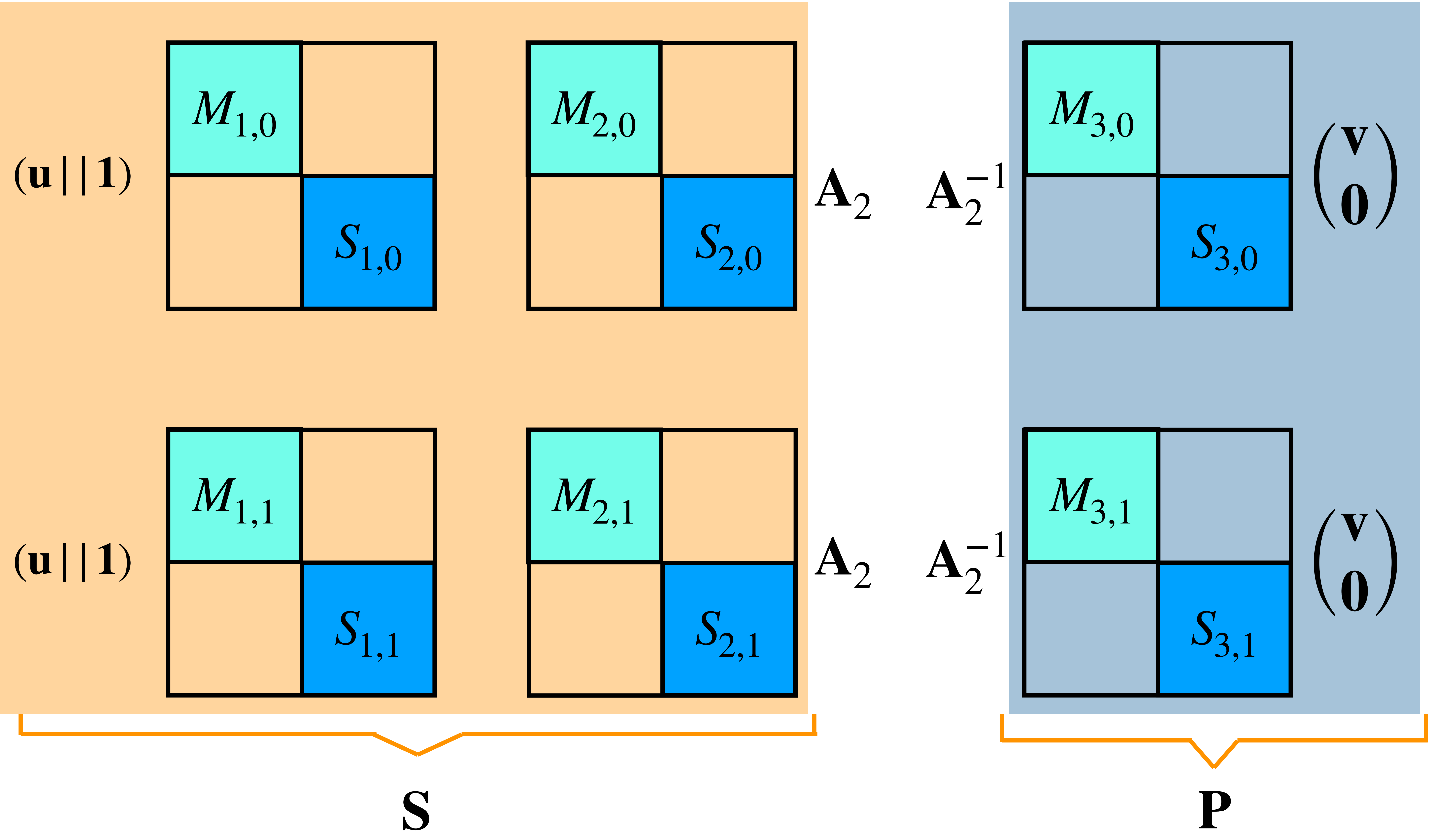

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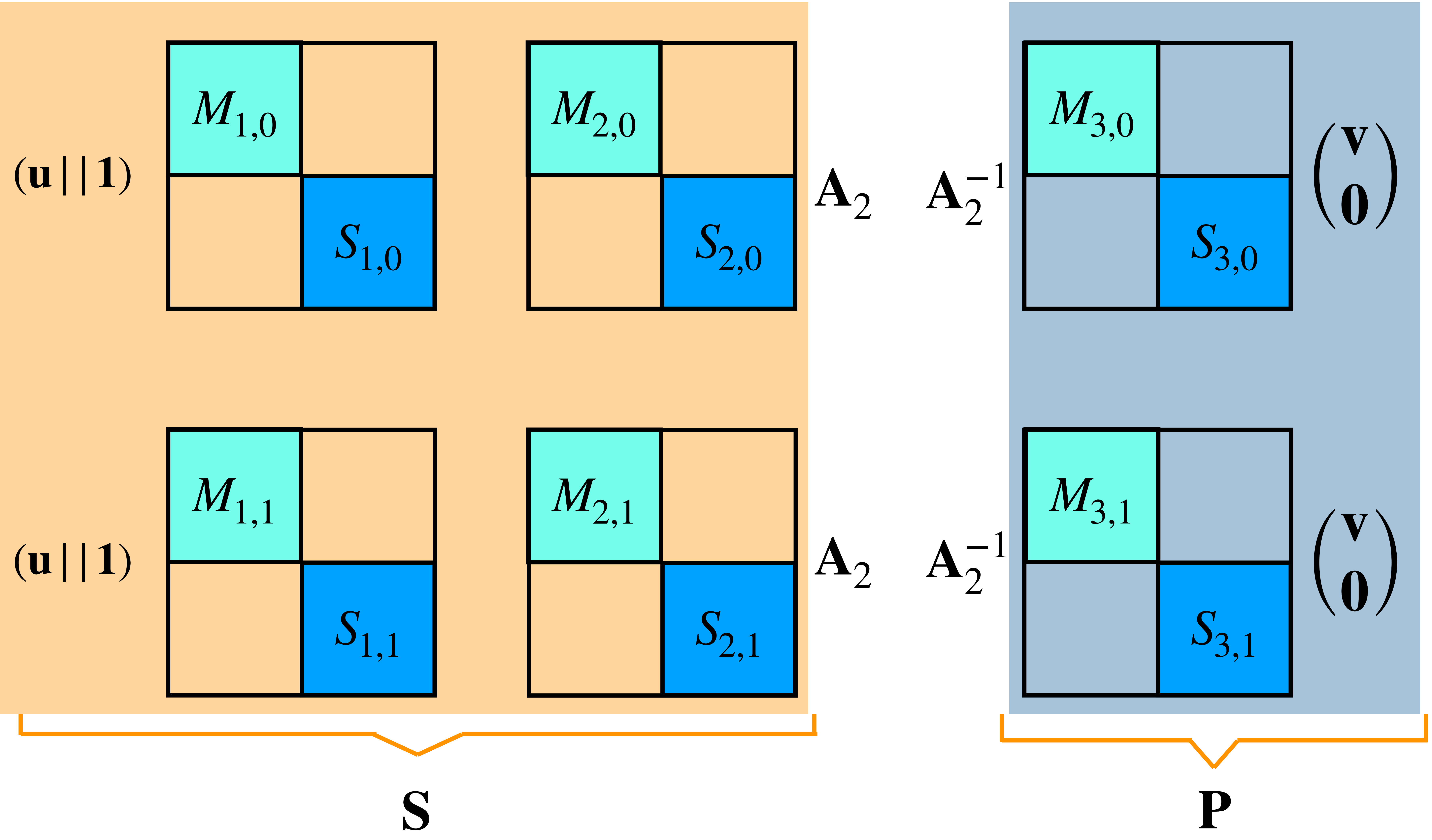

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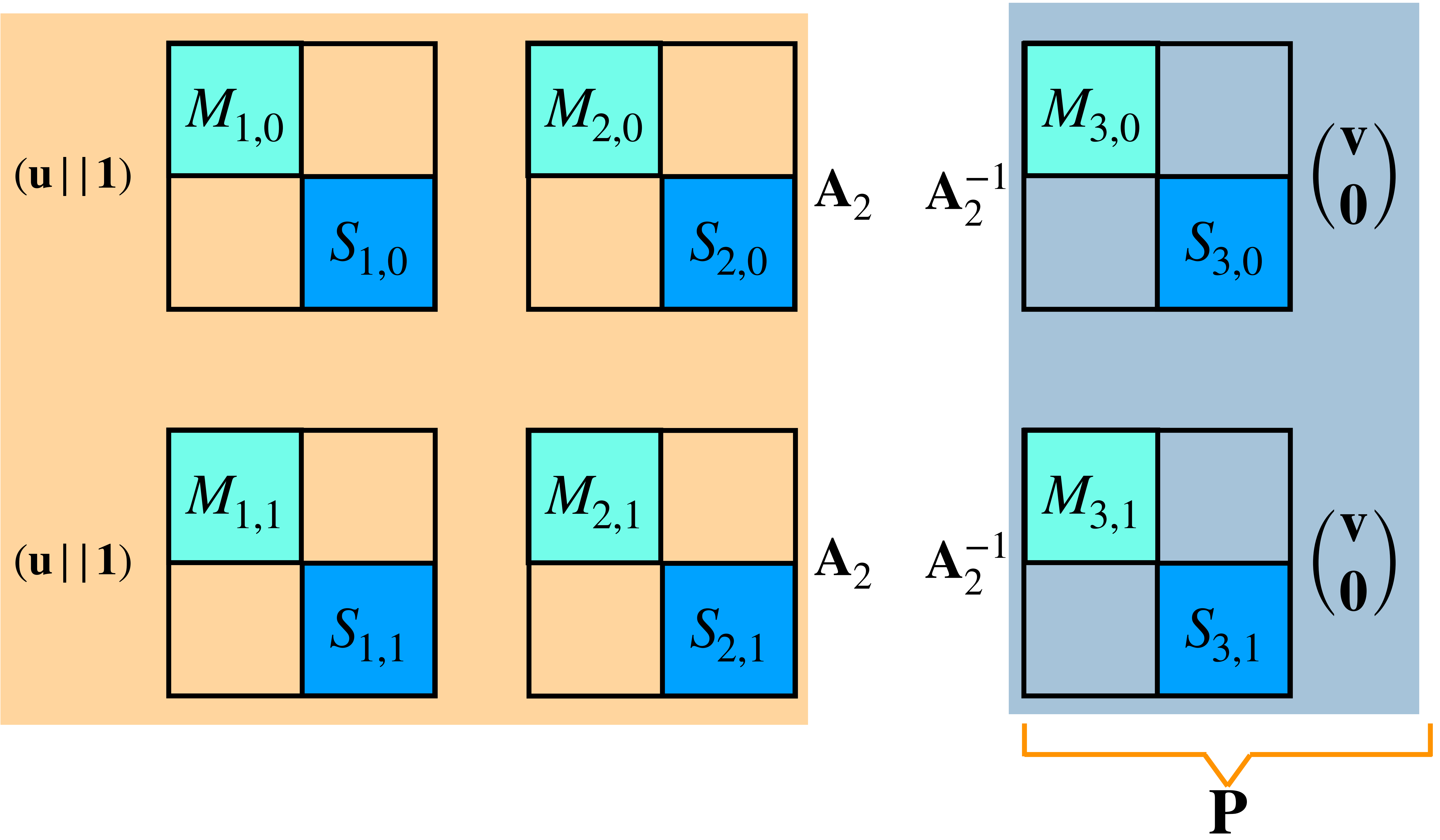

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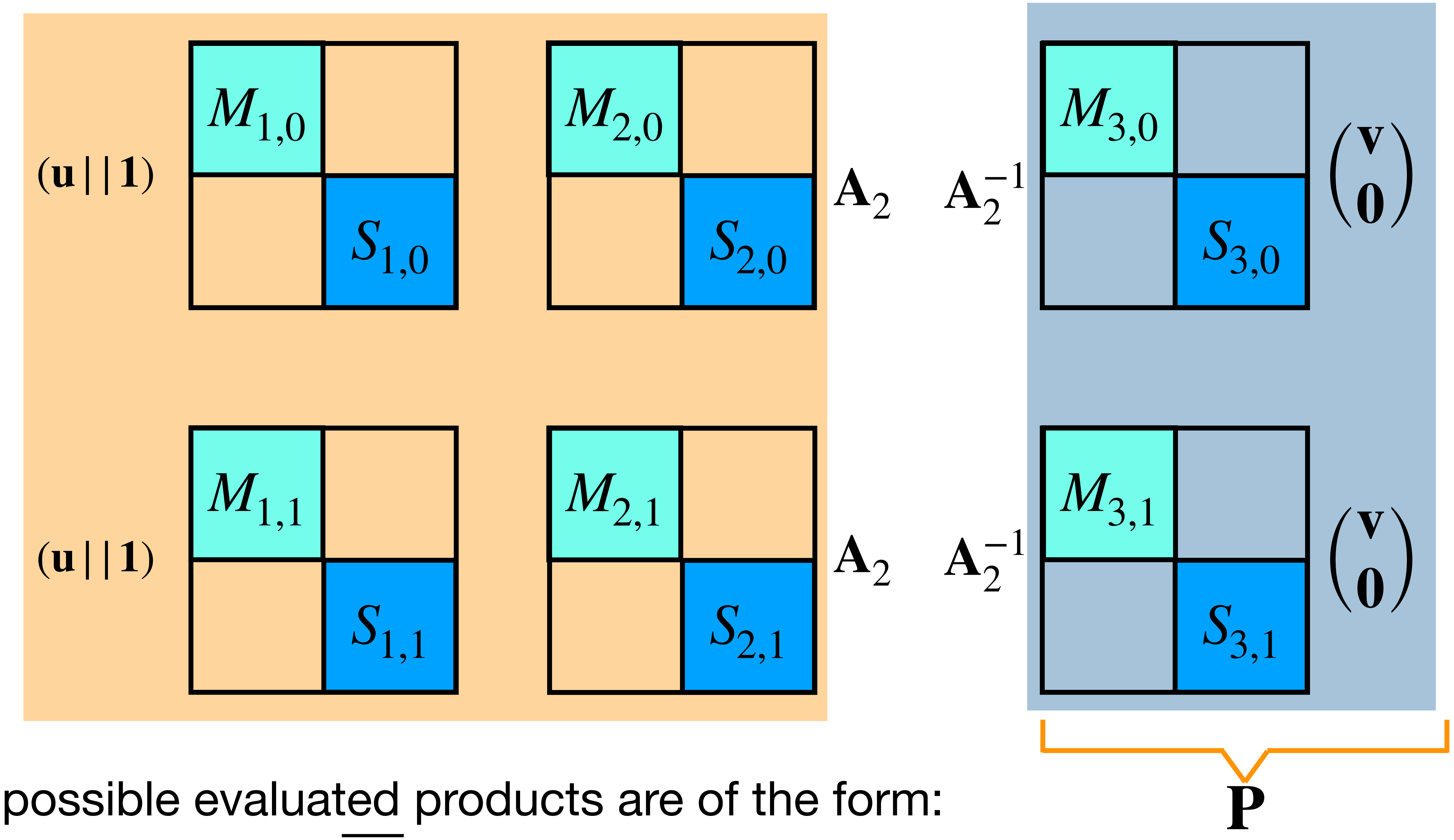

All possible evaluated products are of the form:

$$\mathbf{SB} = \{\mathbf{u}M_{1,x_1}M_{2,x_2}\overline{\mathbf{A}_2} + \mathbf{1}S_{1,x_1}S_{2,x_2}\underline{\mathbf{A}_2}\}_{x_1,x_2\in\{0,1\}}$$

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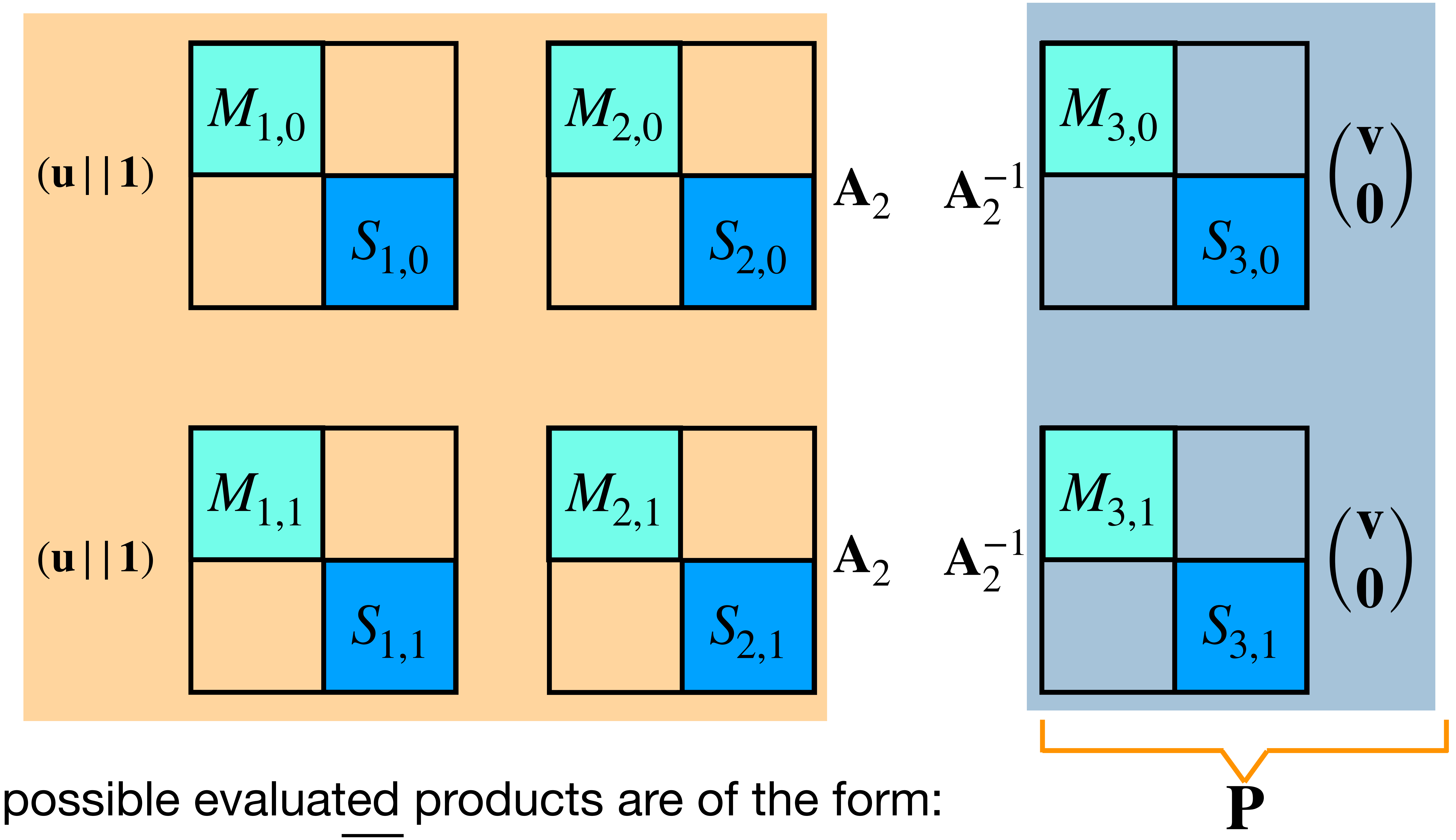

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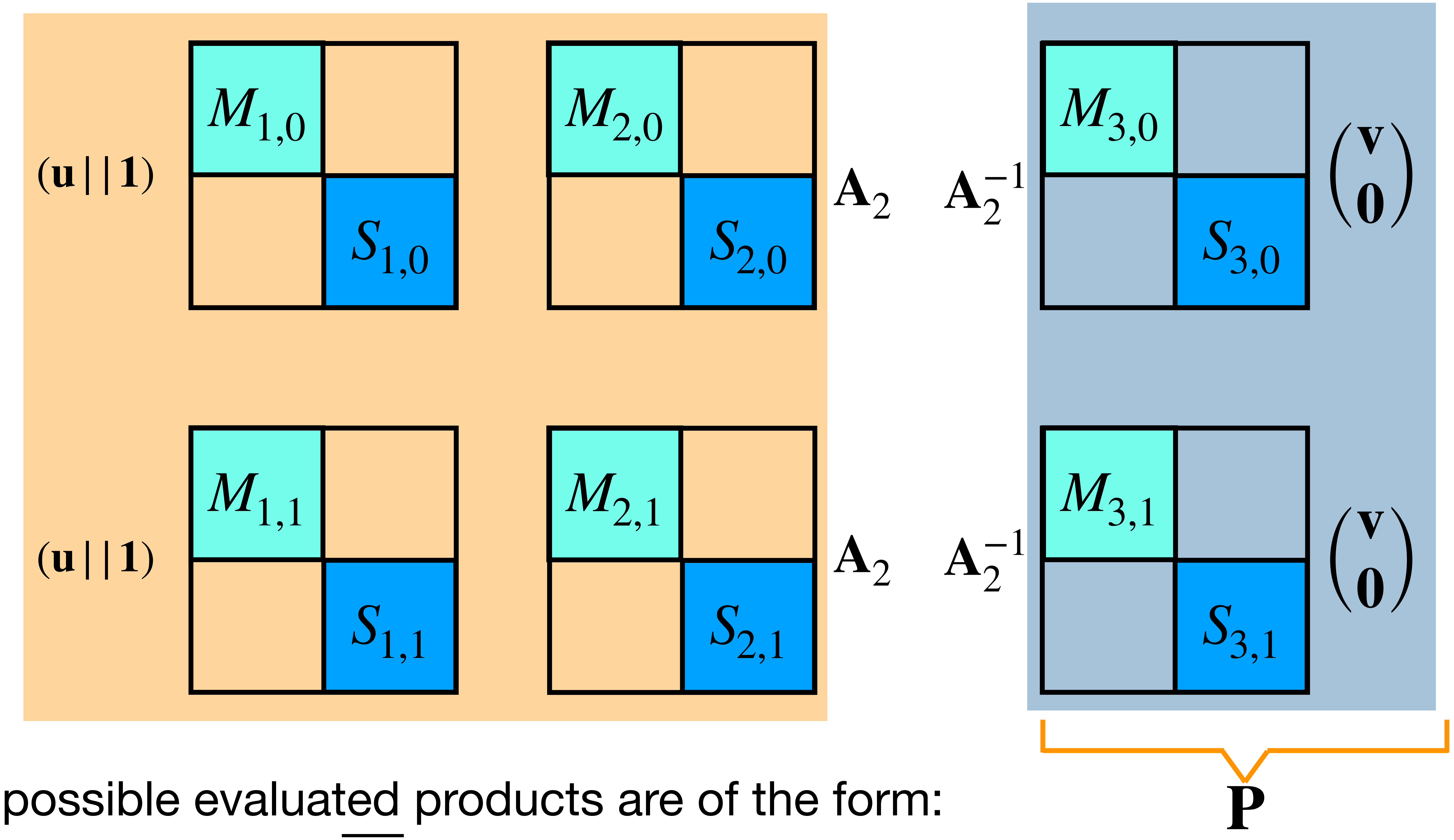
GGH15 Encodings



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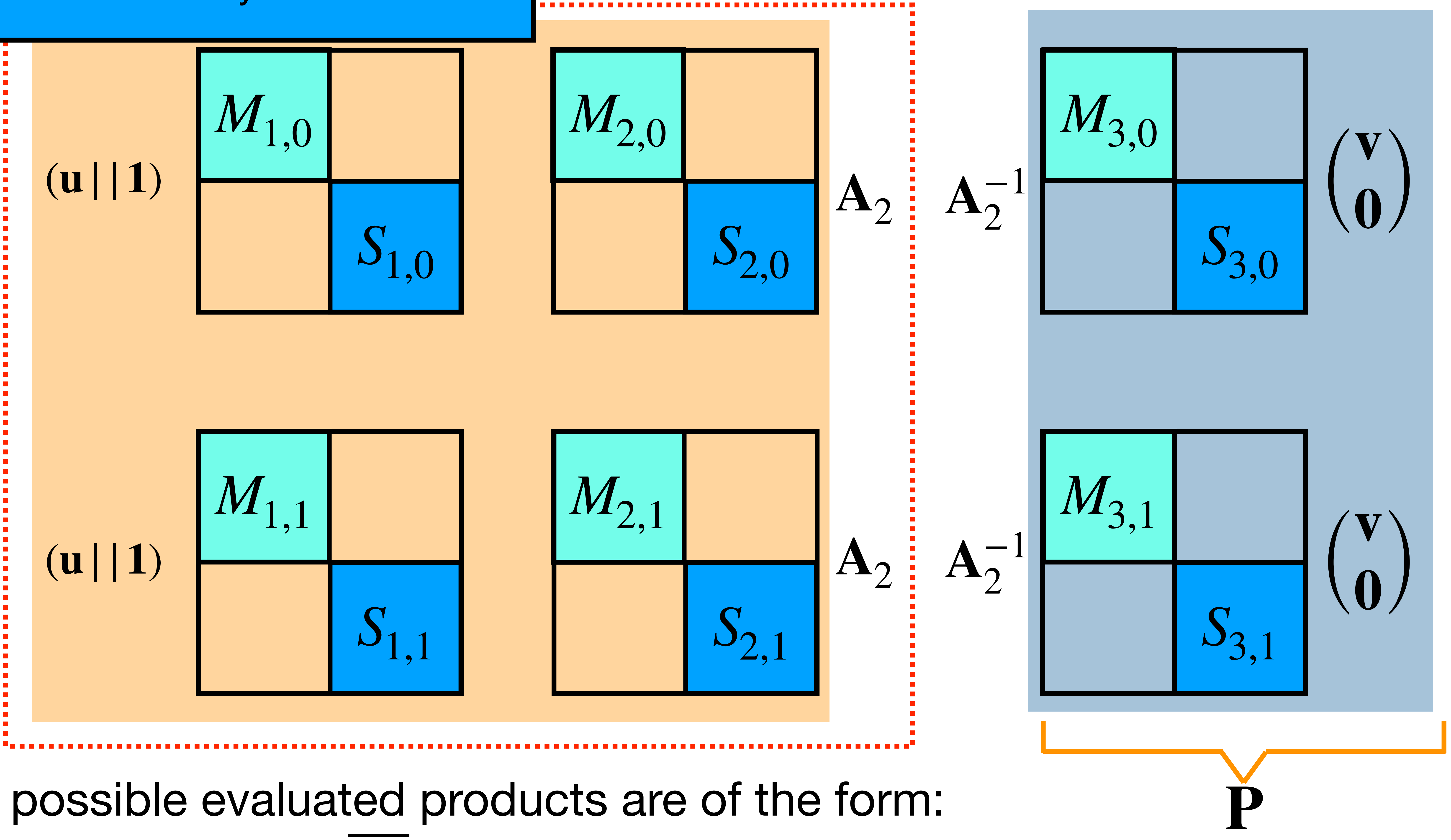
GGH15 Encodings

Pseudorandom by evasive LWE!



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- Repeatedly apply evasive LWE!
- Shrunk the size from 2^h evaluated products to size to $2h$ matrices.

GGH15 Encodings

$$\left((\mathbf{u}||\mathbf{1}) \begin{bmatrix} M_{1,0} & \\ & S_{1,0} \end{bmatrix} \mathbf{A}_1 \right) \mathbf{A}_1^{-1} \left(\begin{bmatrix} M_{2,0} & \\ & S_{2,0} \end{bmatrix} \mathbf{A}_2 \right) \mathbf{A}_2^{-1} \left(\begin{bmatrix} M_{3,0} & \\ & S_{3,0} \end{bmatrix} \begin{pmatrix} \mathbf{v} \\ \mathbf{0} \end{pmatrix} \right)$$

$$\left((\mathbf{u}||\mathbf{1}) \begin{bmatrix} M_{1,1} & \\ & S_{1,1} \end{bmatrix} \mathbf{A}_1 \right) \mathbf{A}_1^{-1} \left(\begin{bmatrix} M_{2,1} & \\ & S_{2,1} \end{bmatrix} \mathbf{A}_2 \right) \mathbf{A}_2^{-1} \left(\begin{bmatrix} M_{3,1} & \\ & S_{3,1} \end{bmatrix} \begin{pmatrix} \mathbf{v} \\ \mathbf{0} \end{pmatrix} \right)$$

- Repeatedly apply evasive LWE!
- Shrunk the size from 2^h evaluated products to size to $2h$ matrices.