

# MacORAMa: Optimal Oblivious RAM with Integrity

To appear at CRYPTO 2023

**Surya Mathialagan**

MIT



**Neekon Vafa**

MIT



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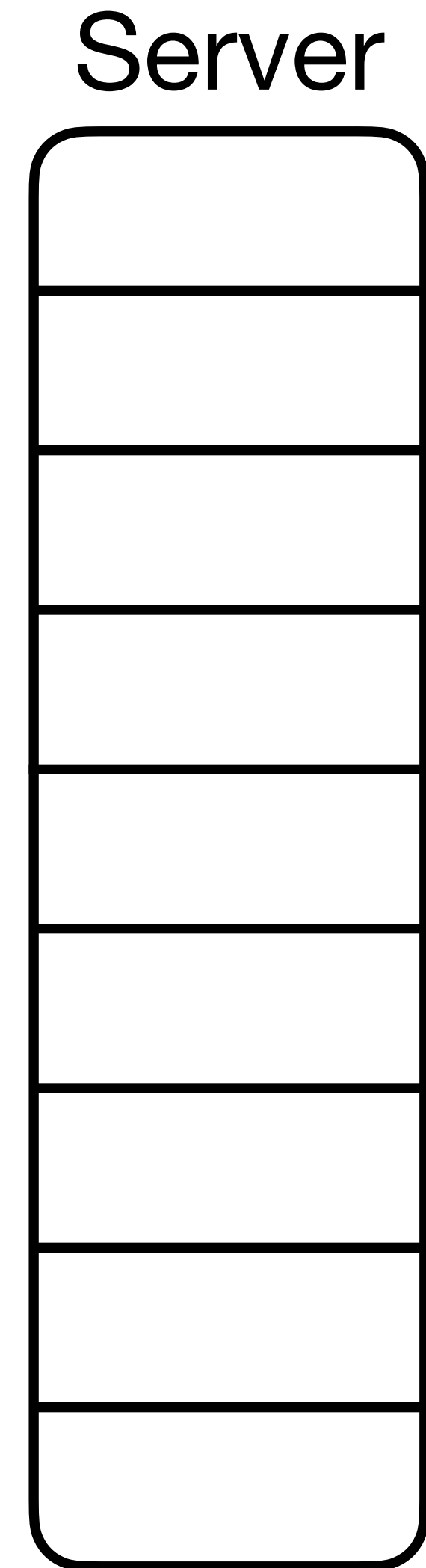
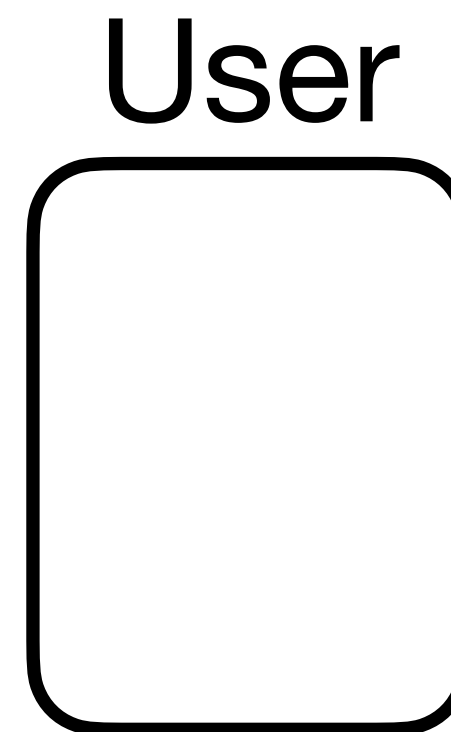
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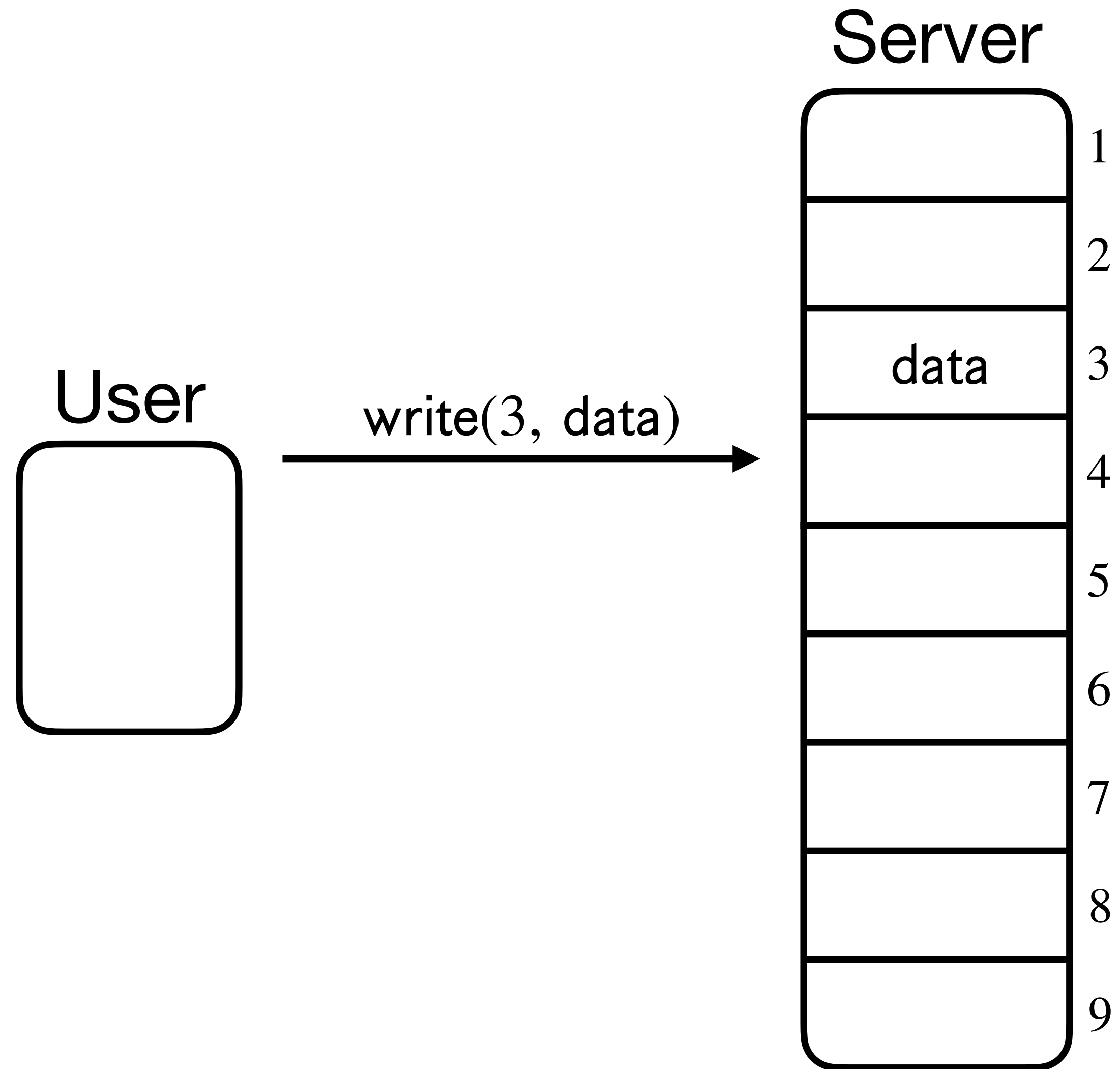
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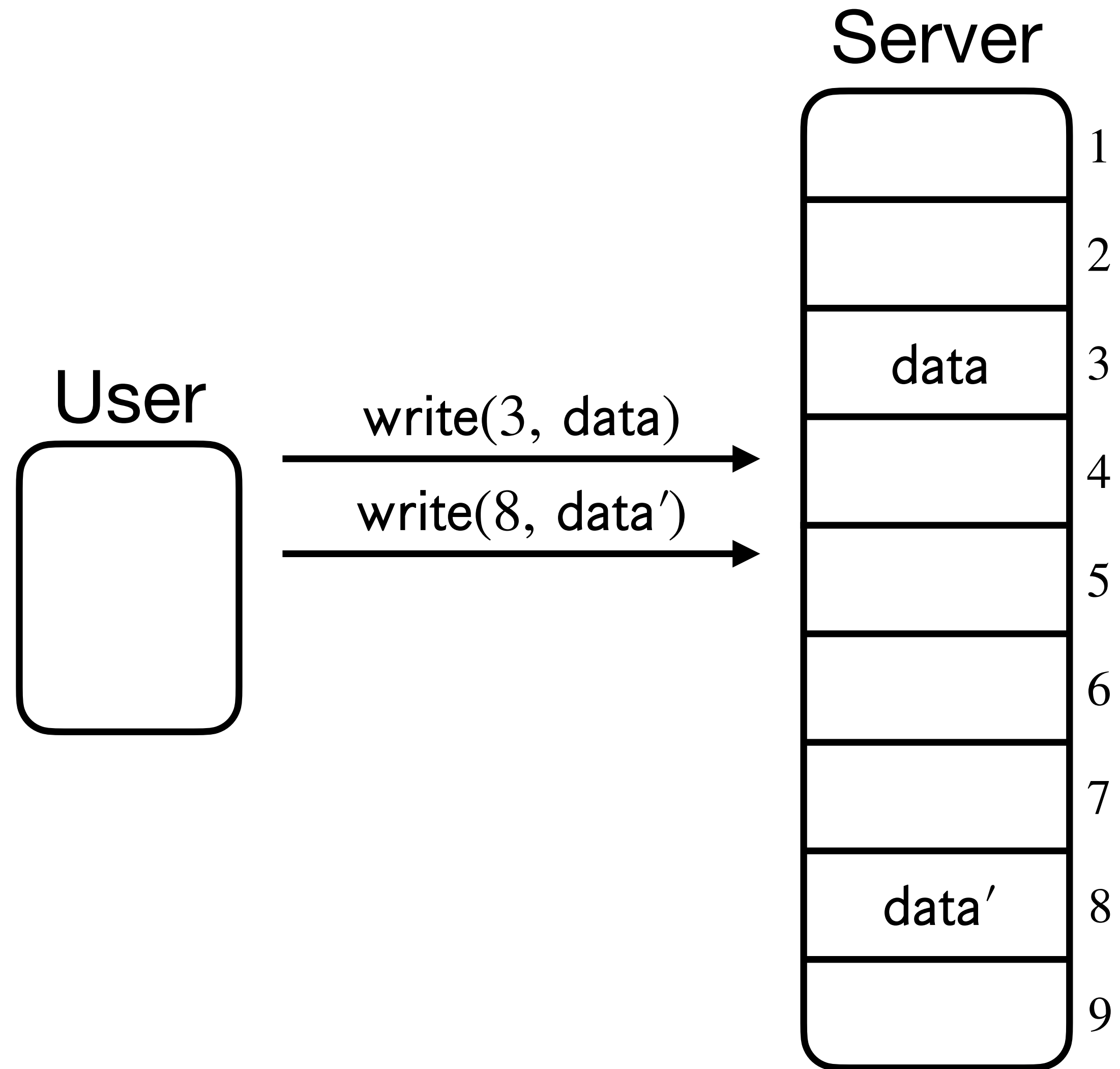
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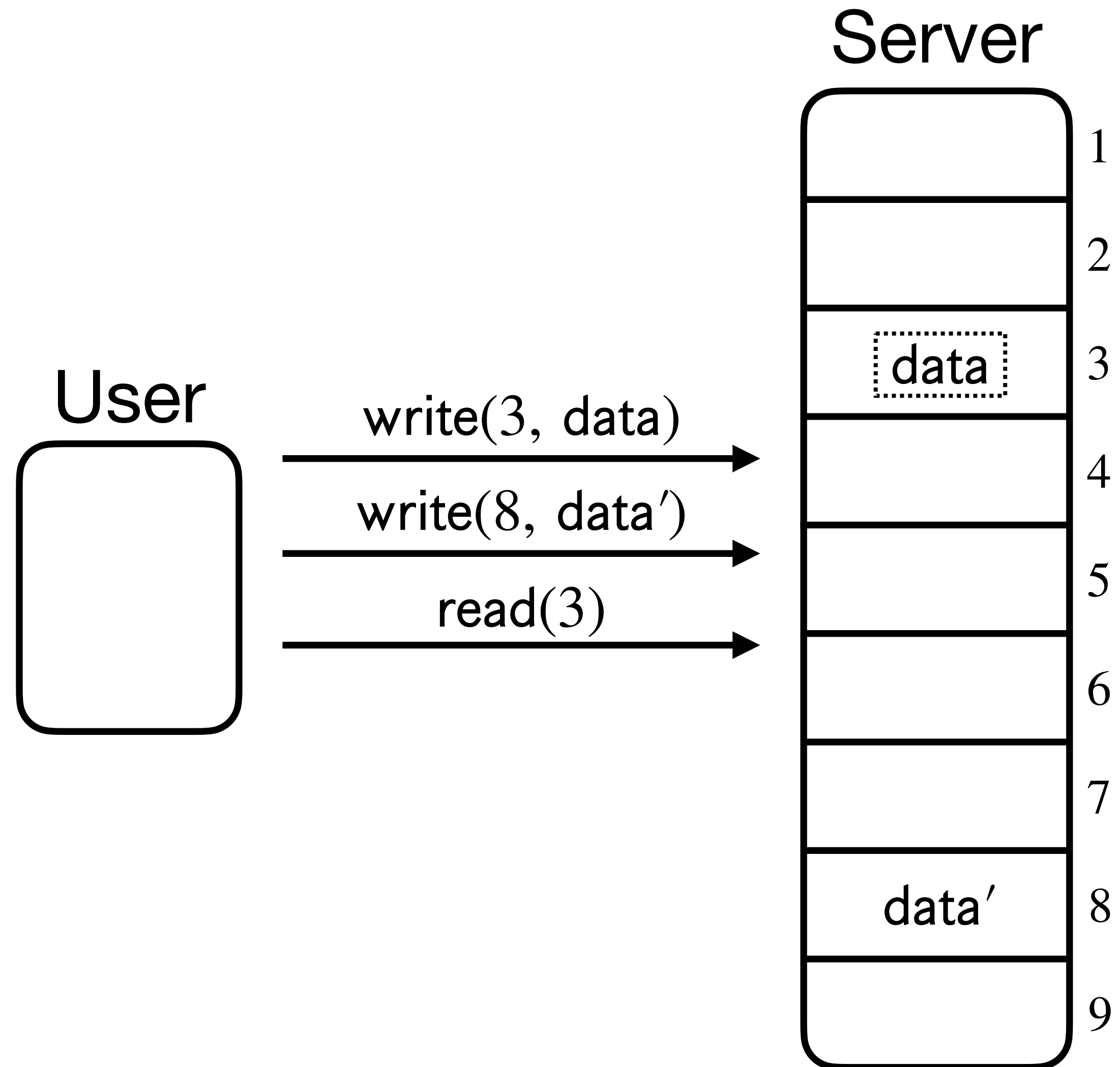
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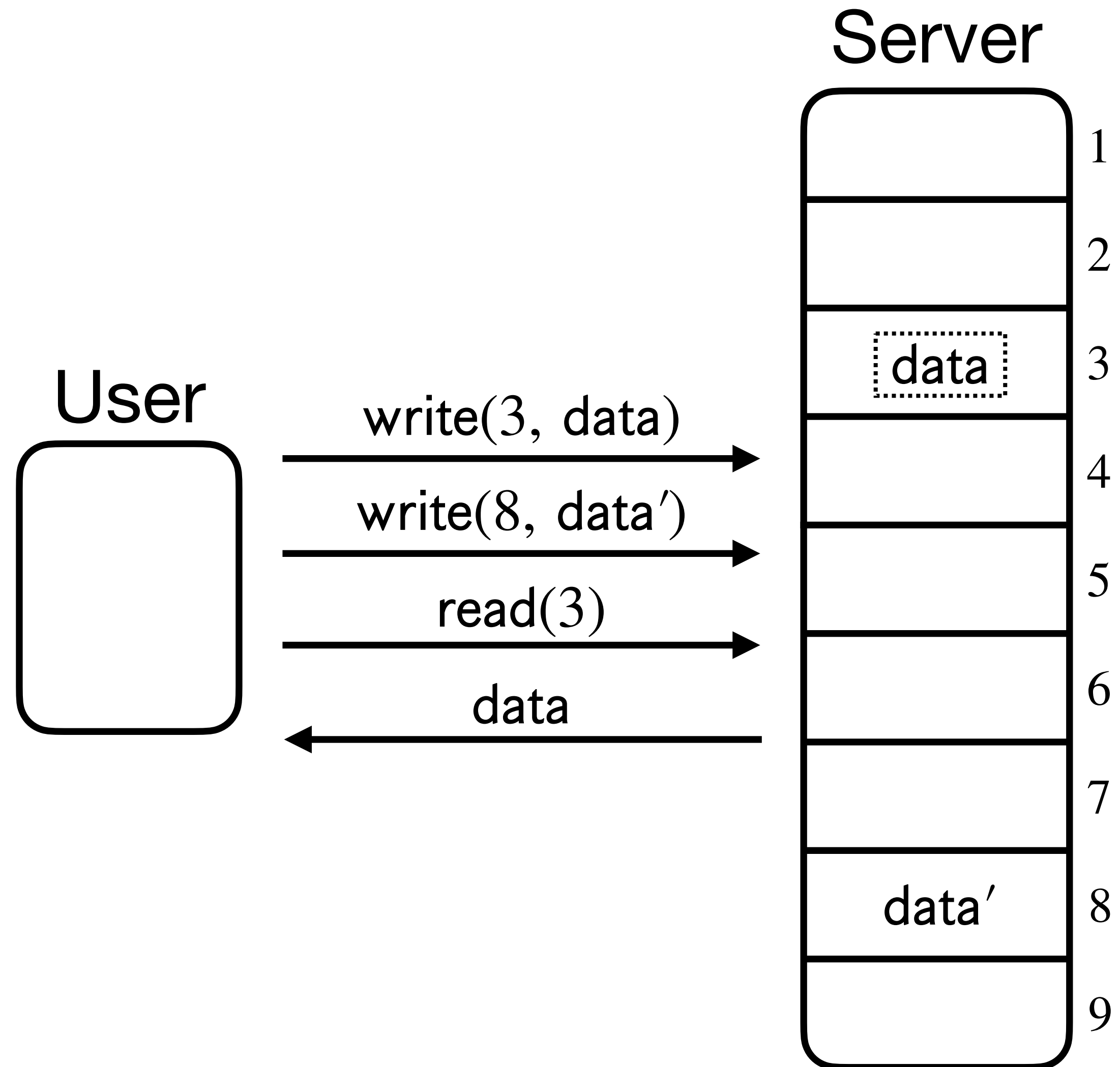
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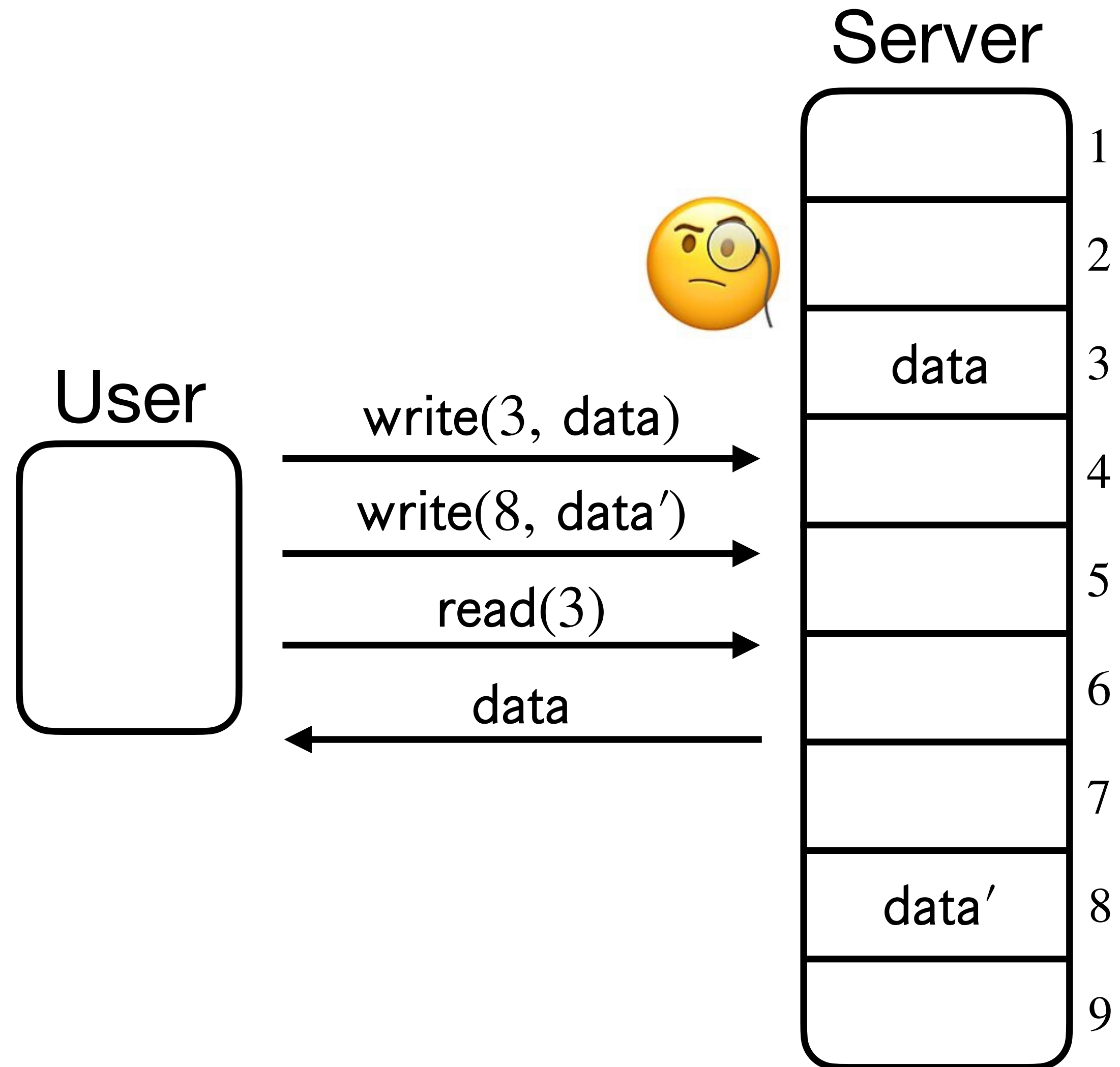
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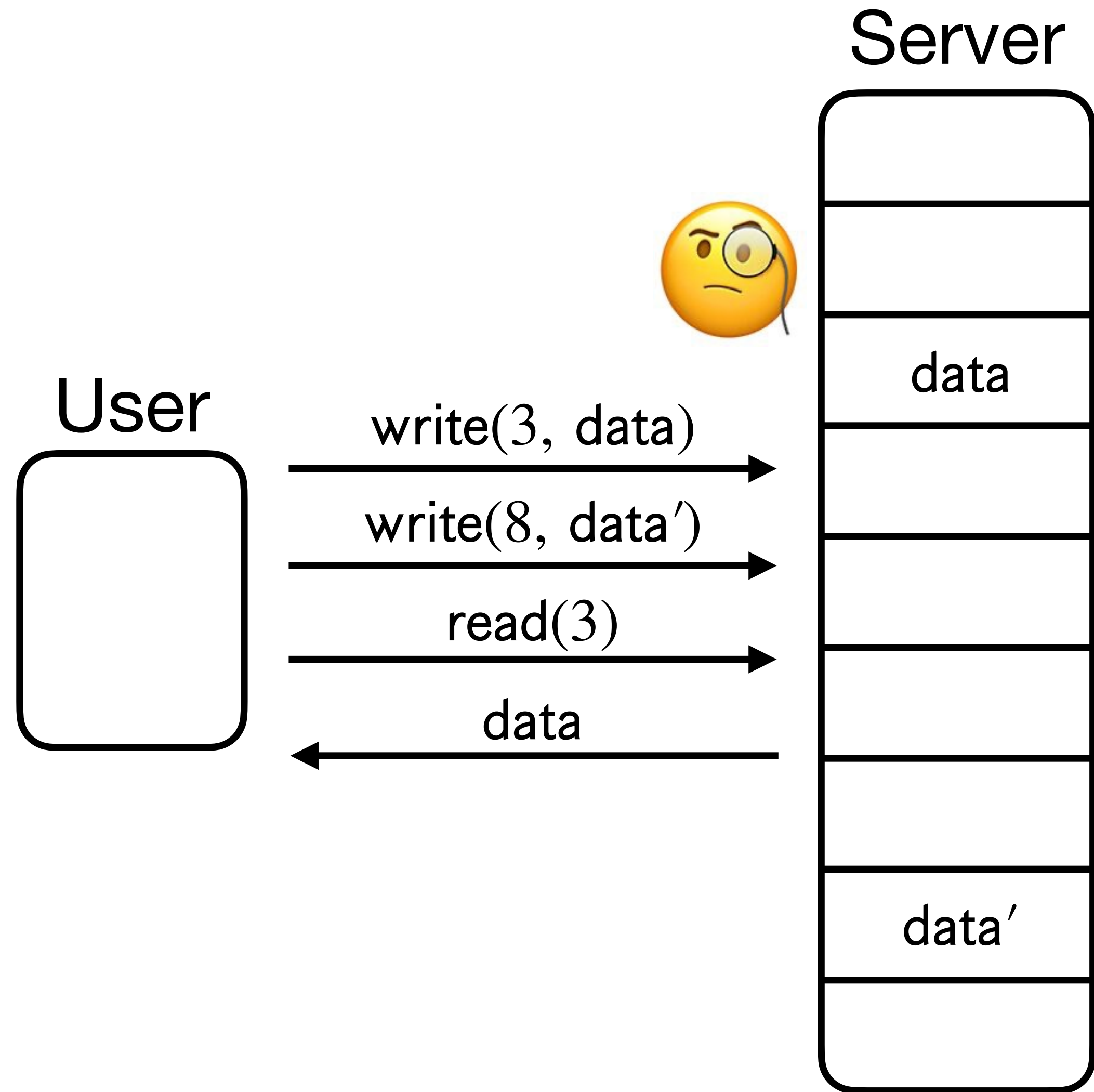
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- How can the user ensure privacy of its computation against a curious server?



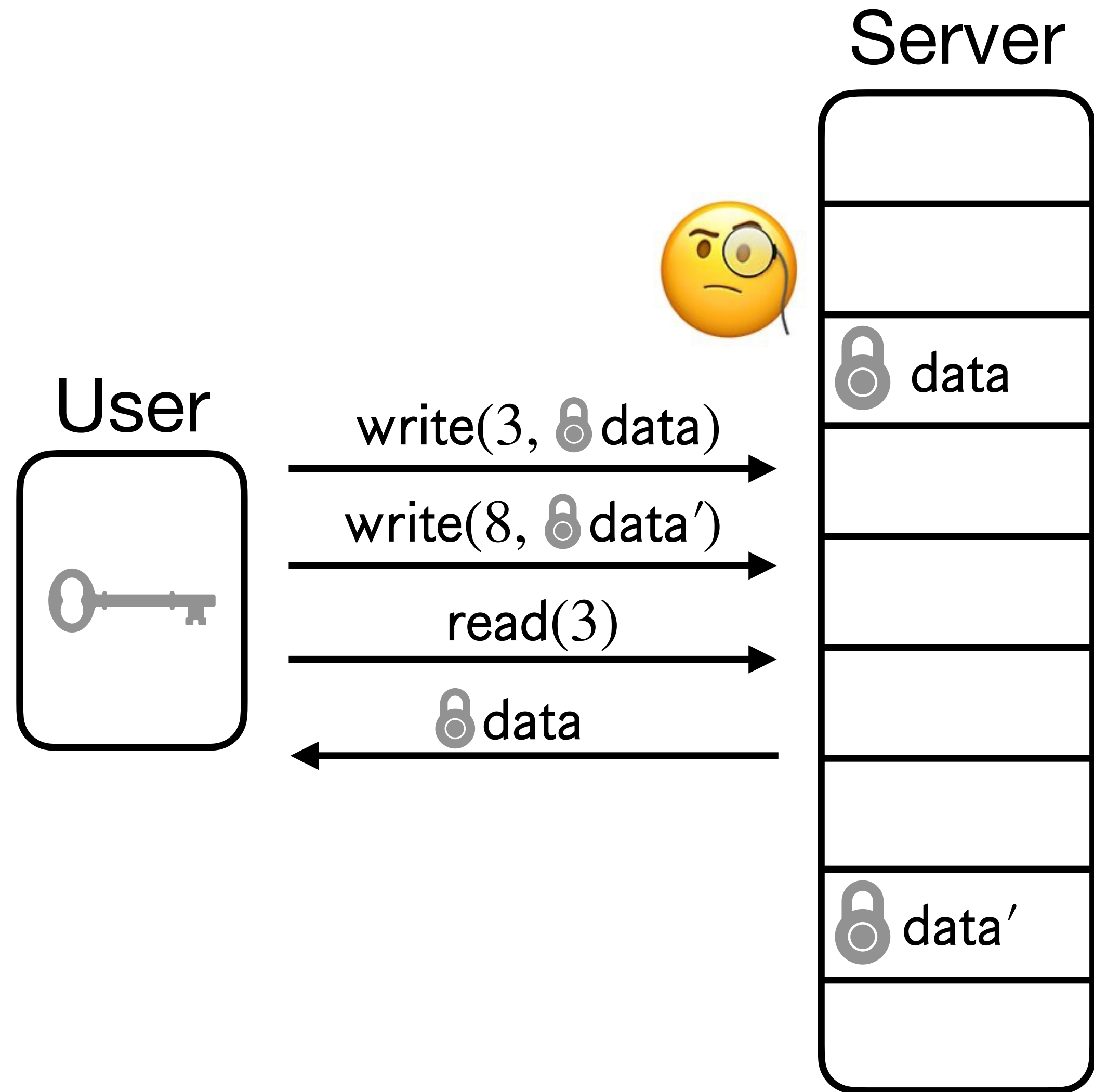
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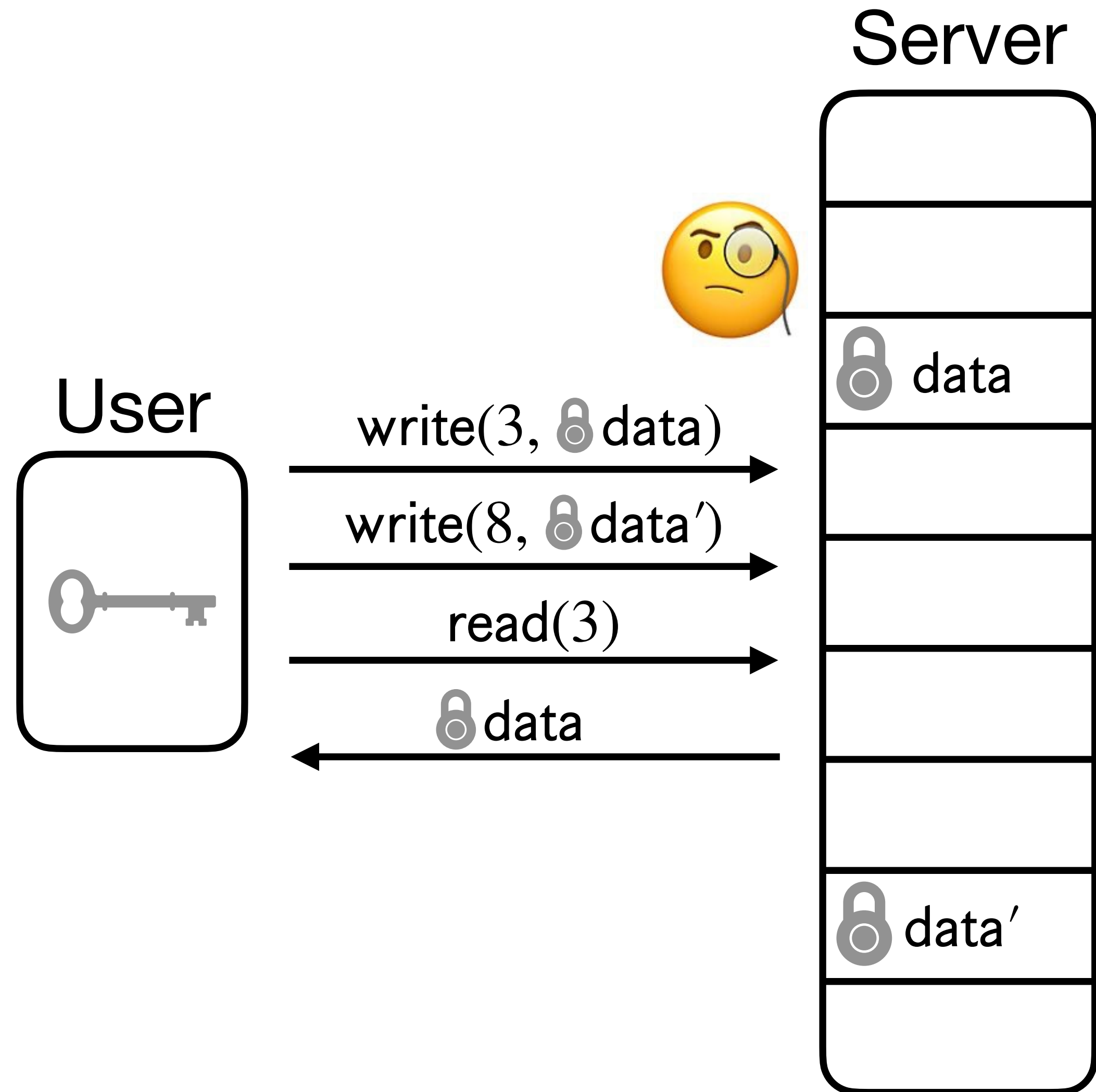
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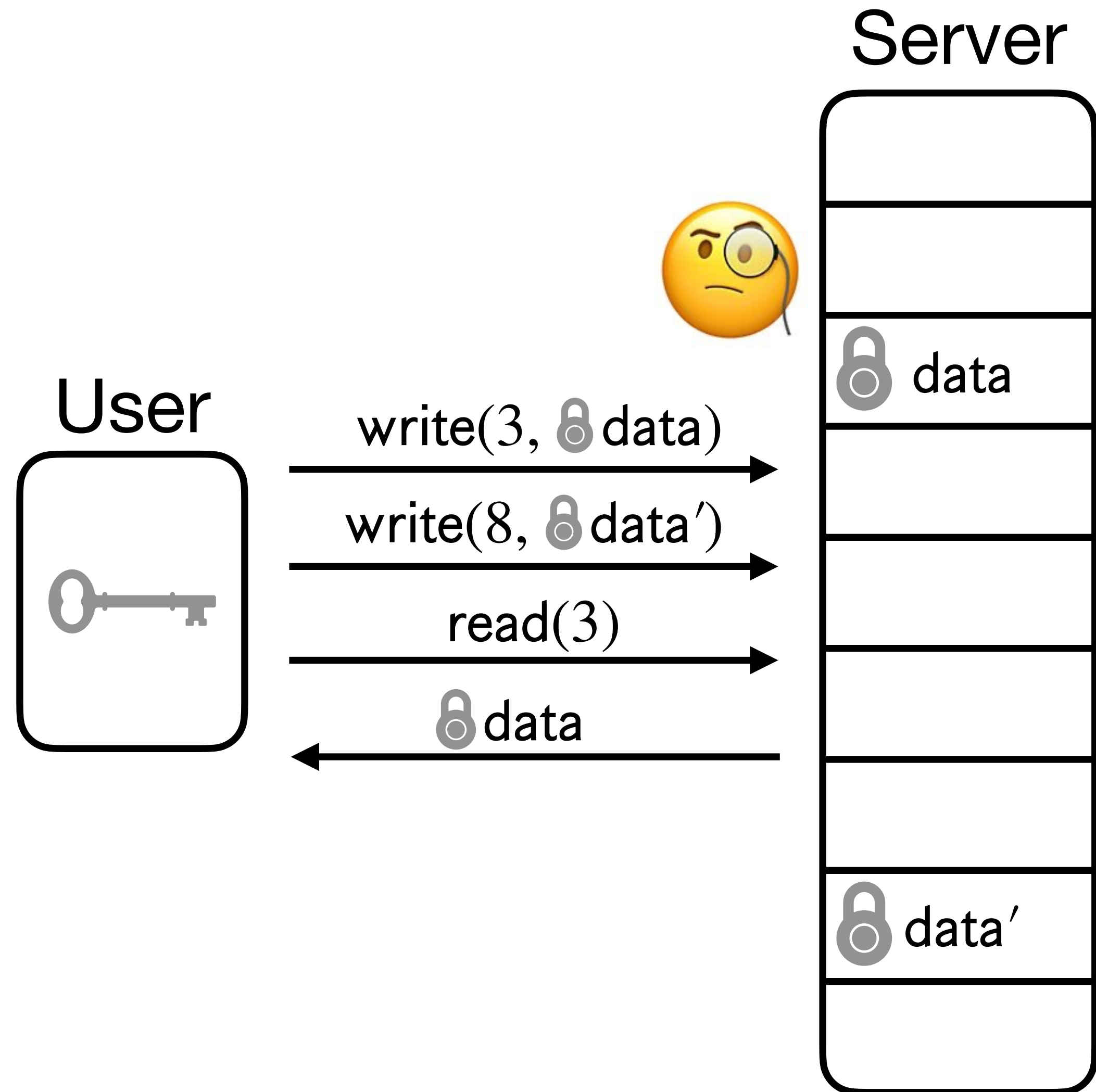
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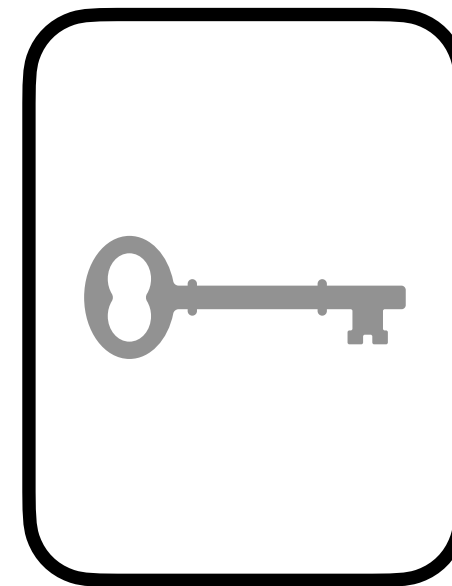
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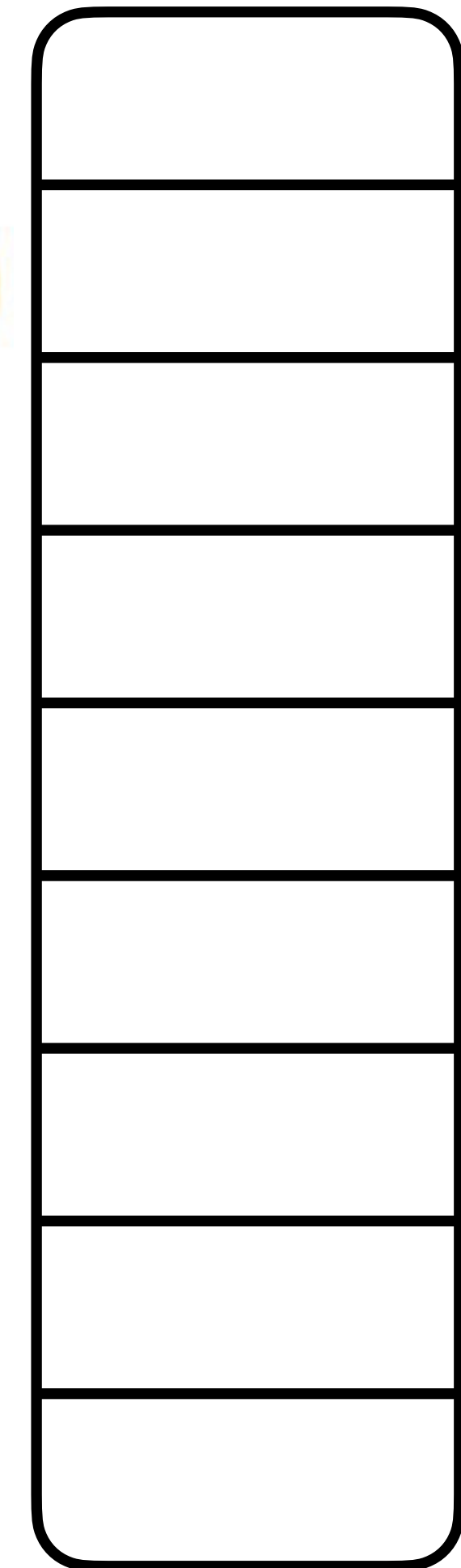
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Scientist



Server



Brain  
Data

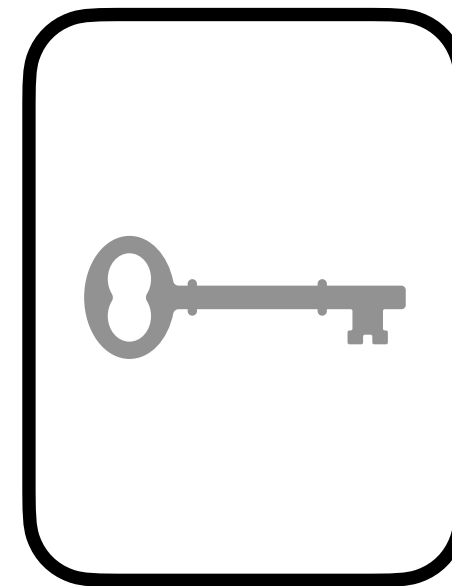
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Data

Heart  
Data

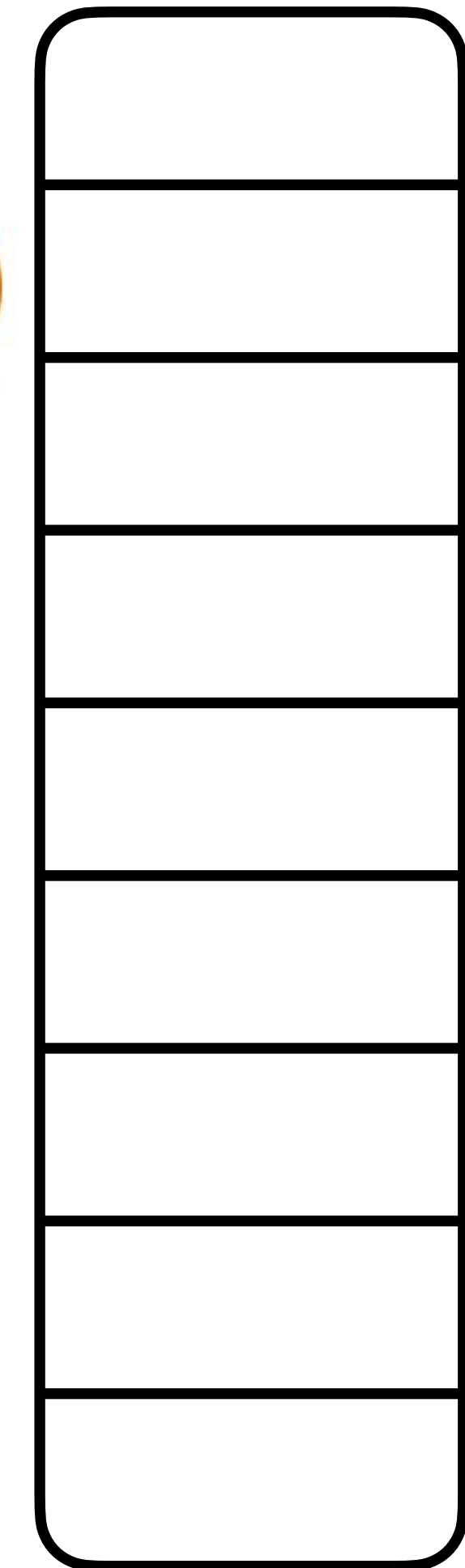
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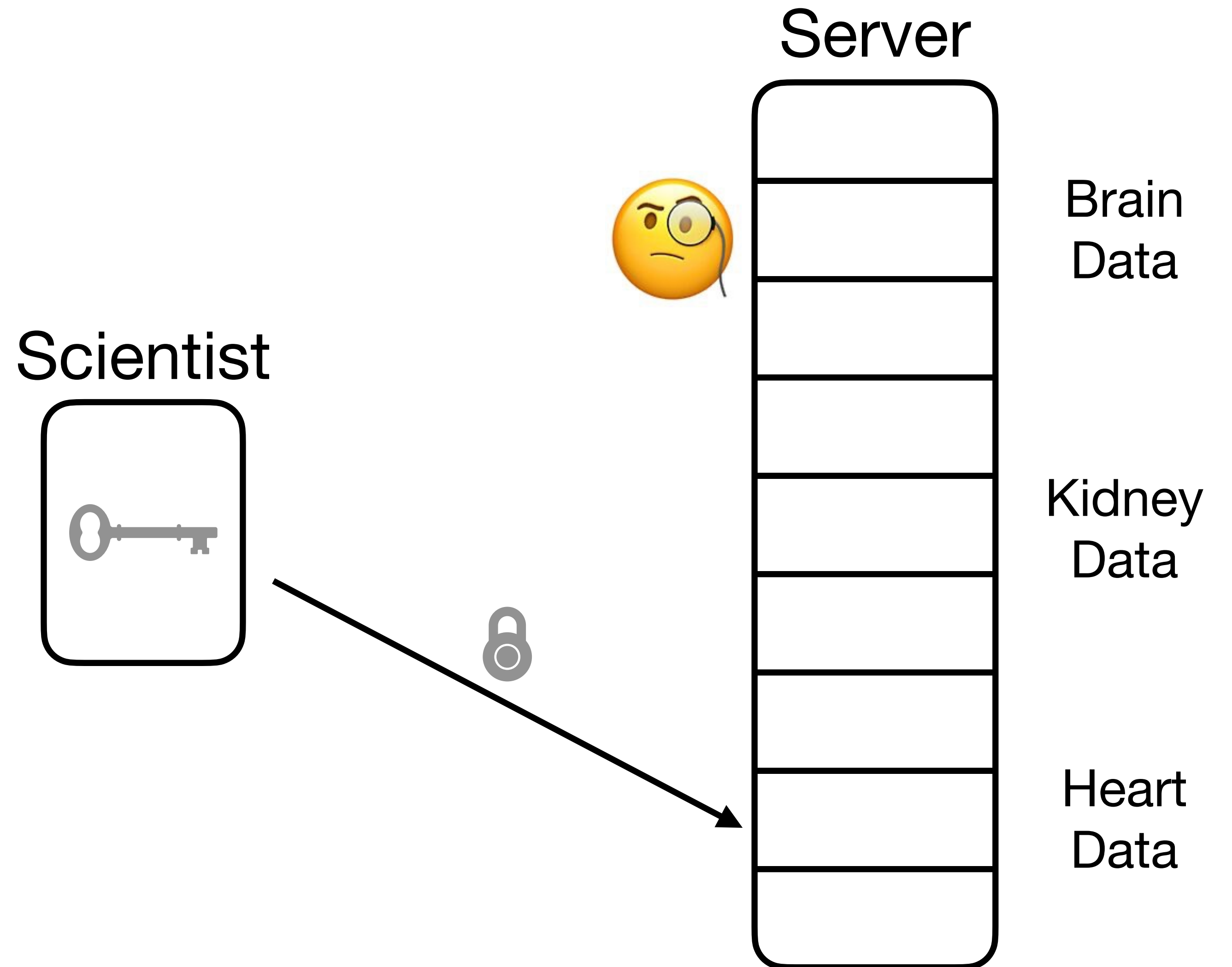
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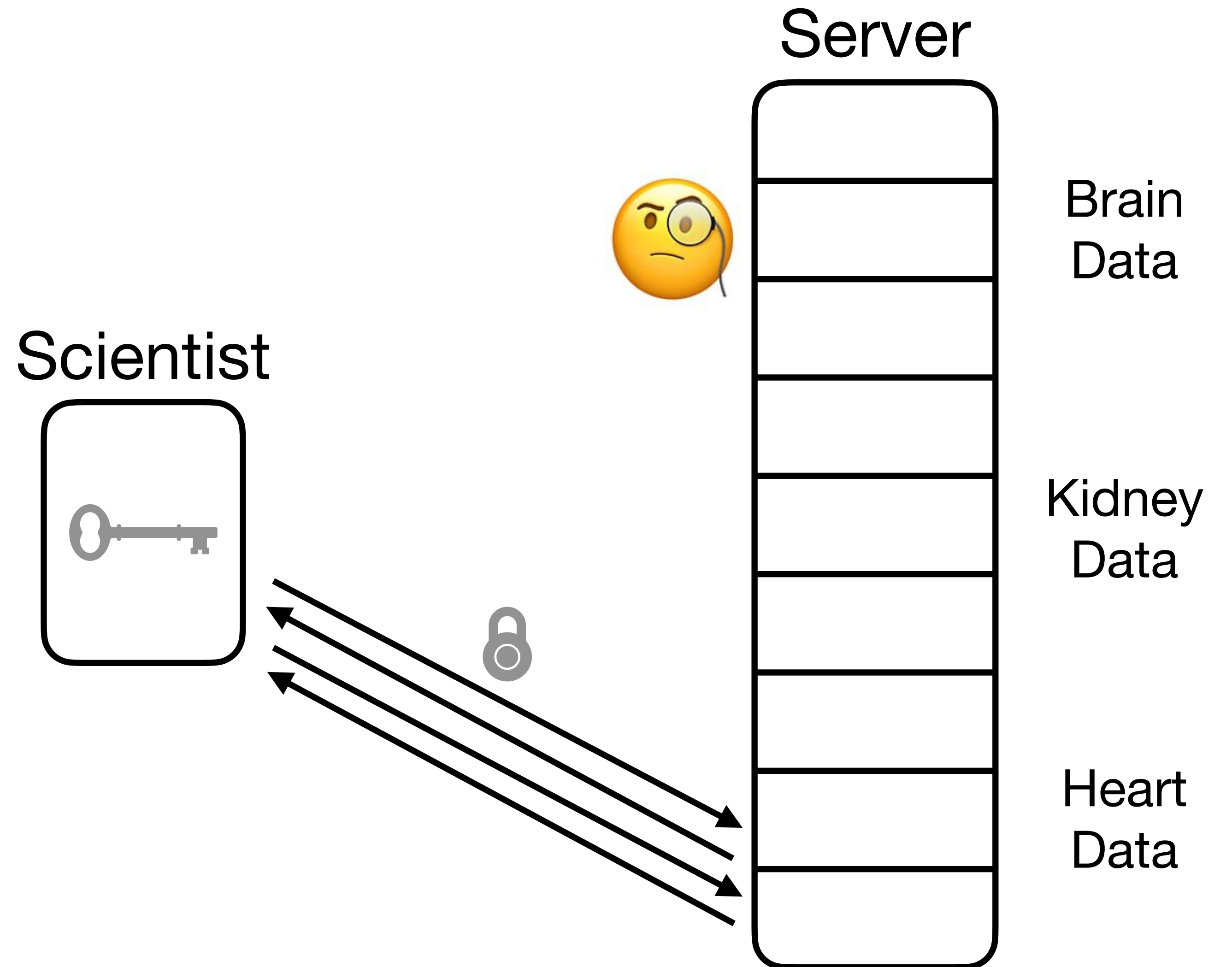
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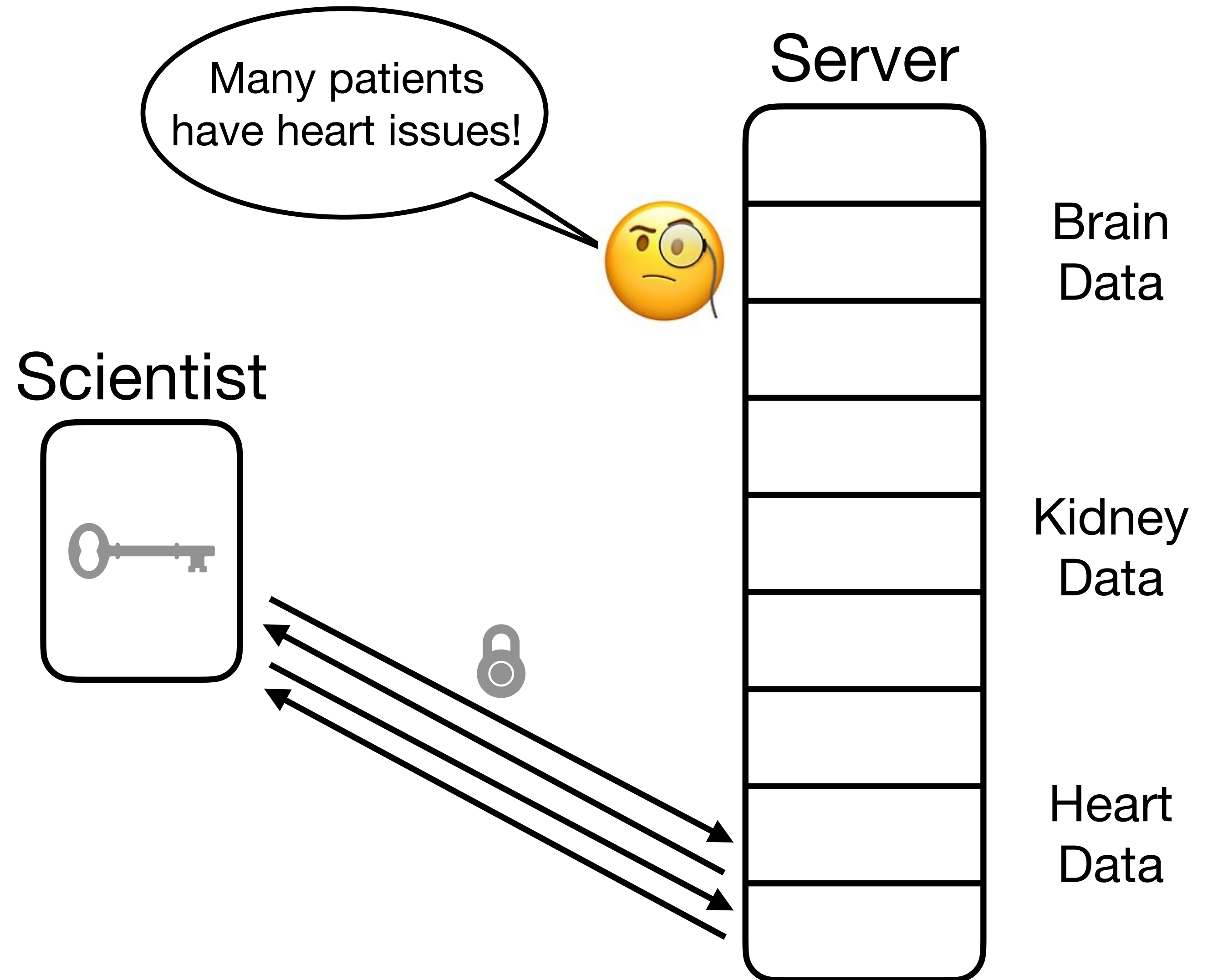
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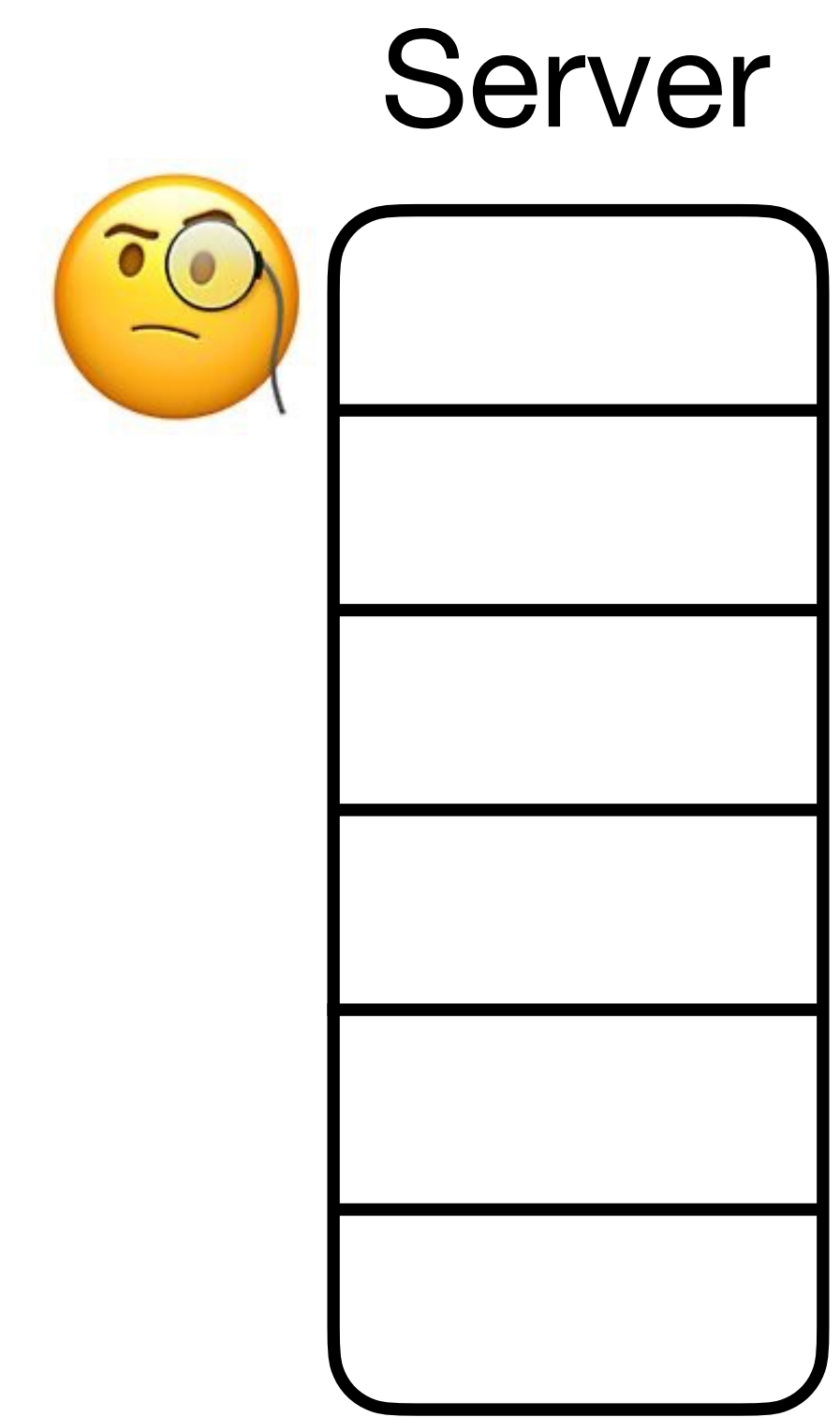
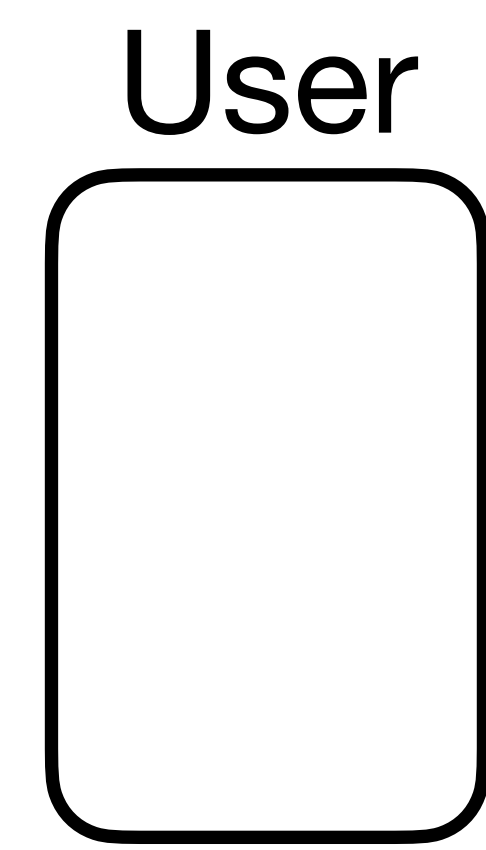
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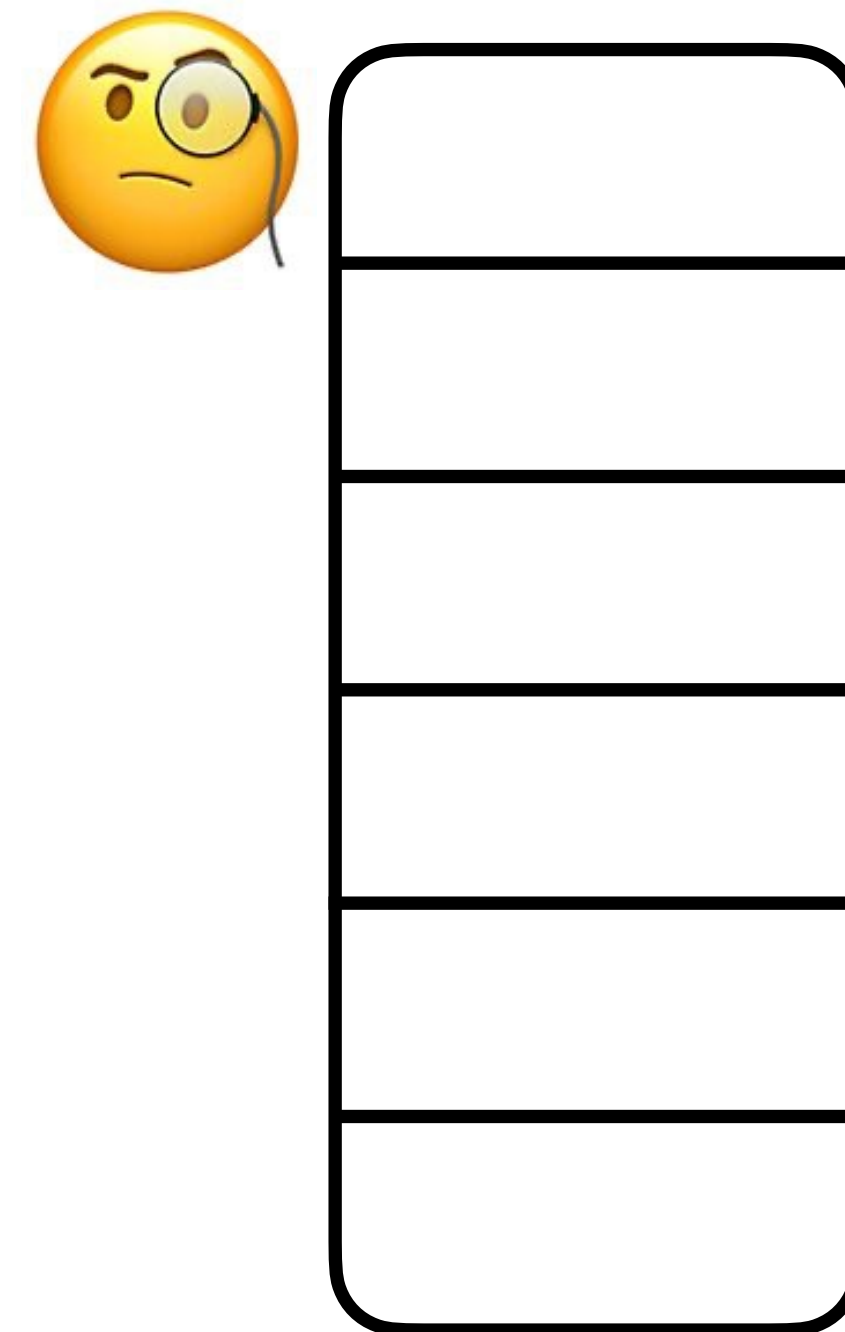
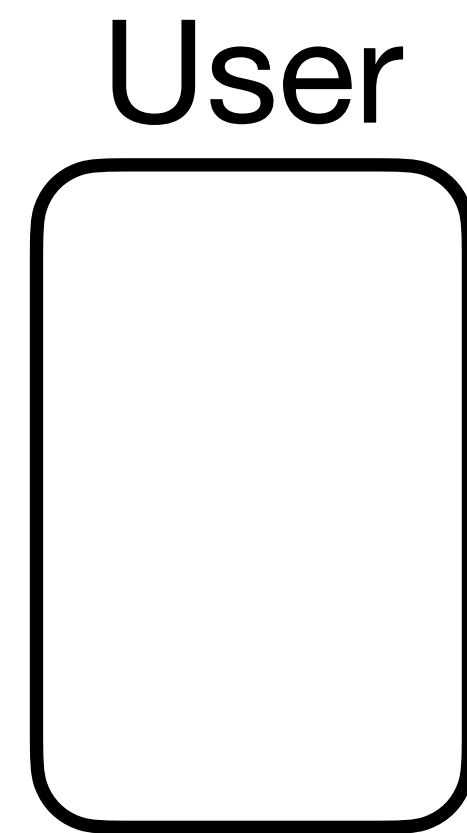
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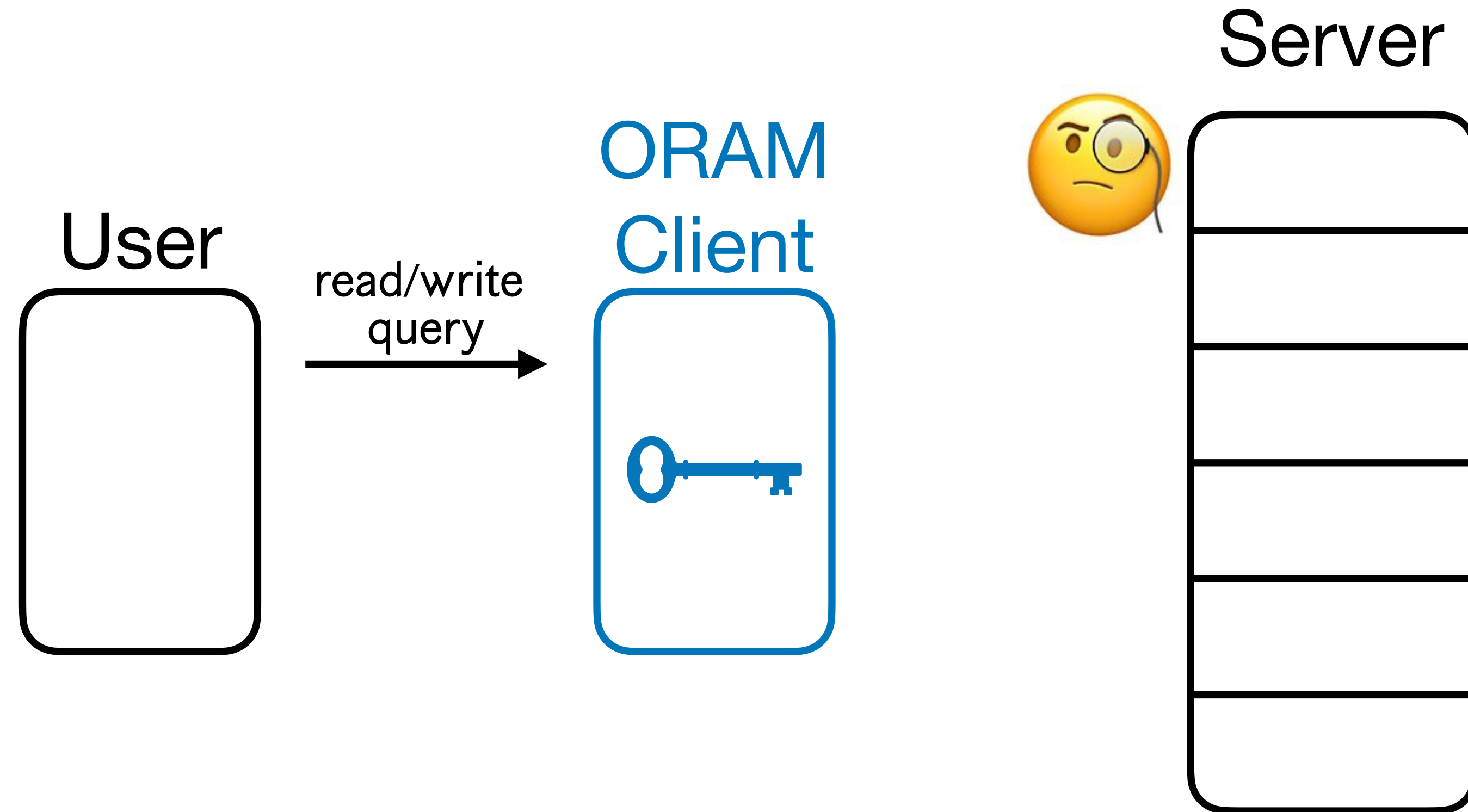
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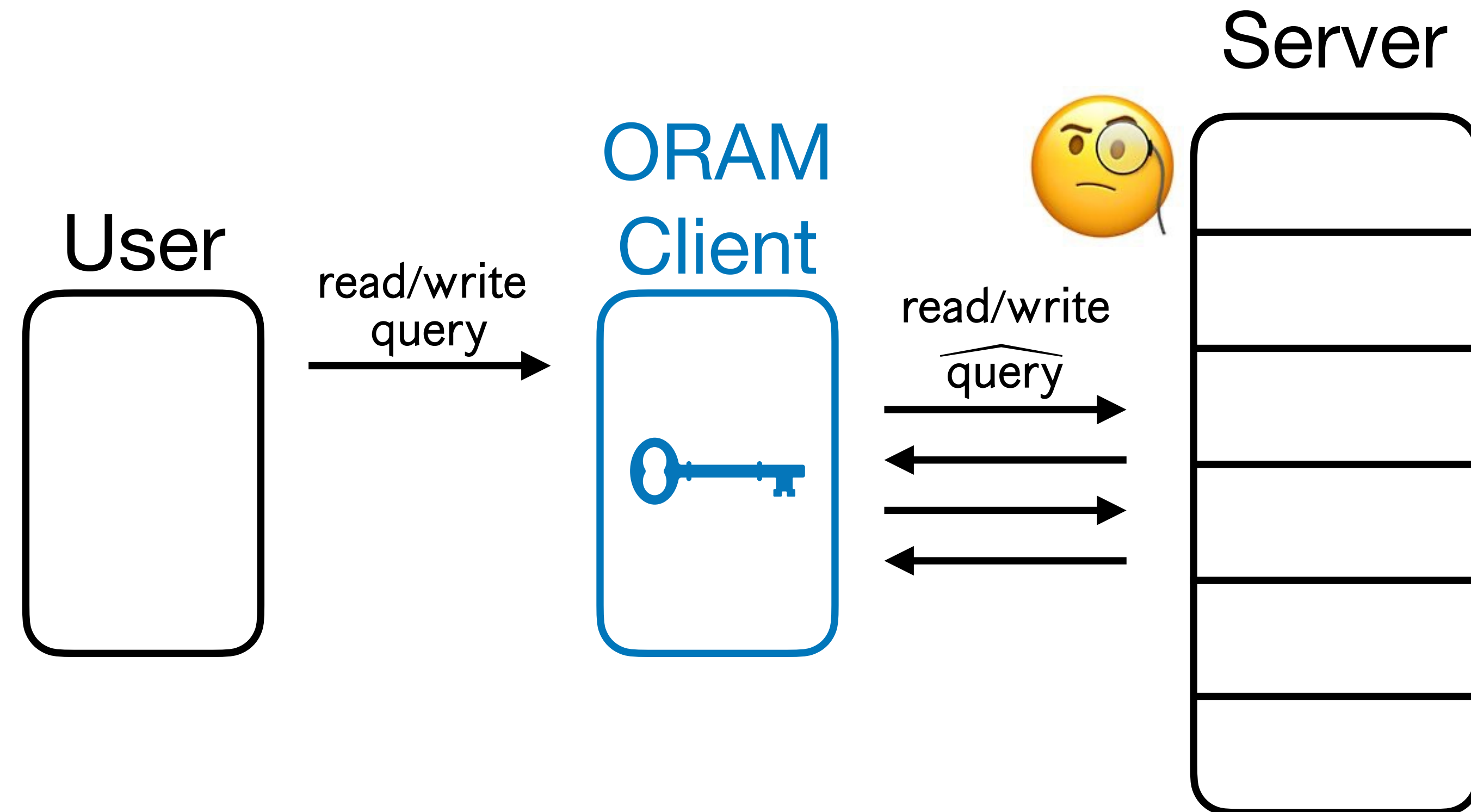
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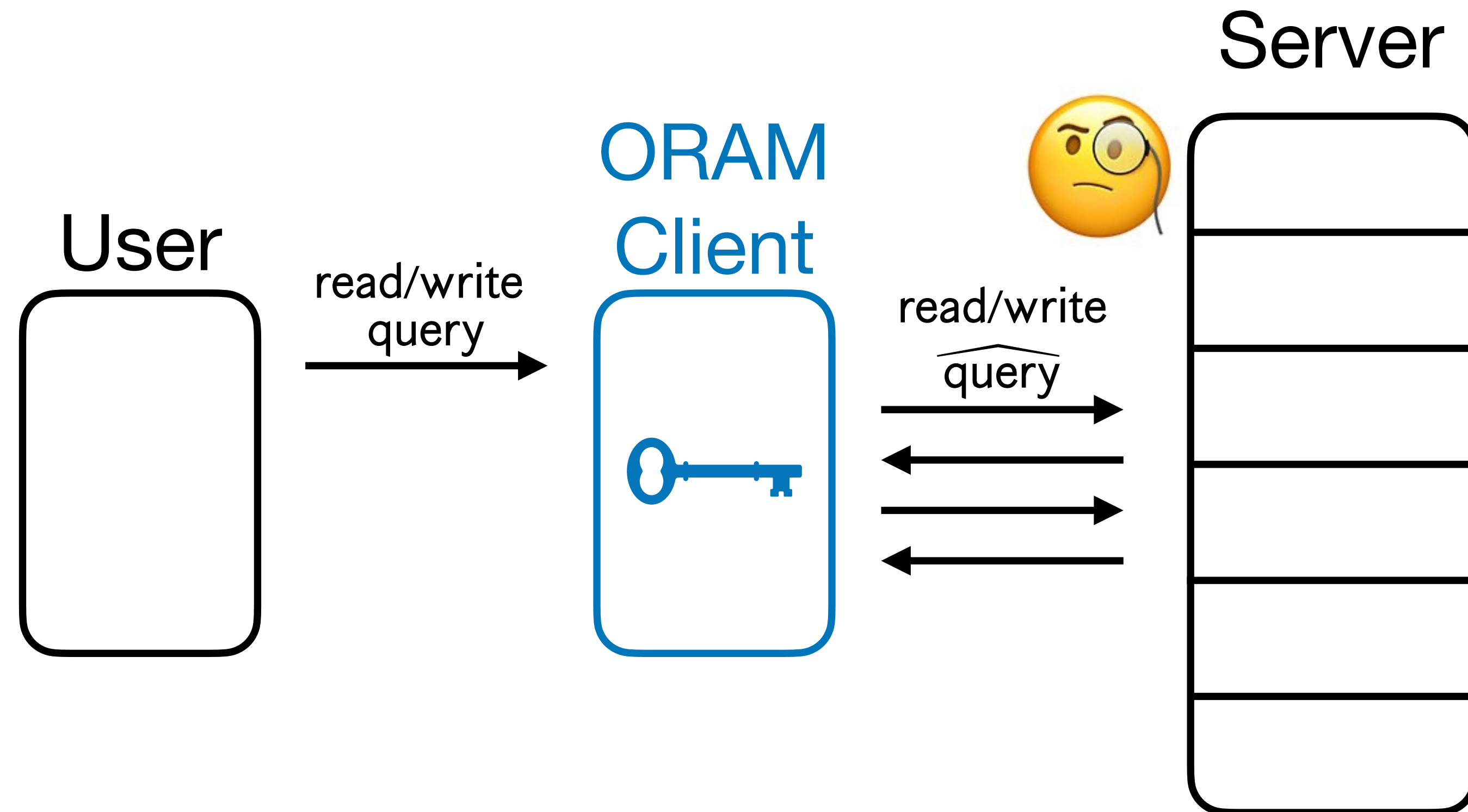
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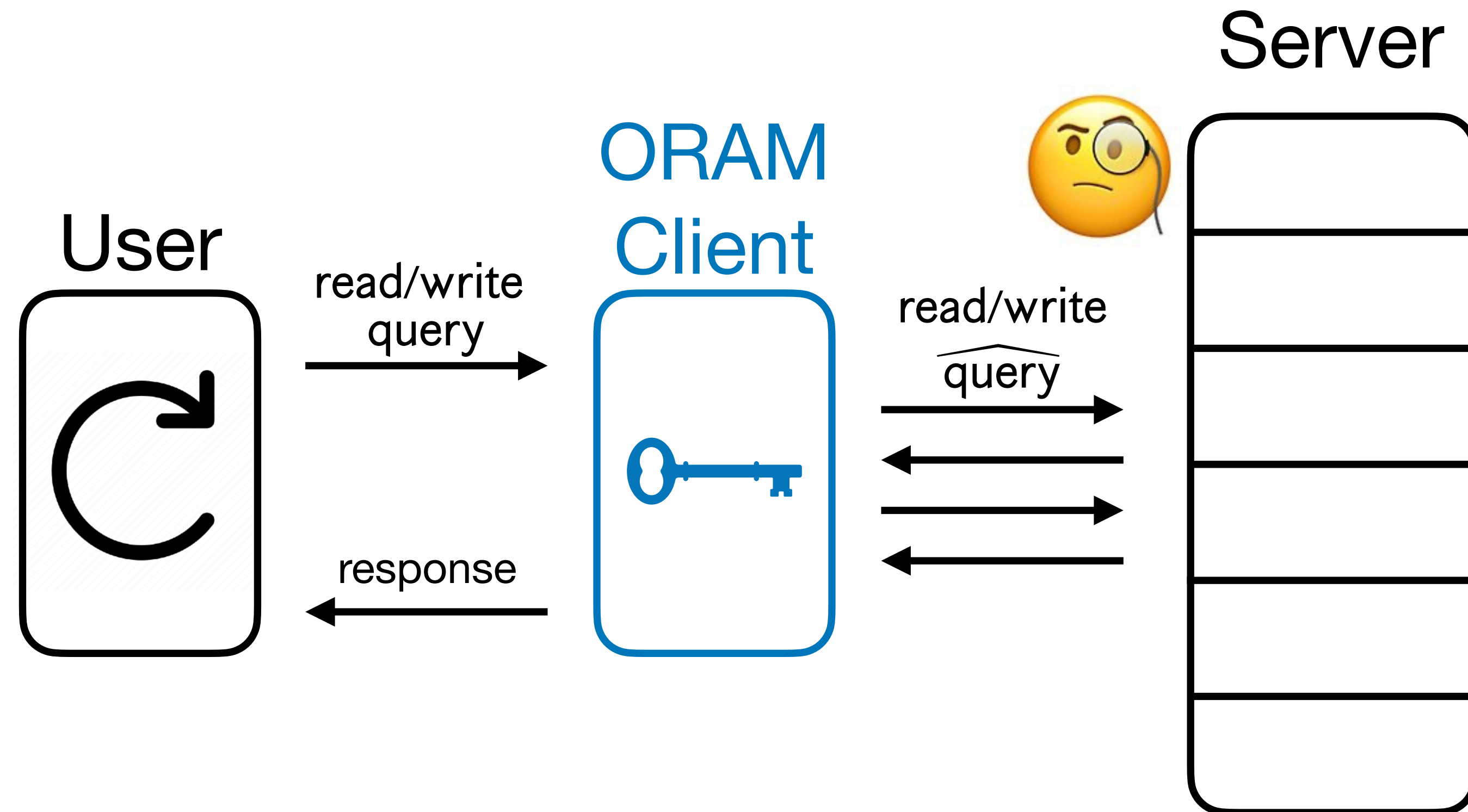


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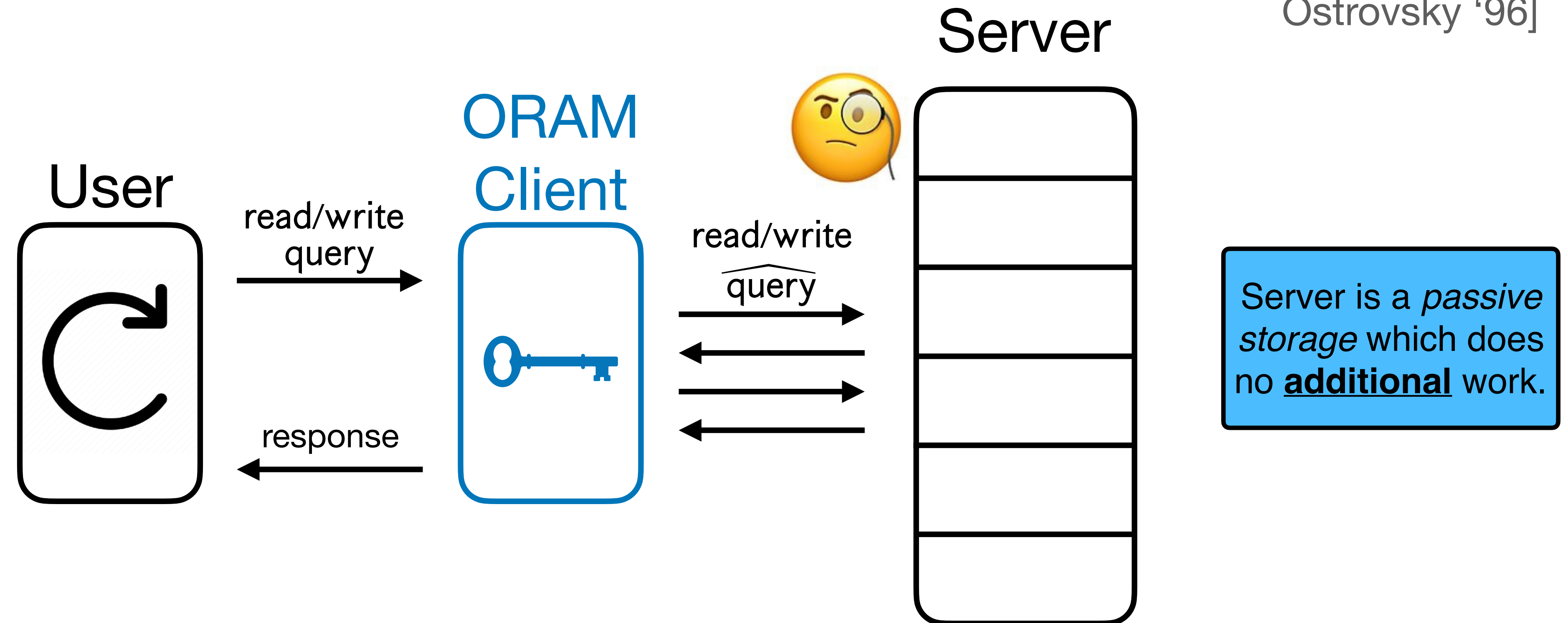
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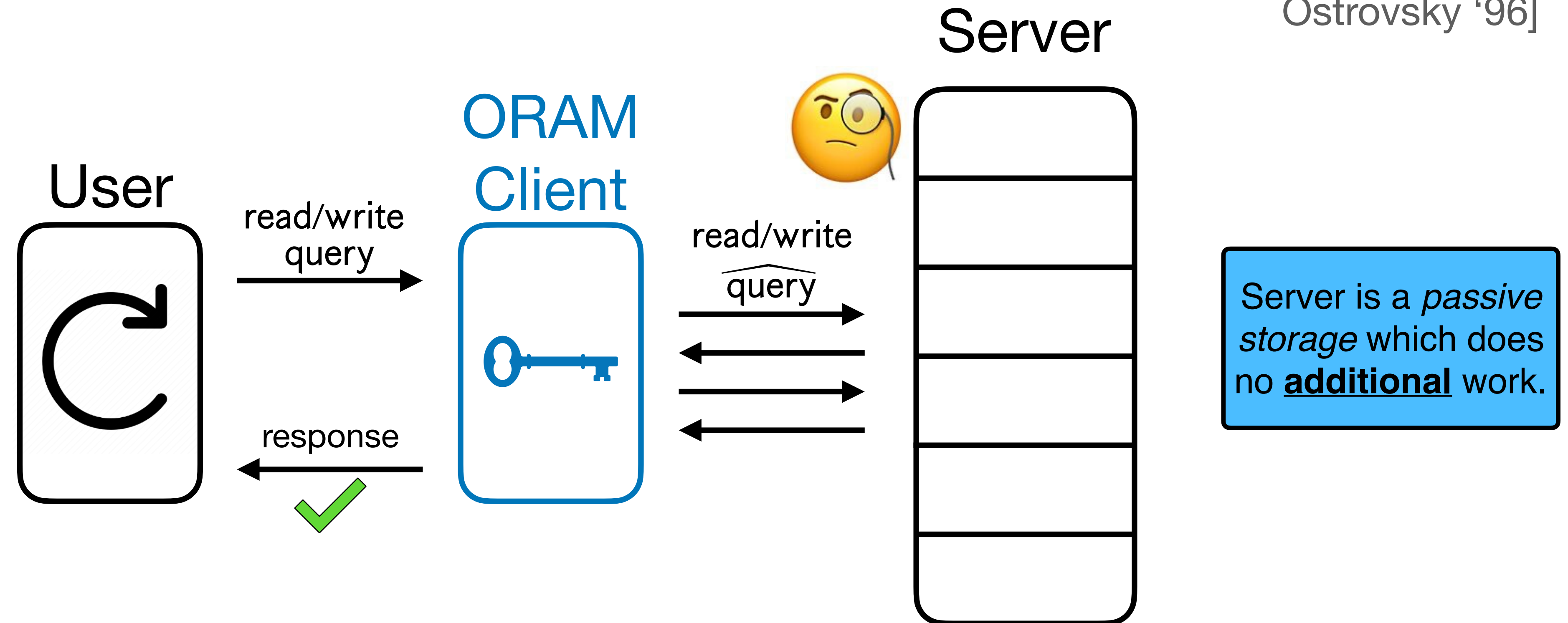
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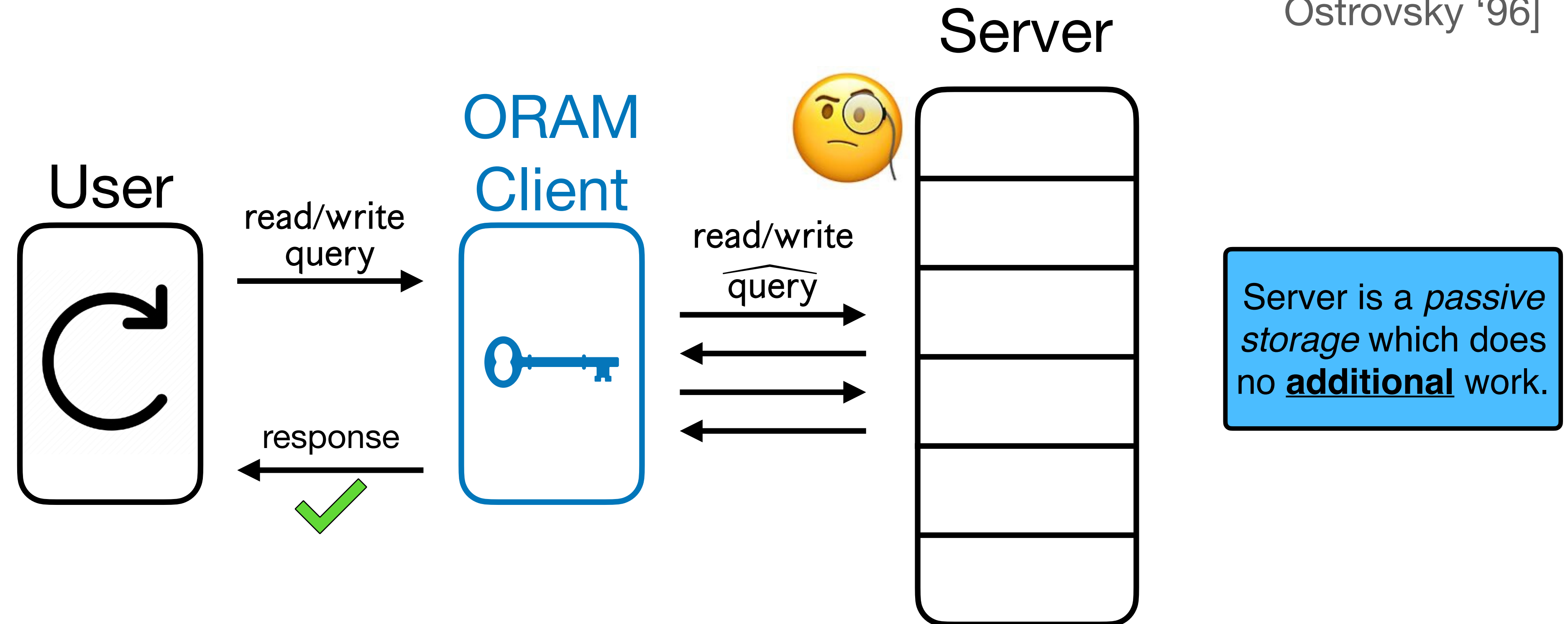
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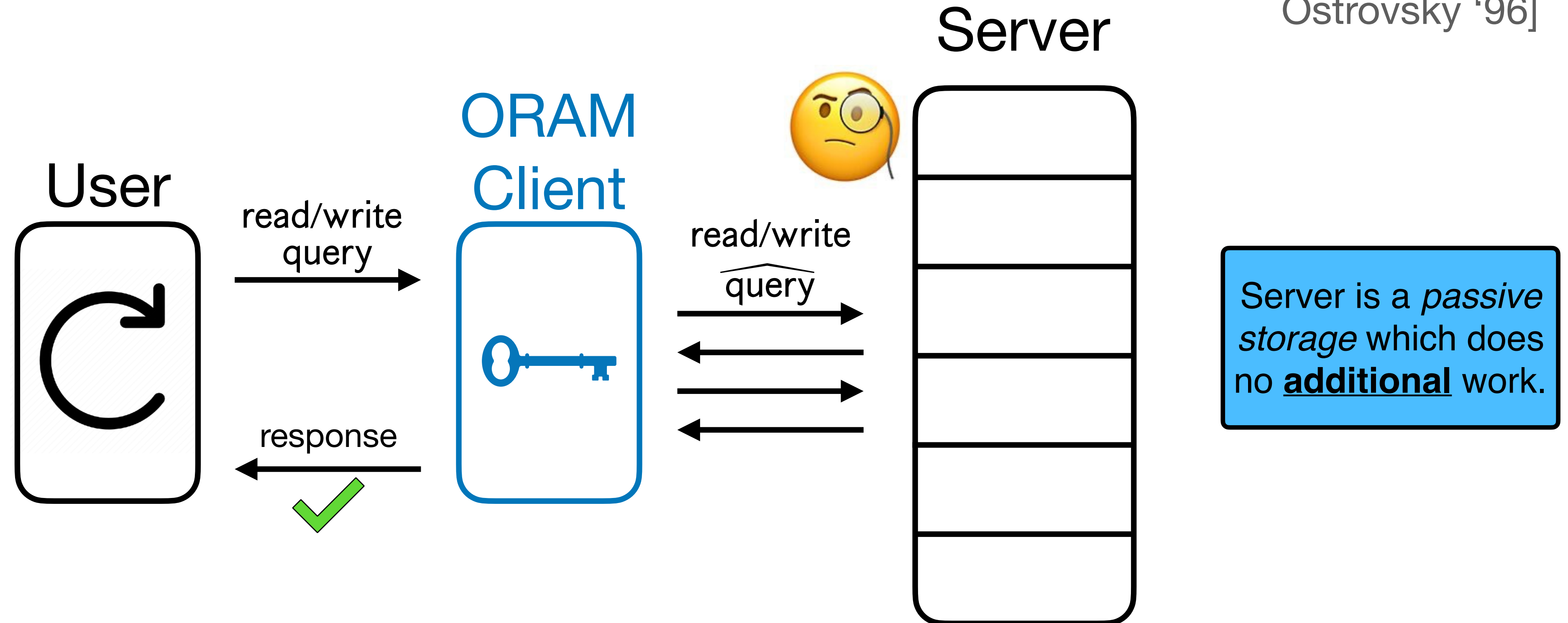
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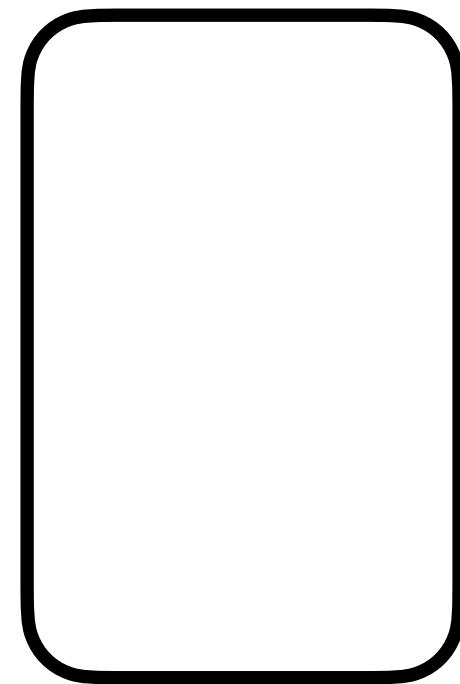
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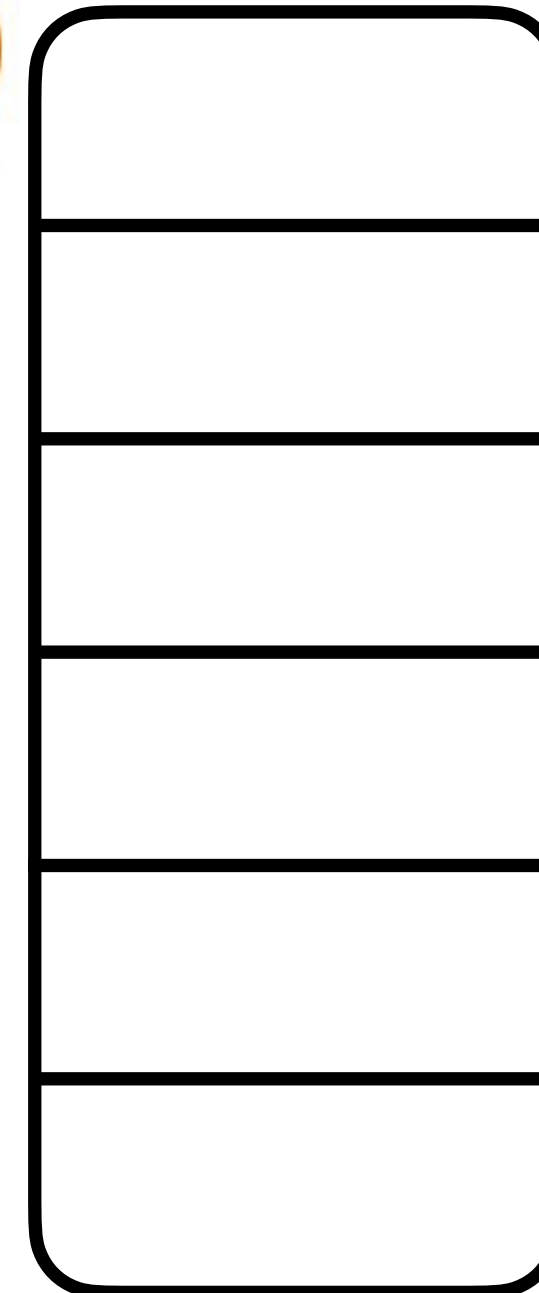
# Application: File Storage Platforms



User



Server



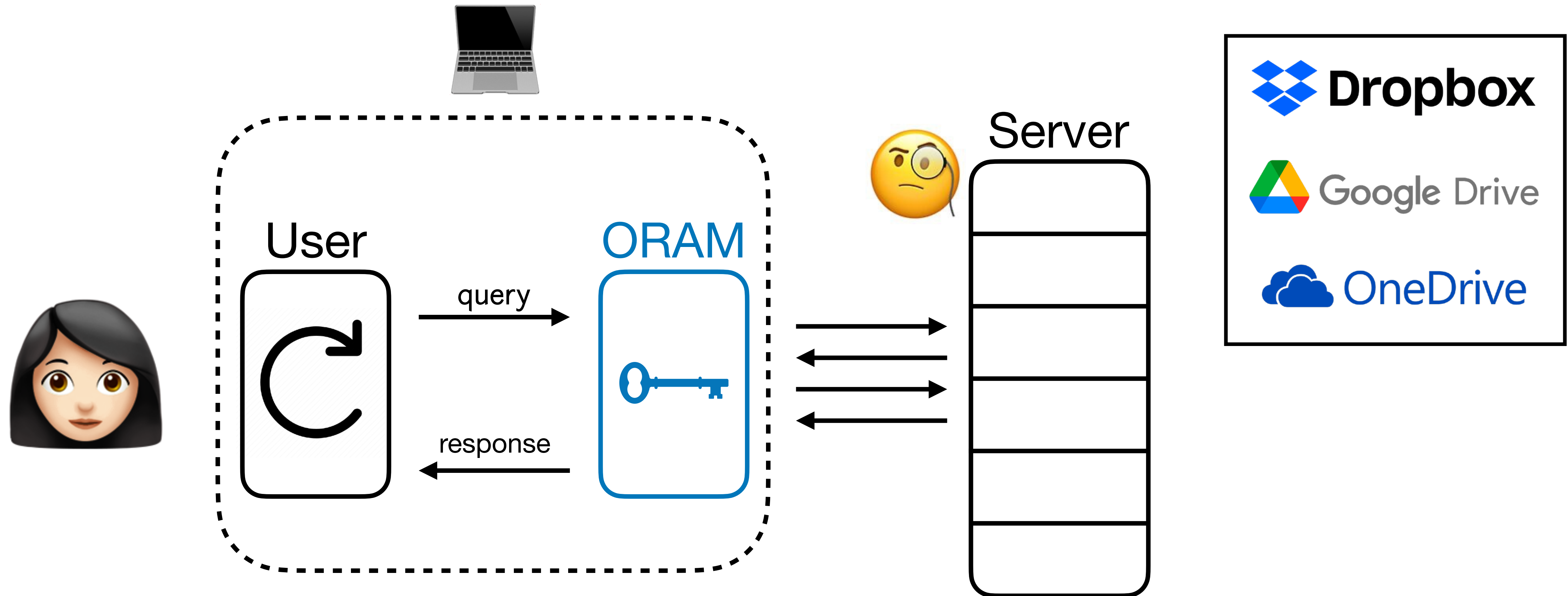
 **Dropbox**

 Google Drive

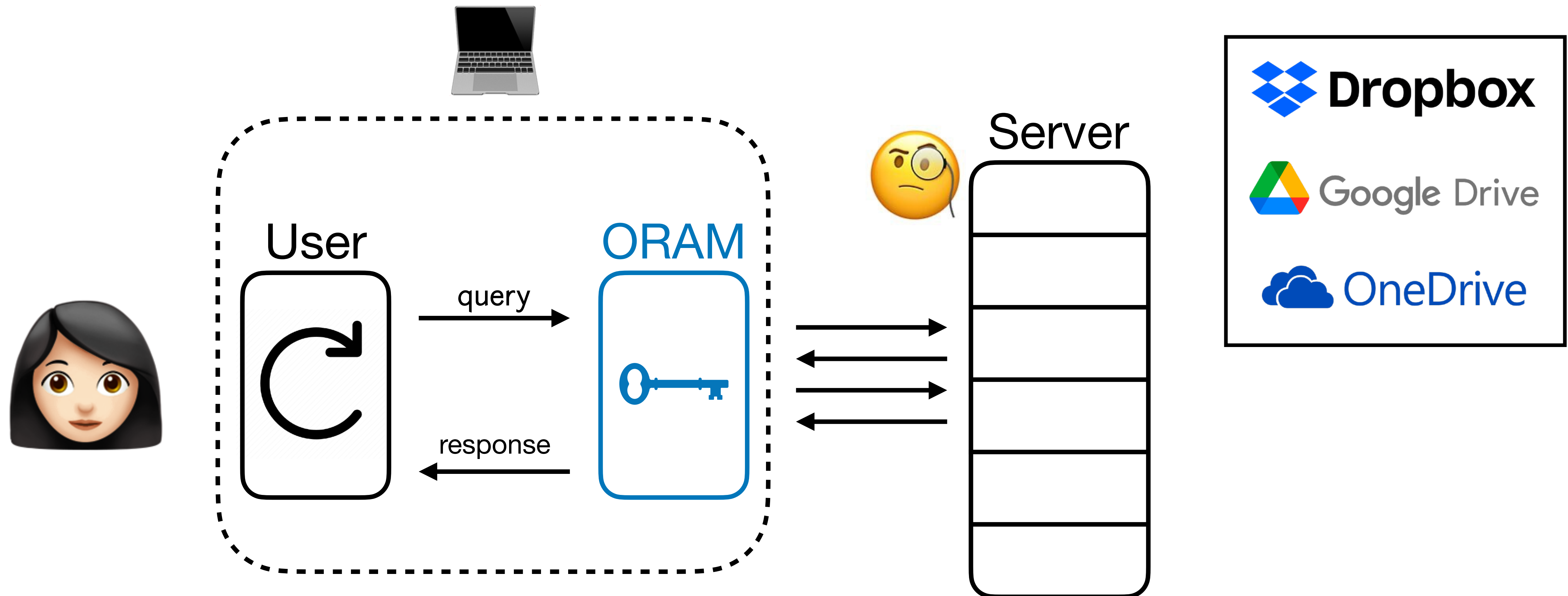
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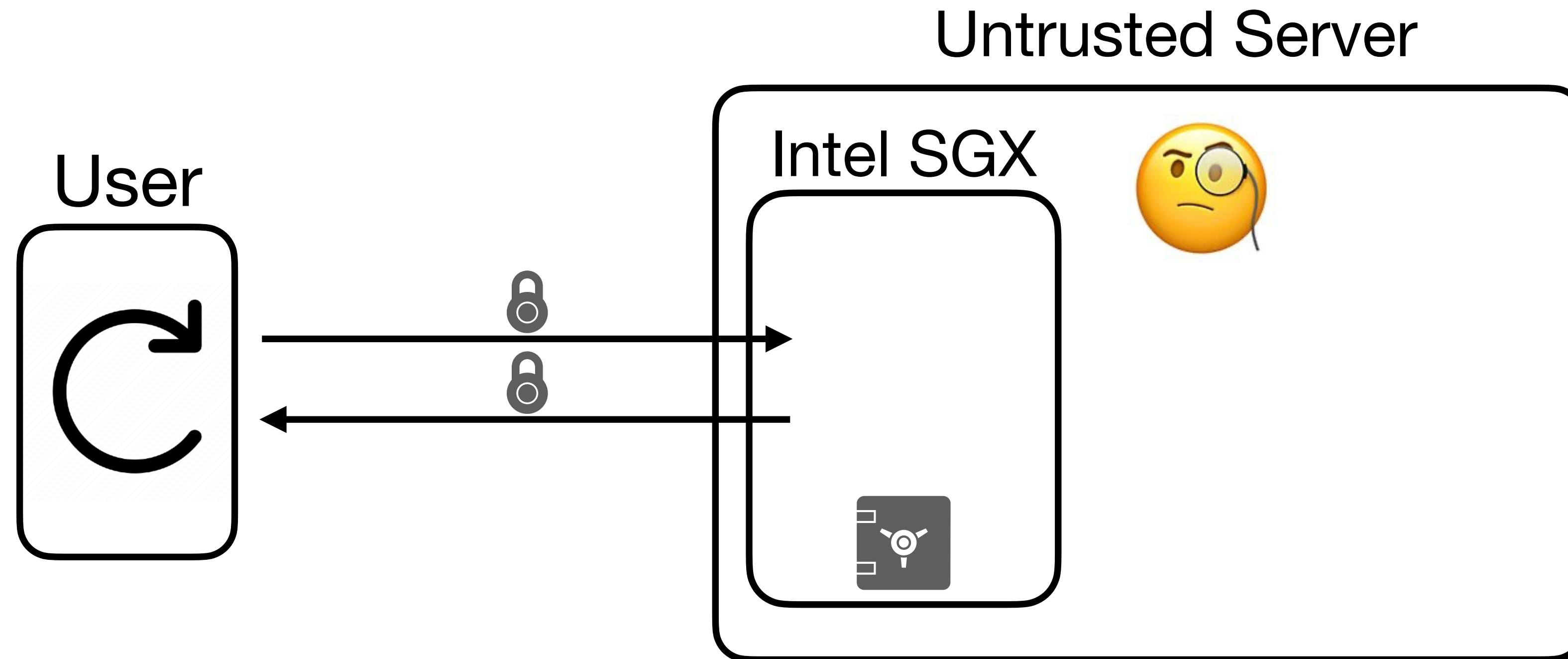


With ORAM, storage platform can't learn anything.



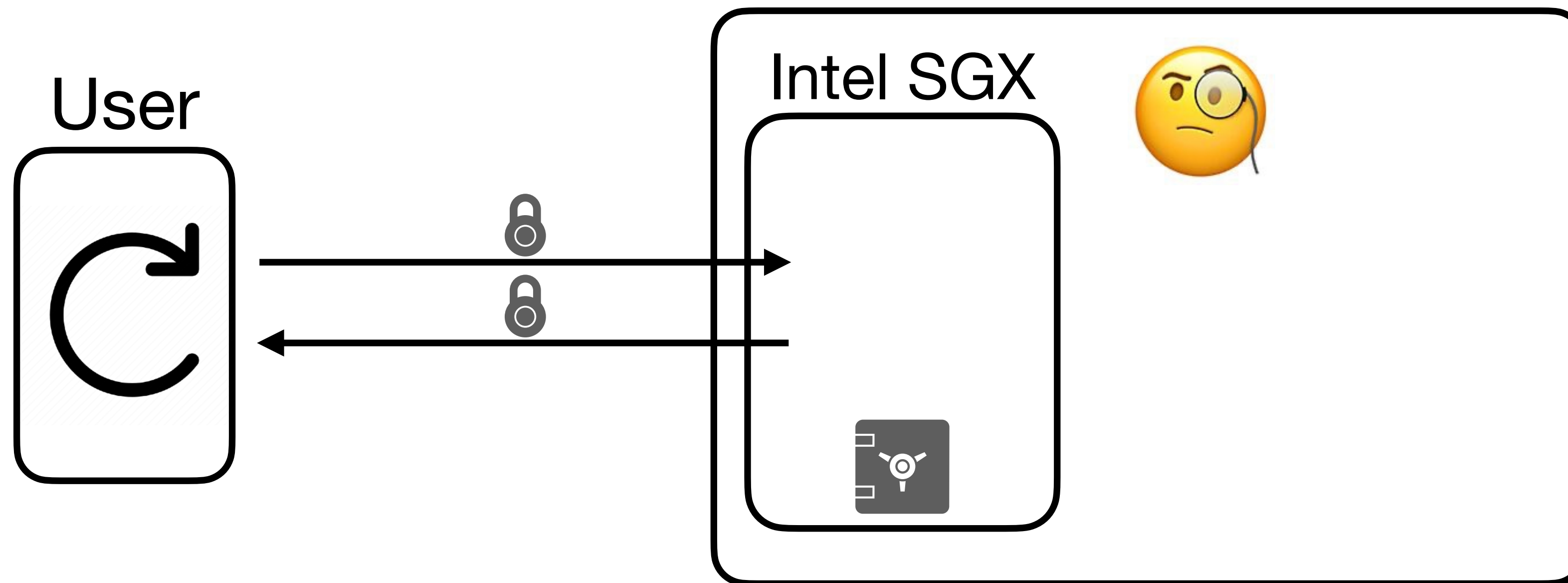
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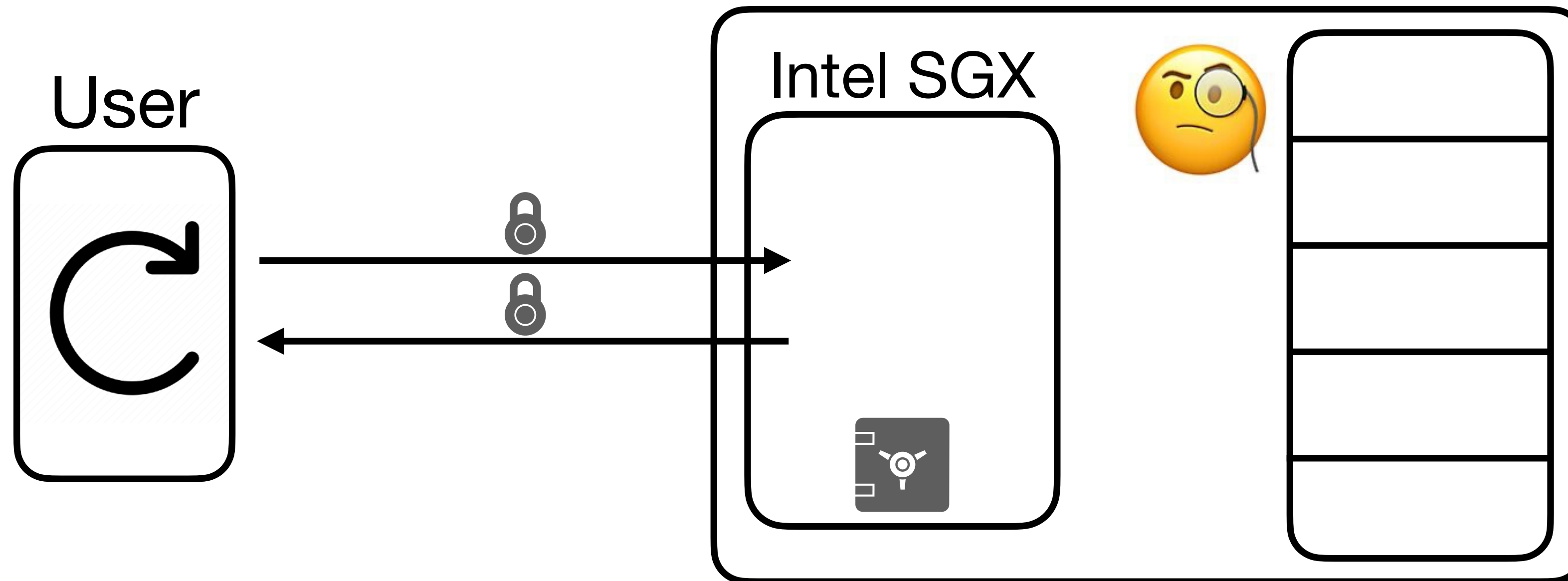
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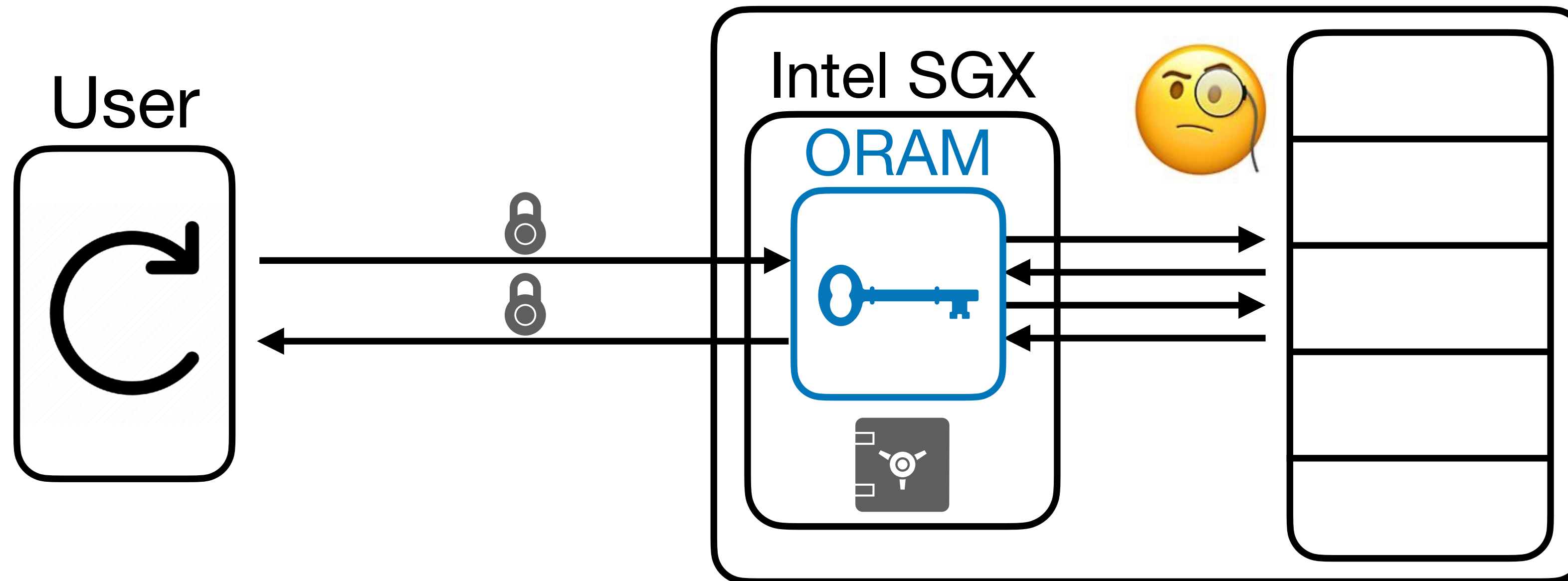
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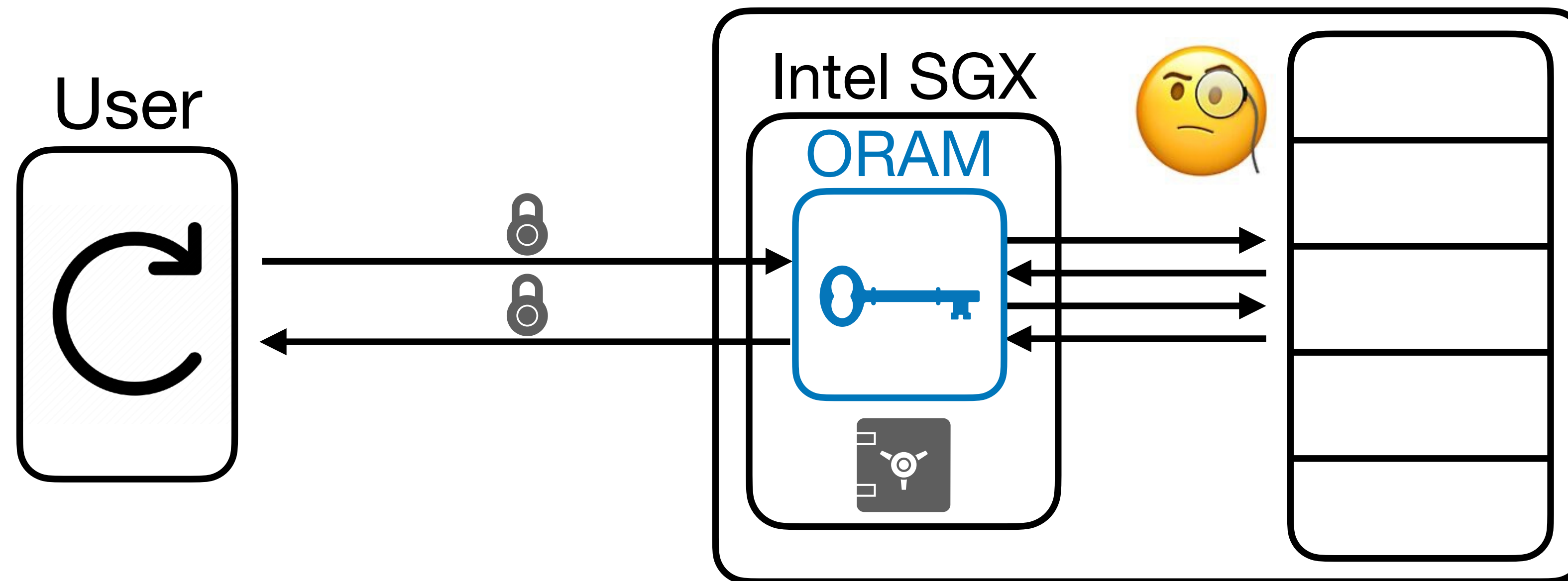
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- **Real World:** Signal very recently implemented ORAM for private contact discovery!



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  - PIR (usually) not stateful, and is typically **read-only** (not updatable).

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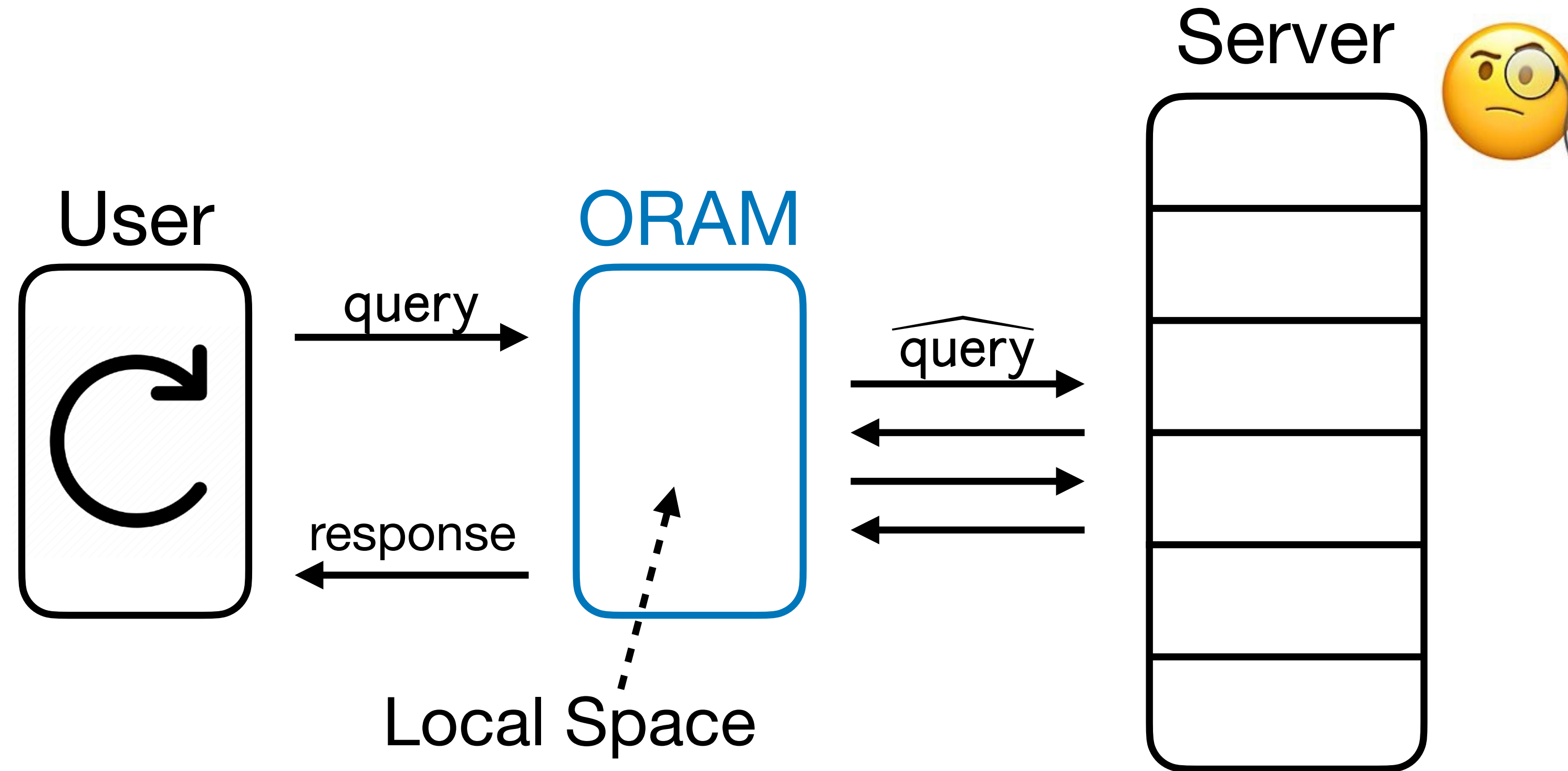
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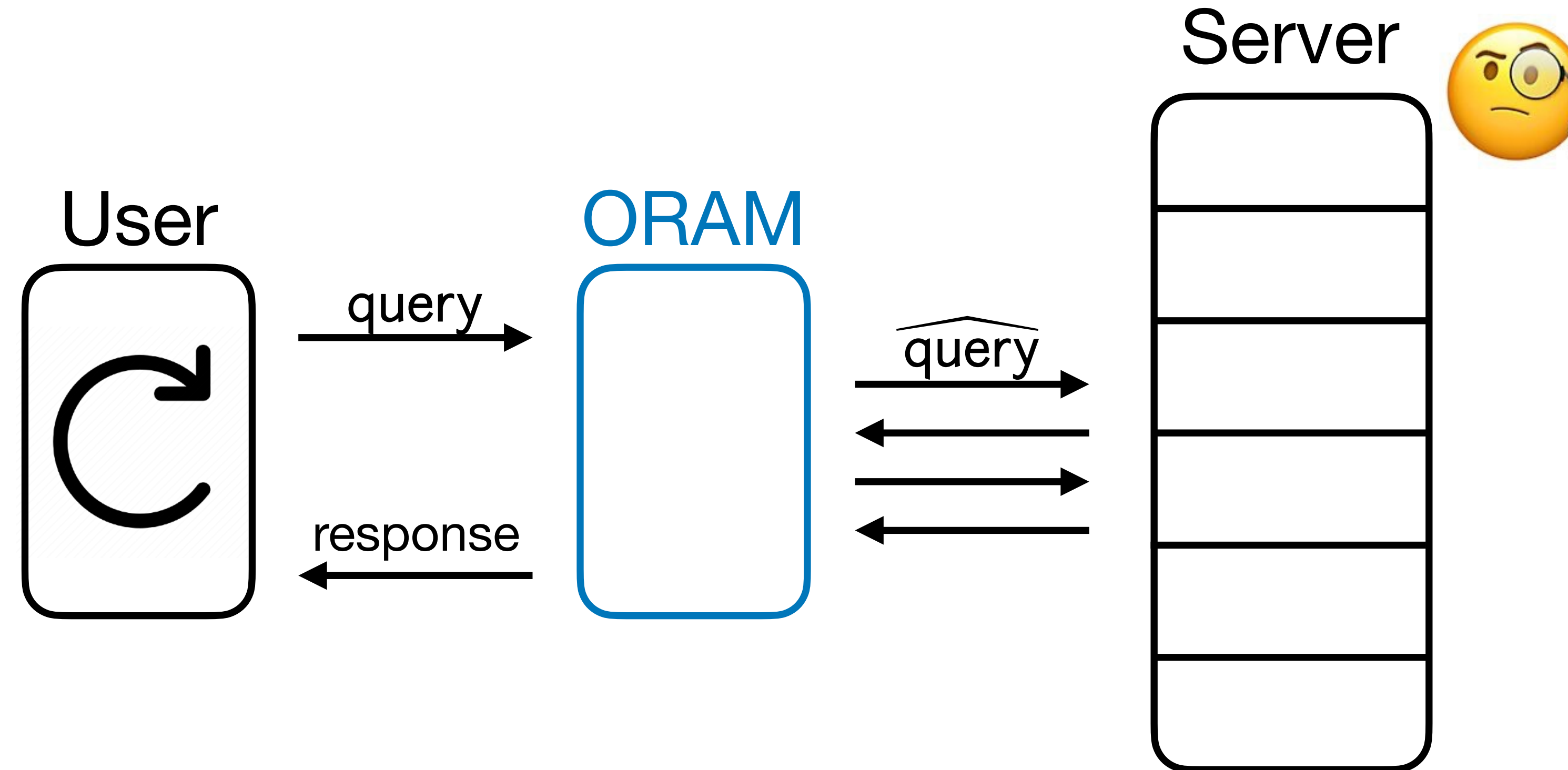
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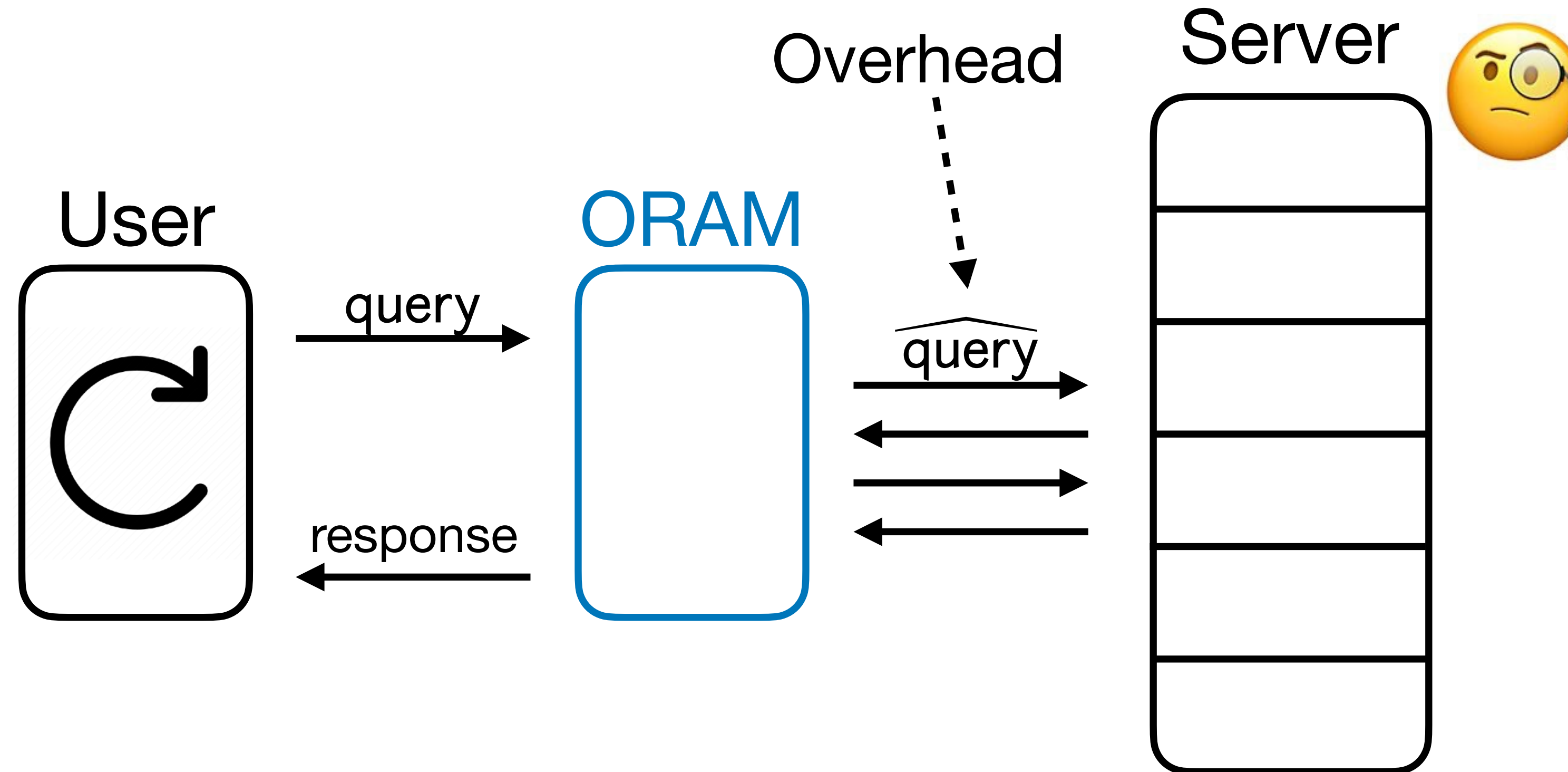


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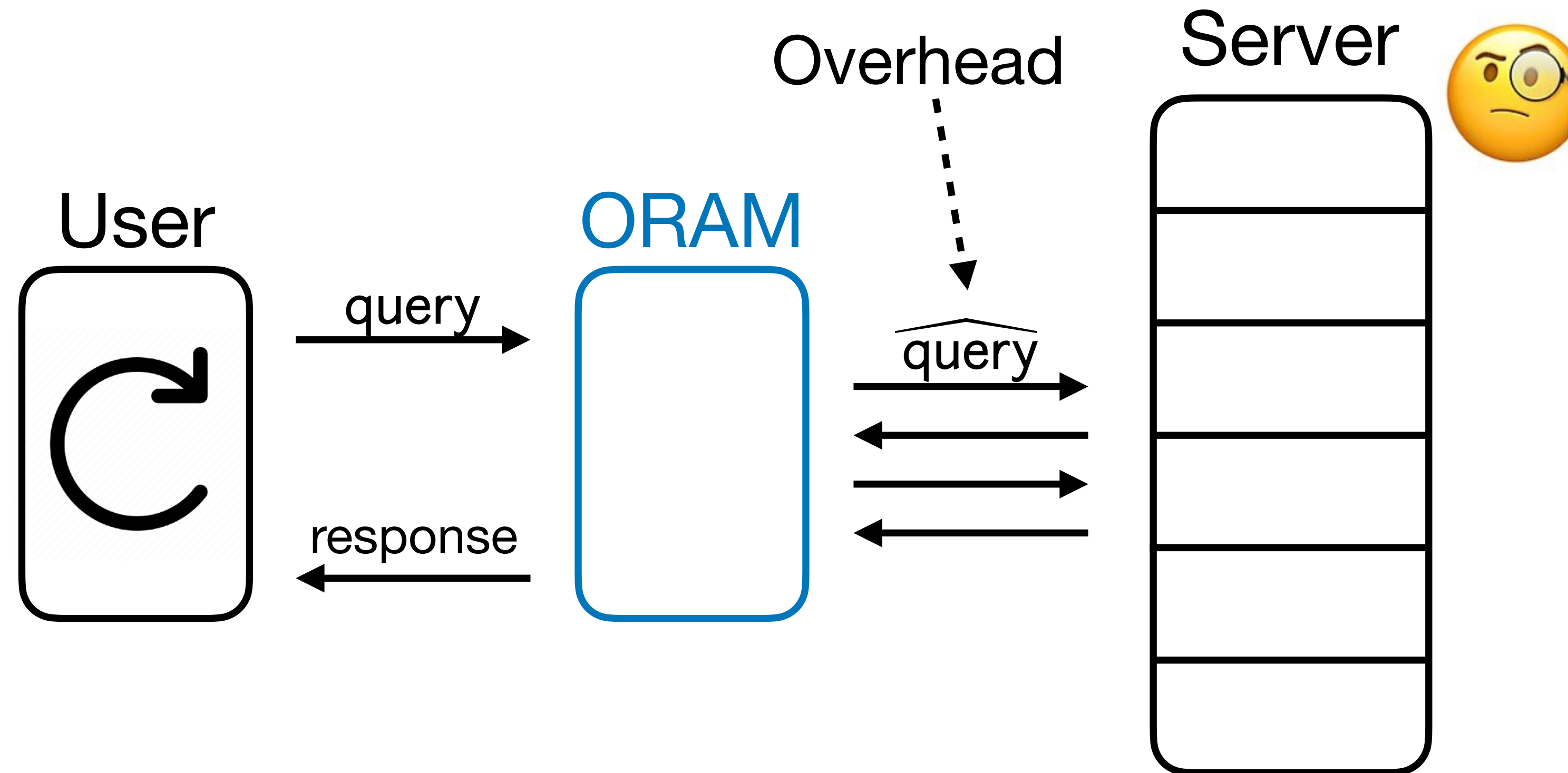
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Work	Overhead
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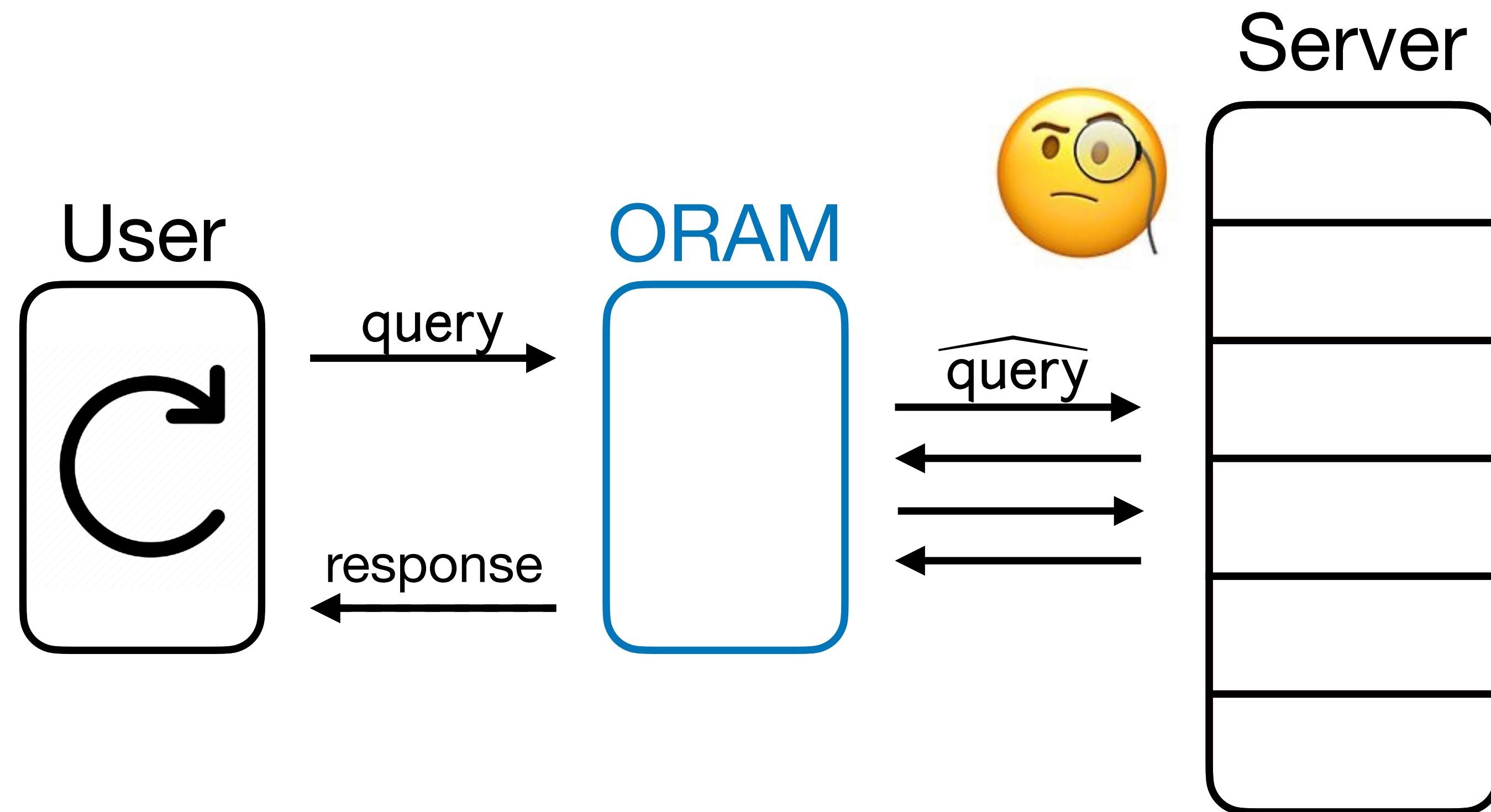
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Optimal!

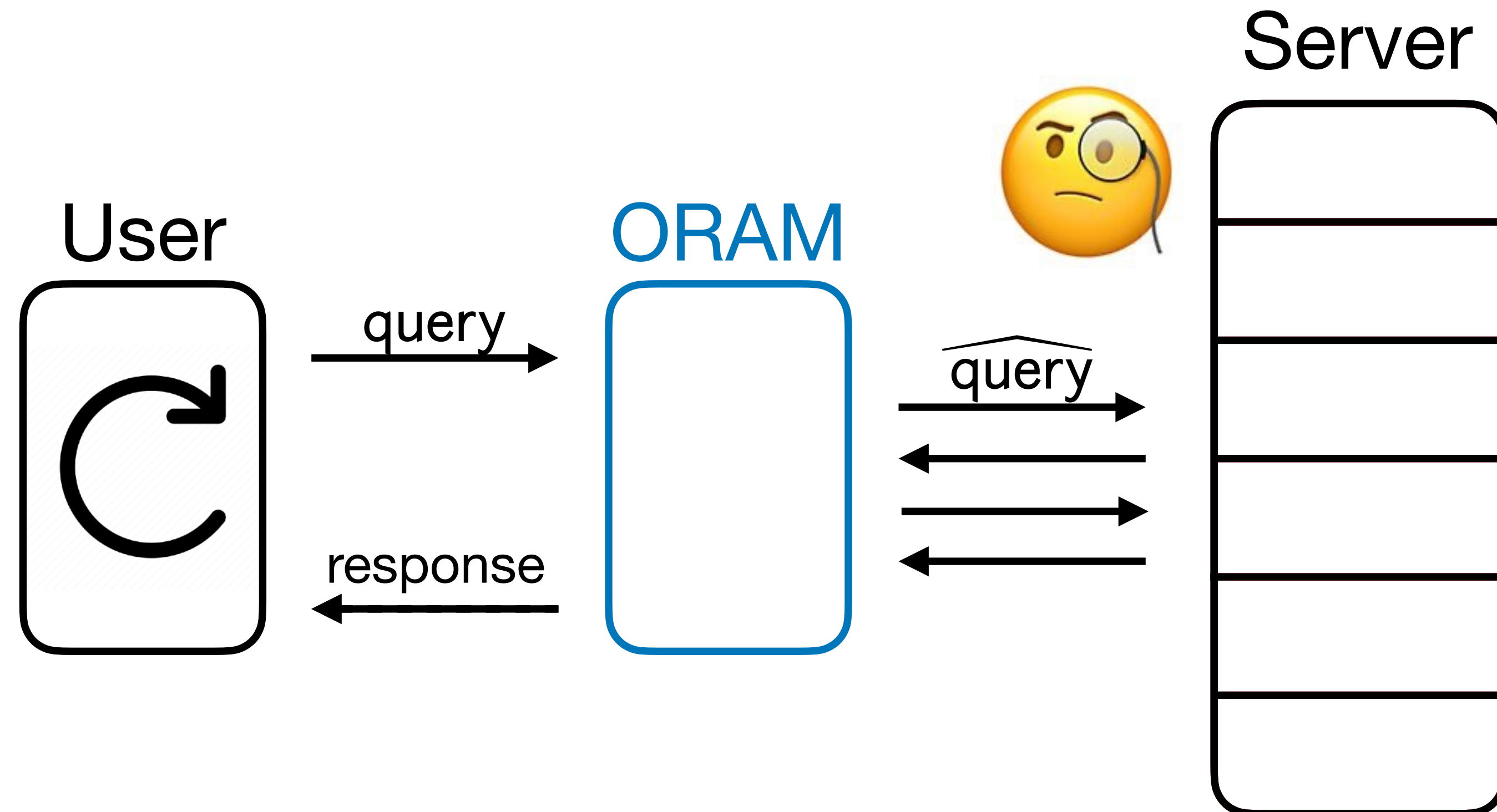
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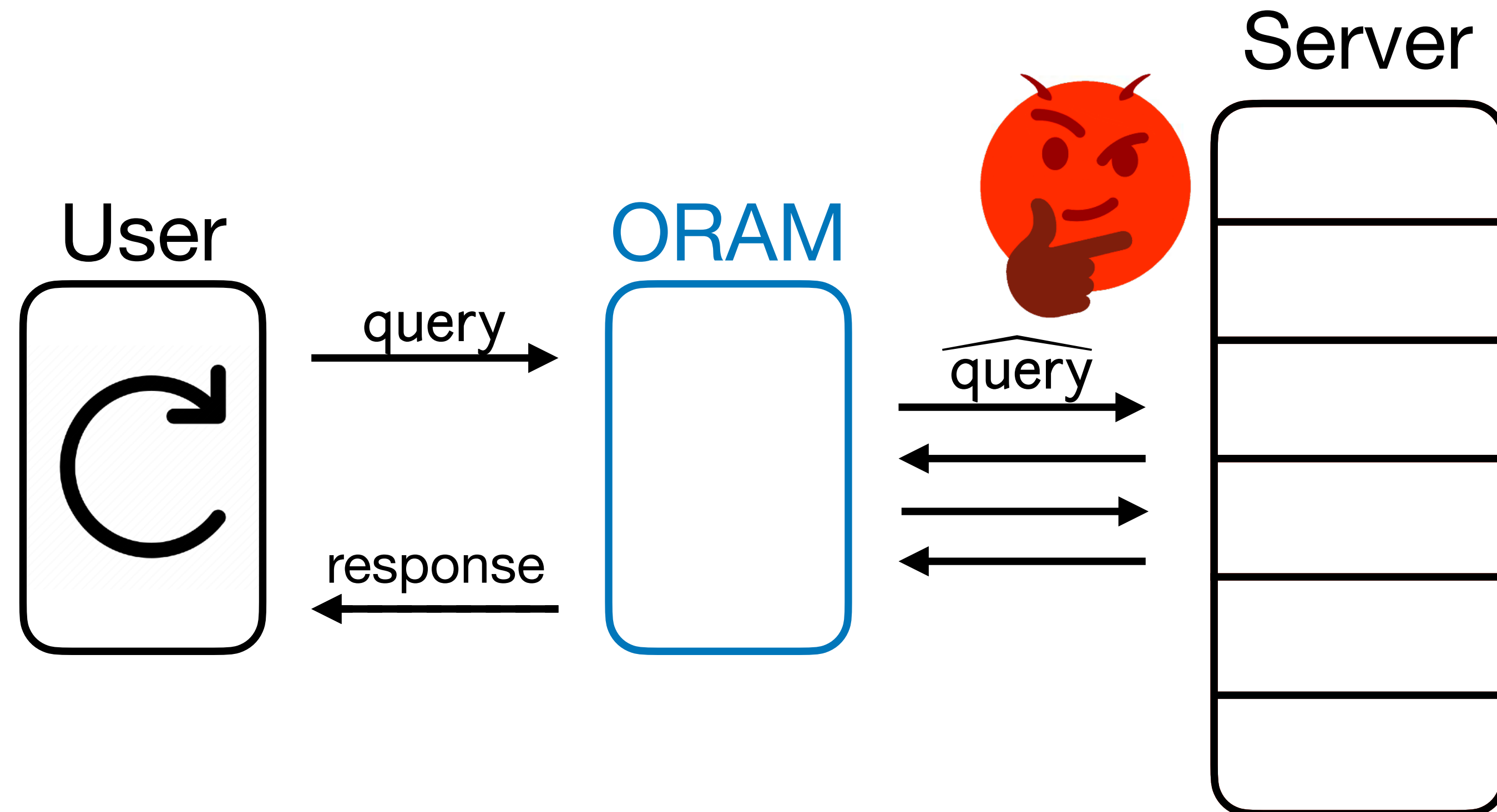
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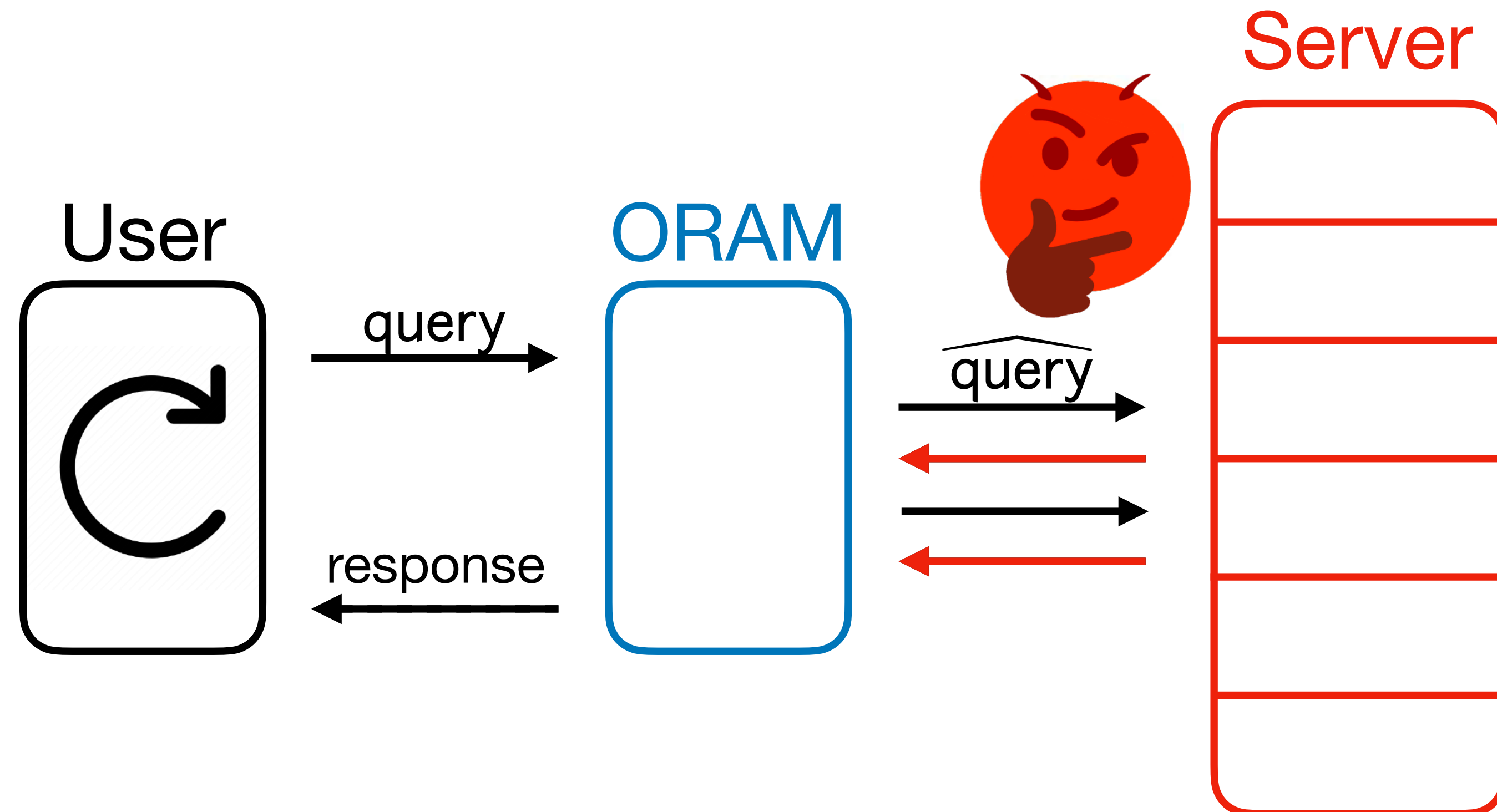
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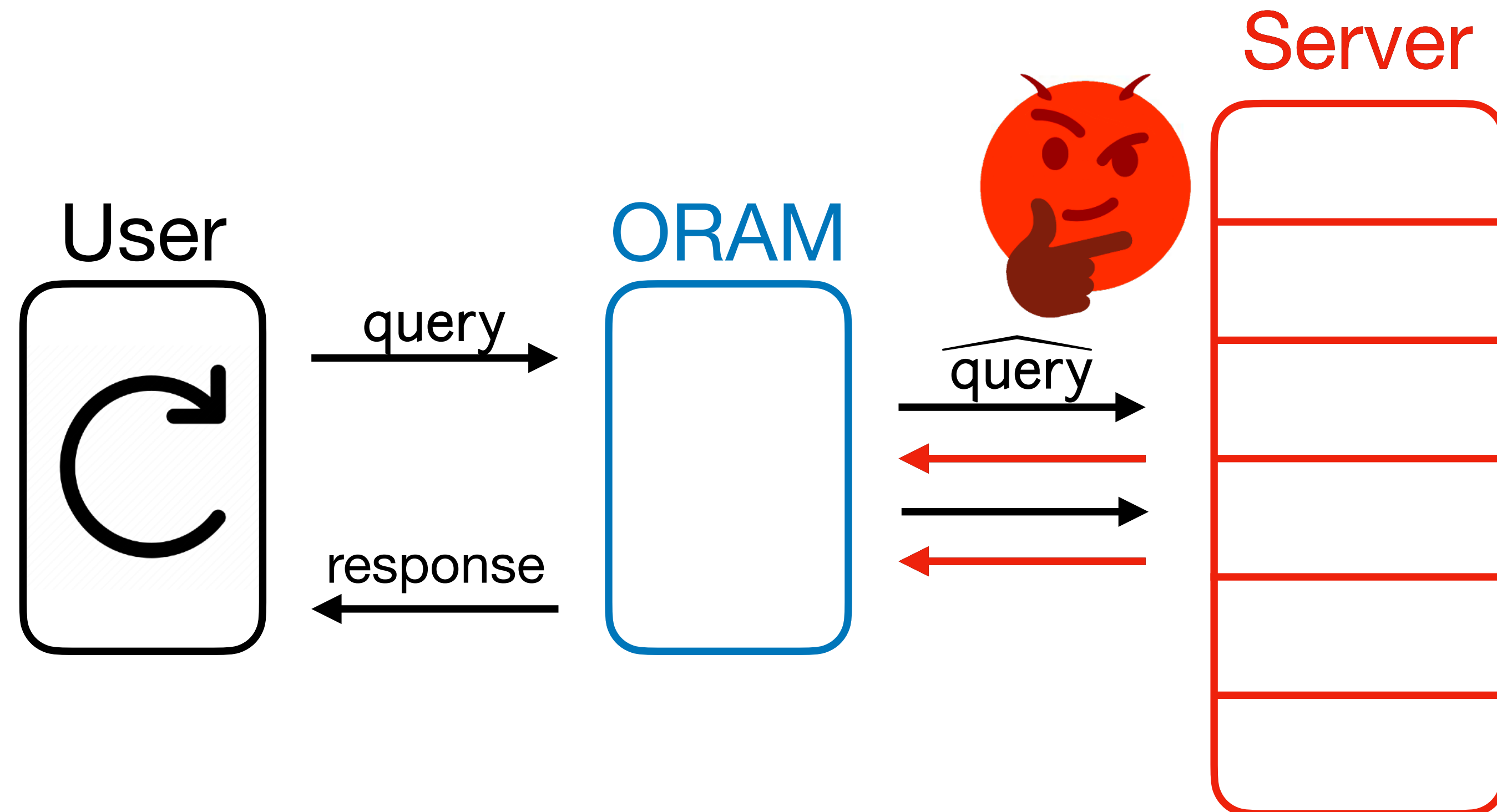
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# Malicious Security

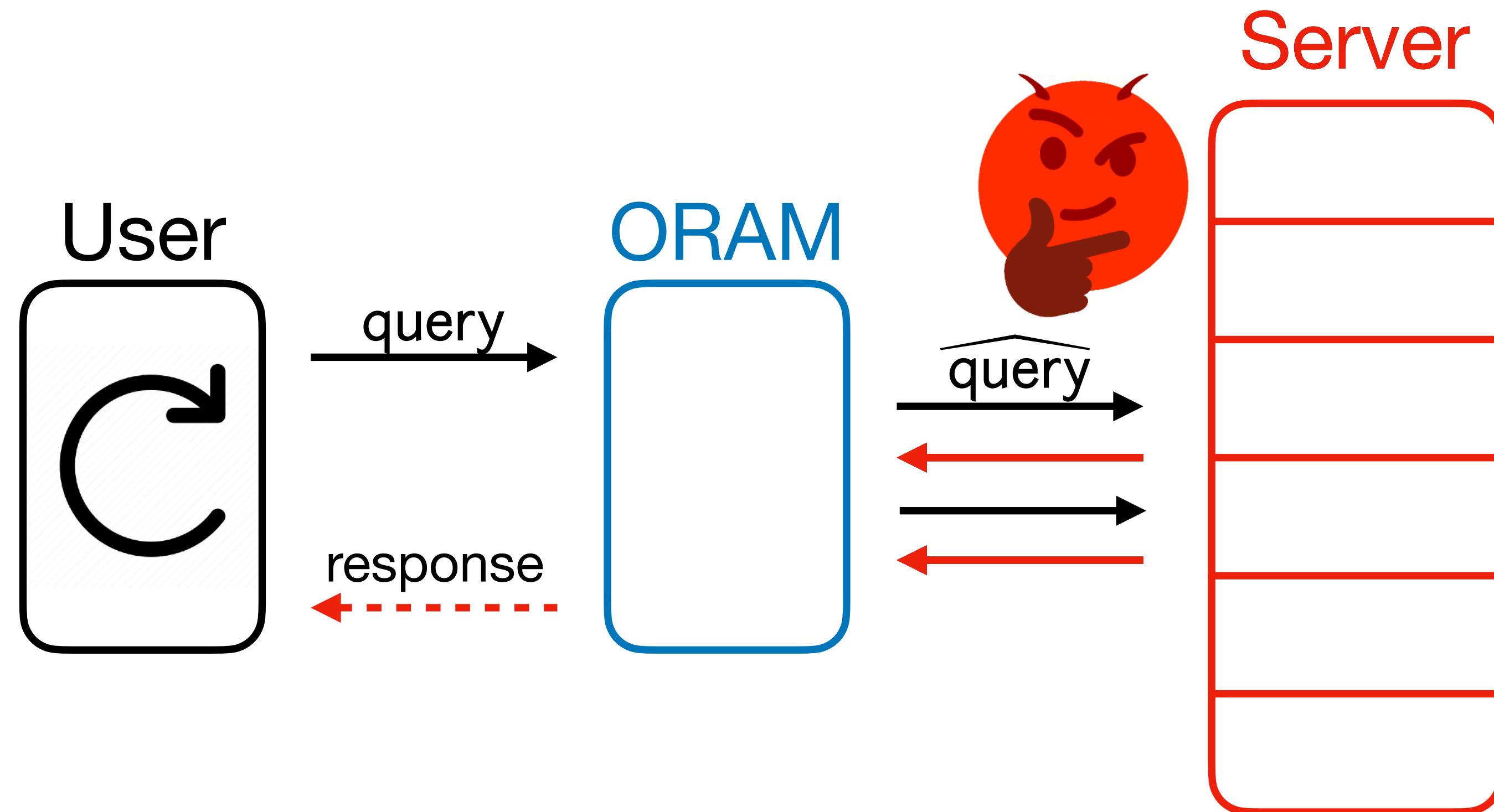
- A malicious server breaks correctness





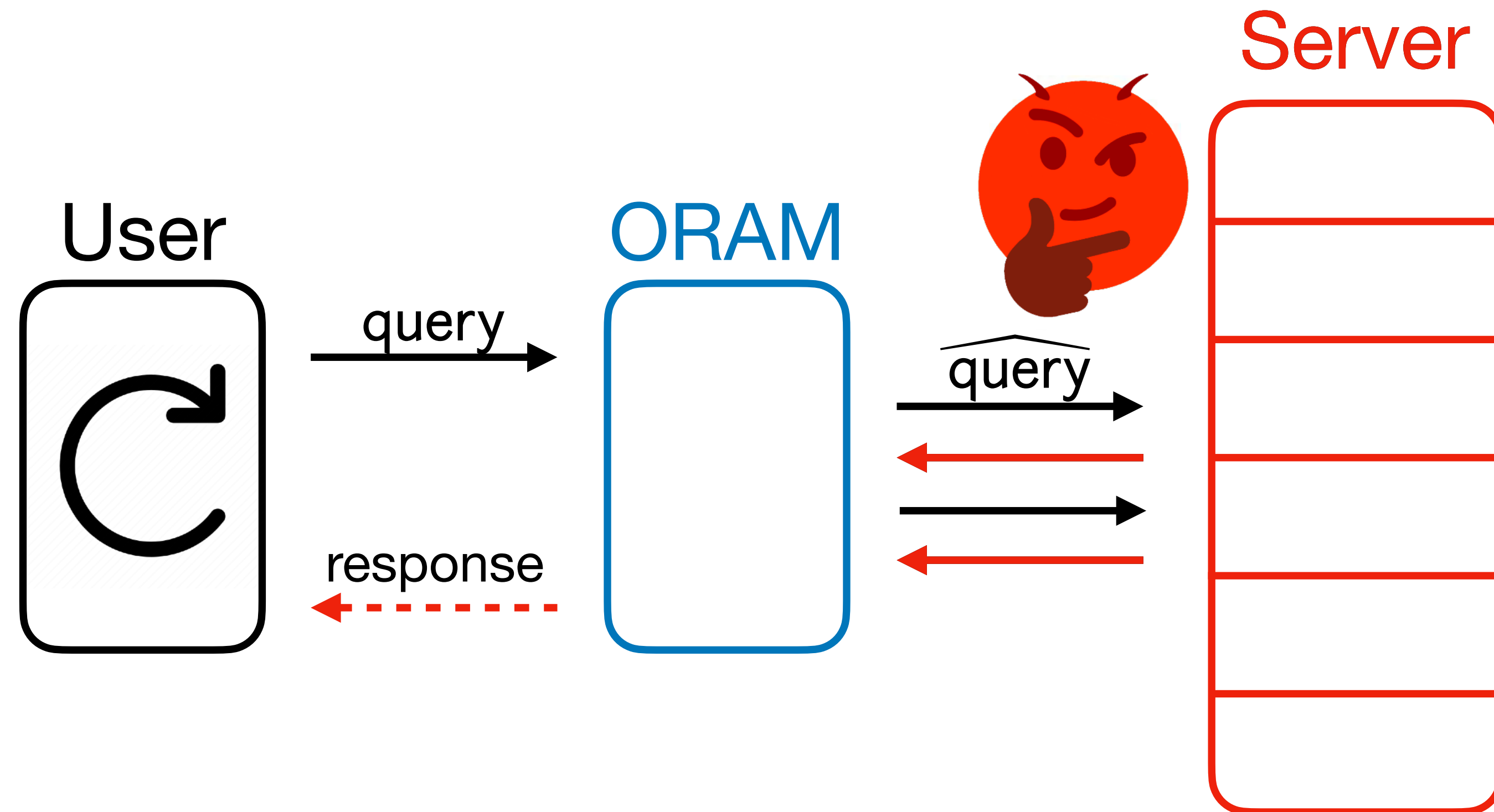
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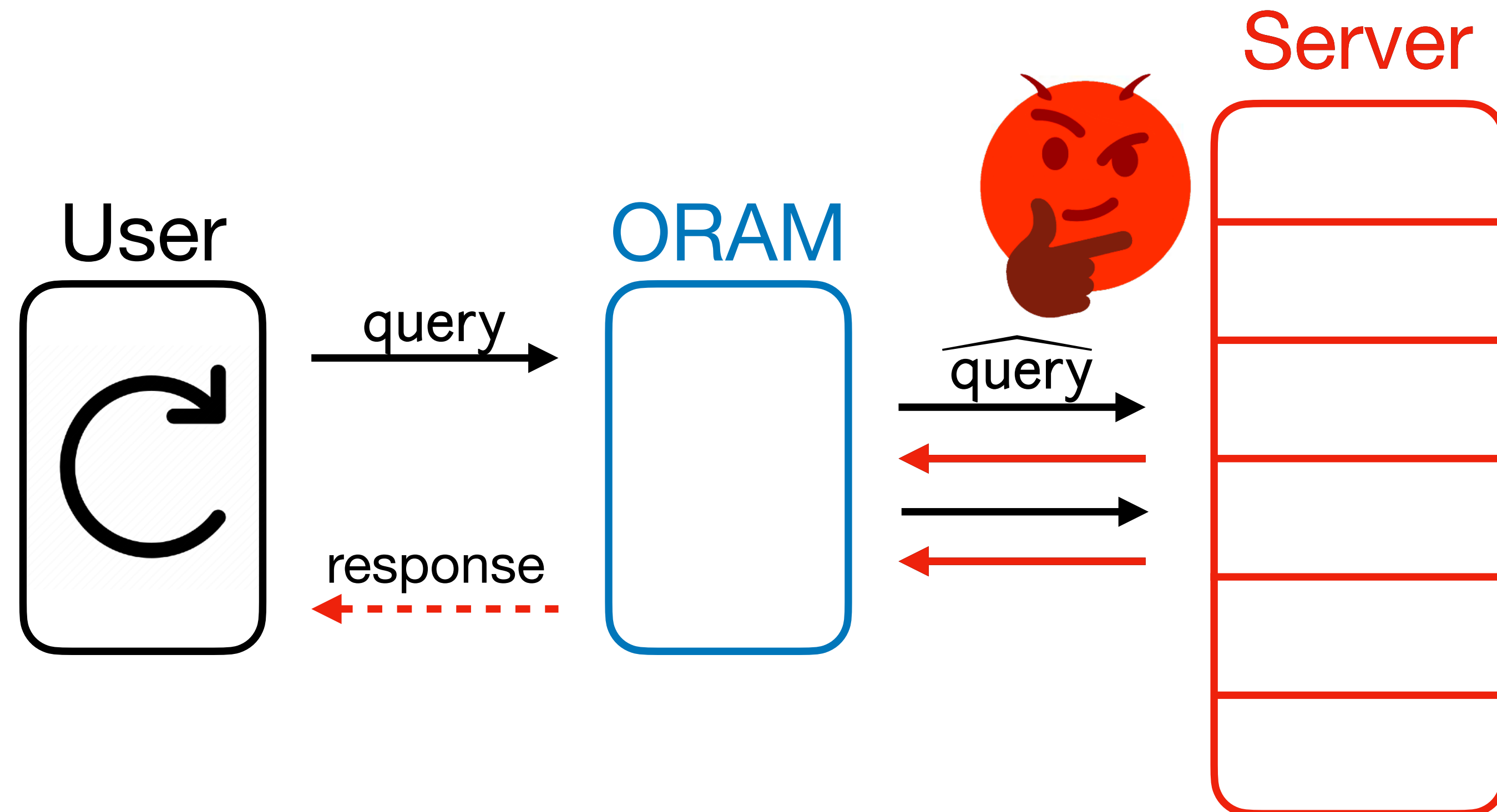
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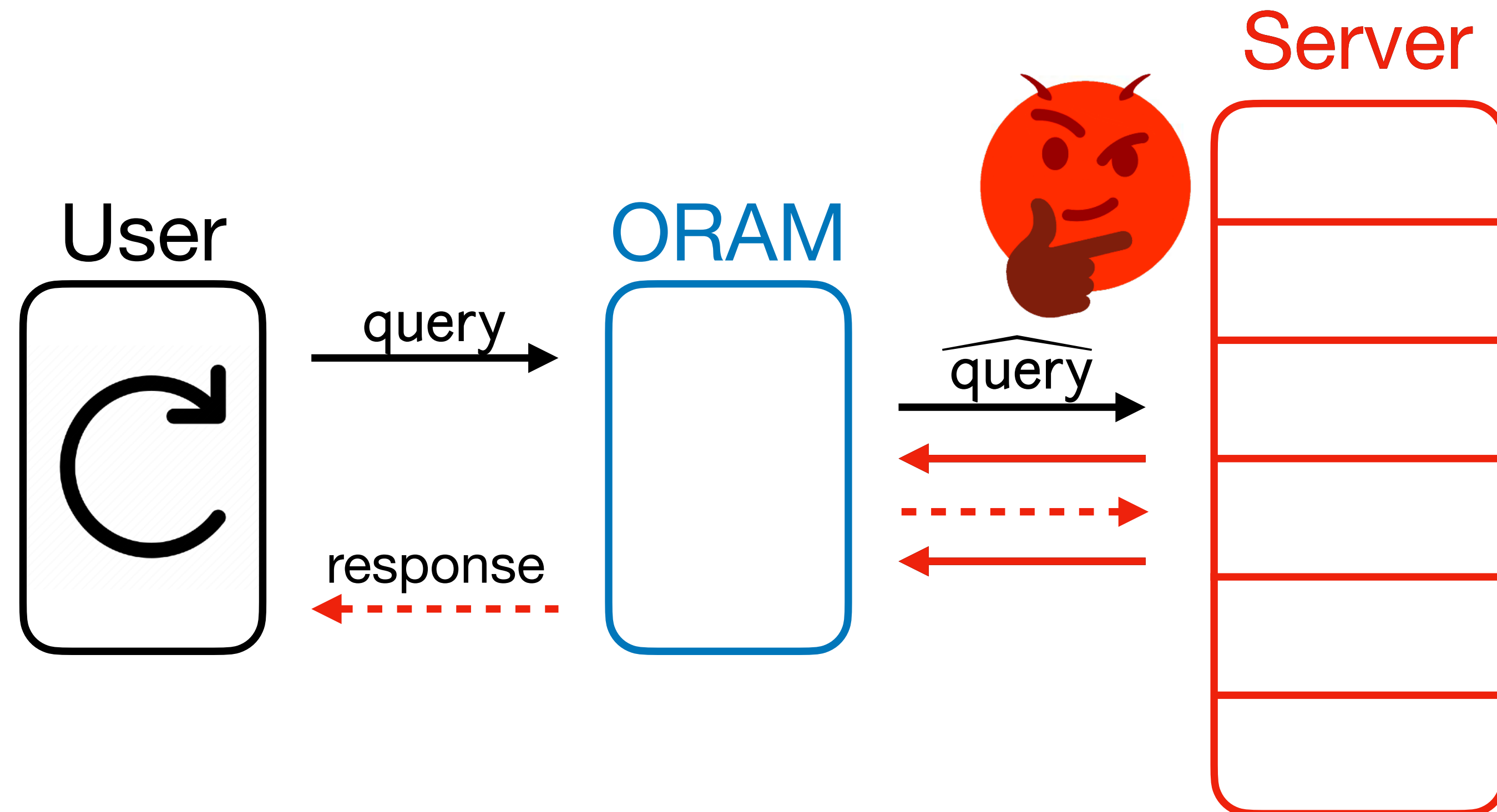
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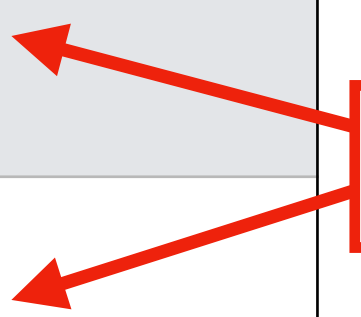


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Assuming one-way functions, we construct **MacORAMa**, a maliciously secure ORAM with  $O(\log N)$  overhead and  $O(1)$  local space\*.

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- OWFs are also *necessary* for maliciously secure ORAM. [Naor, Rothblum '05]
- In private random oracle model, we get *statistical* malicious security against *unbounded adversaries*.

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# Starting Point

We start with **OptORAMa** [Asharov, Komargodski, Lin, Nayak, Peserico, Shi] - a **honest-but-curious** ORAM with **optimal**  $O(\log N)$  overhead.

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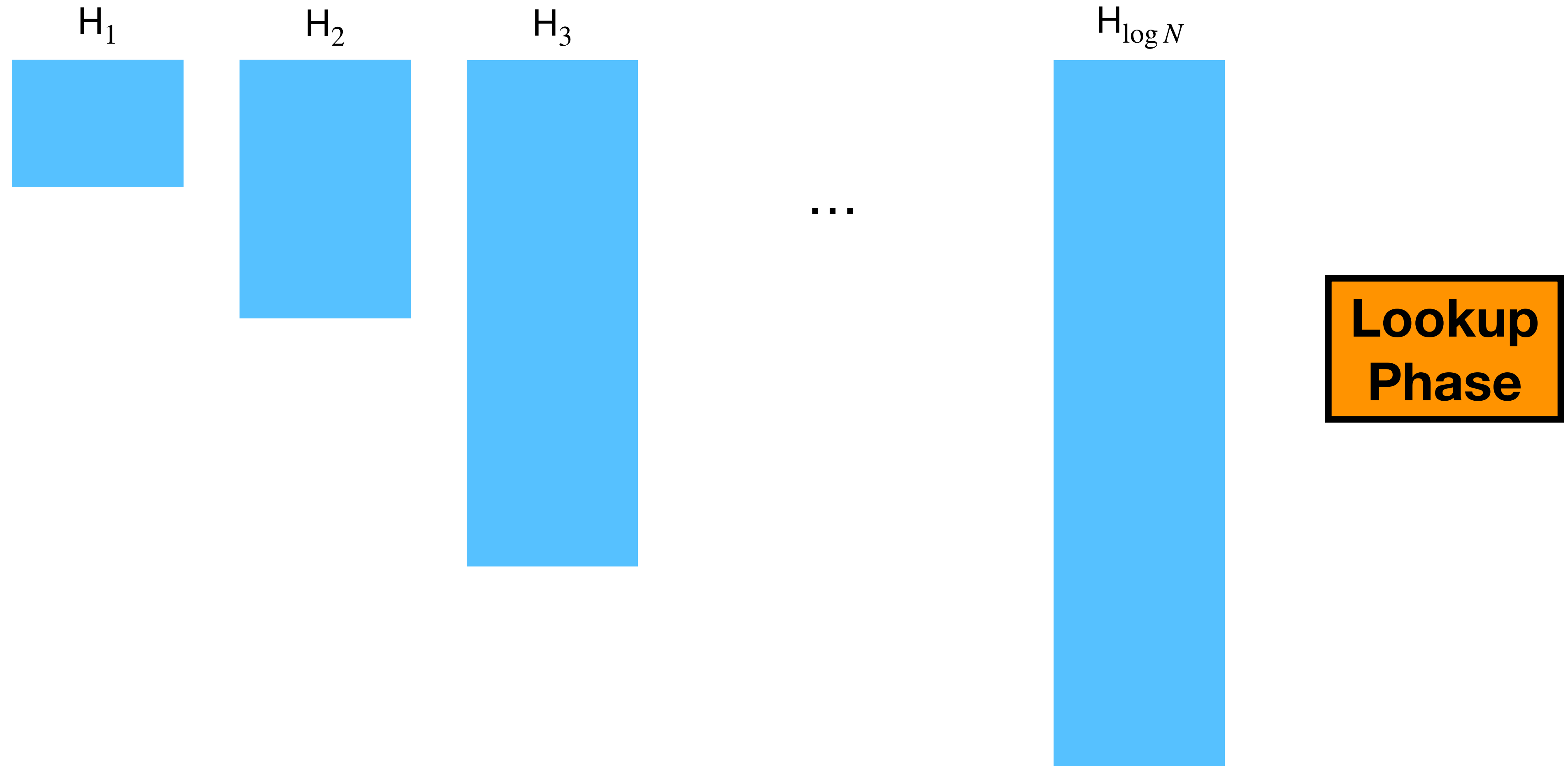
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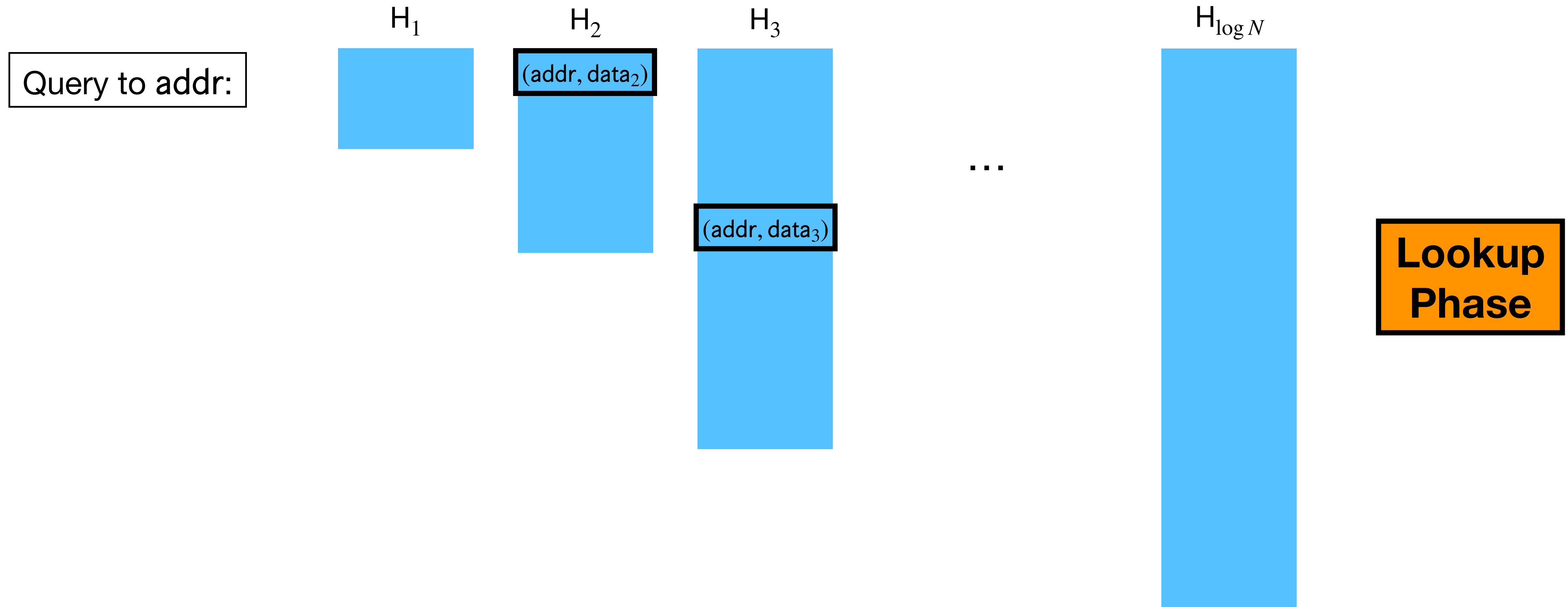
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  - **Rebuild Phase:** Every  $2^i$  queries, obliviously merge  $H_1 \rightarrow H_2 \rightarrow \dots \rightarrow H_{i+1}$  into new  $H_{i+1}$ , removing duplicate addresses by keeping the version from the smaller  $H_j$ .

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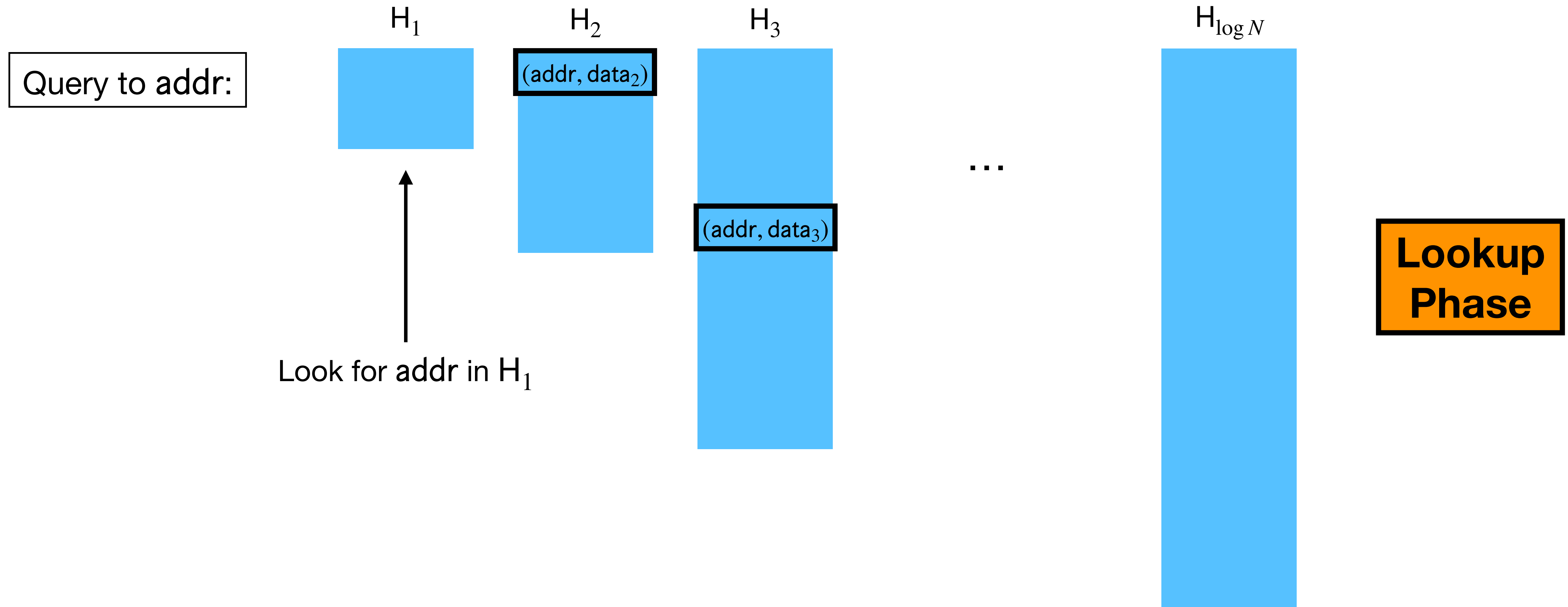




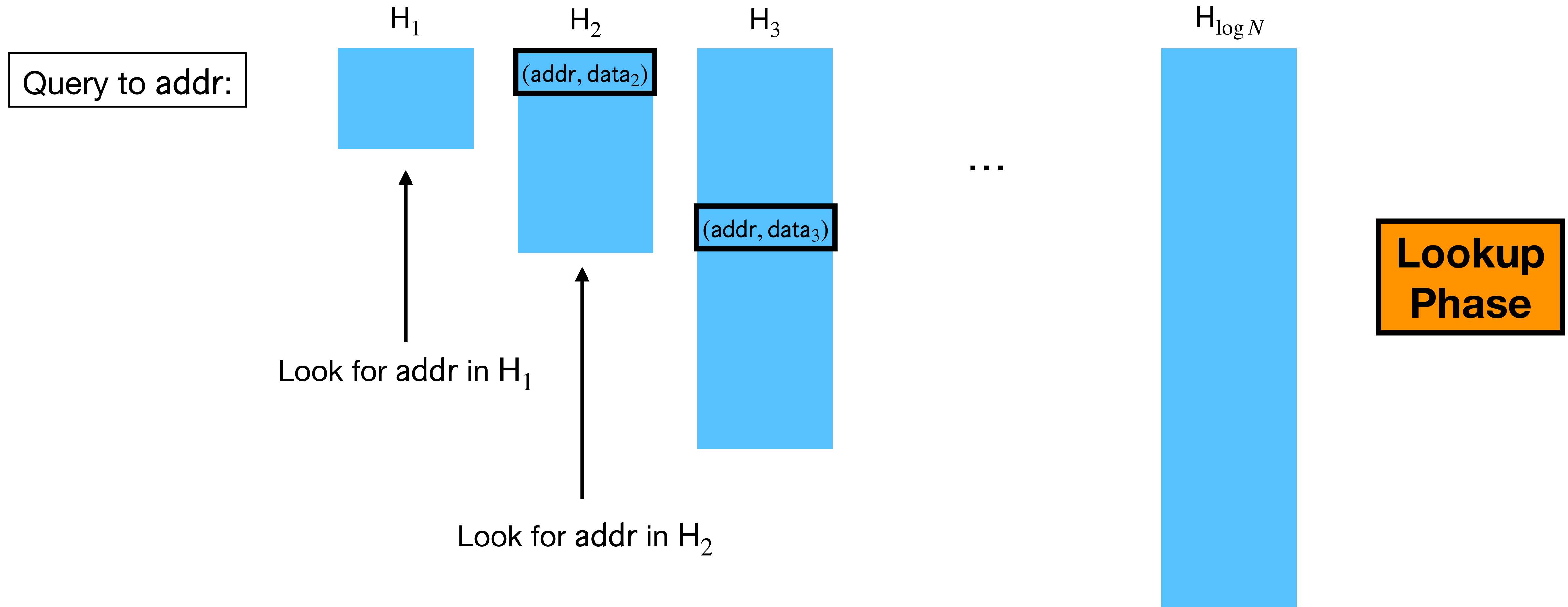
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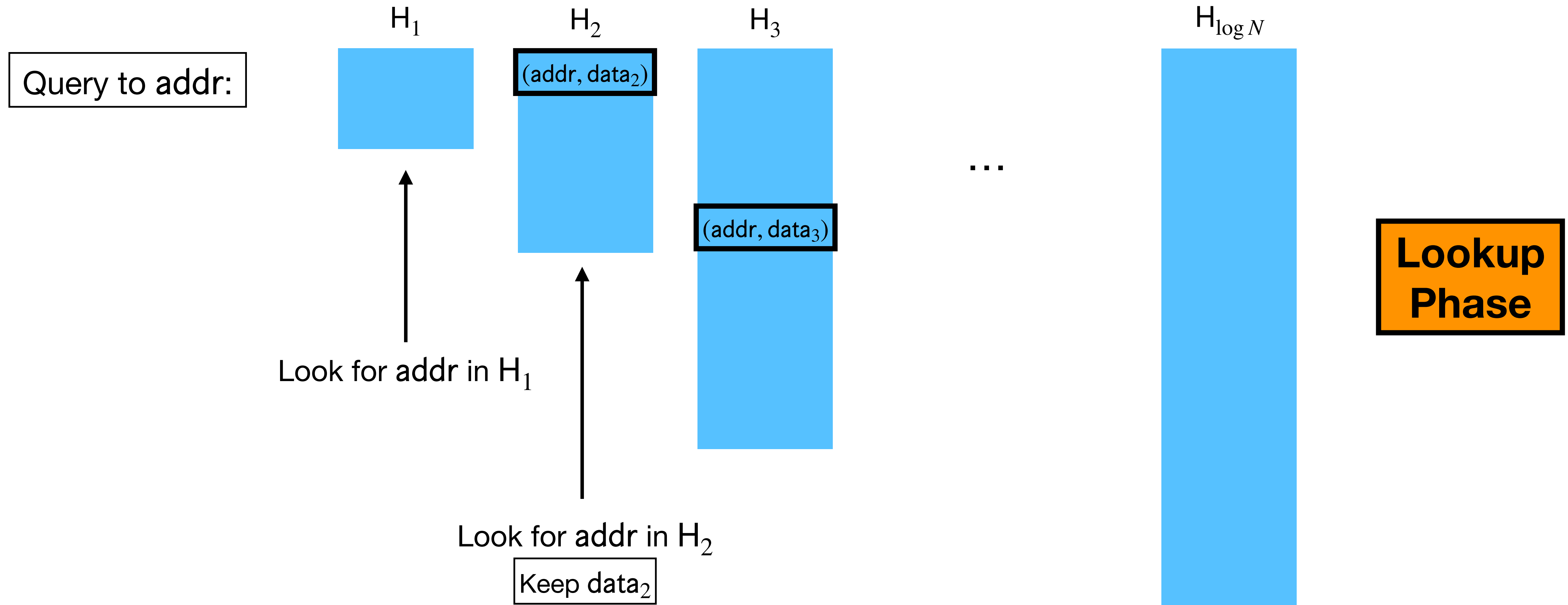
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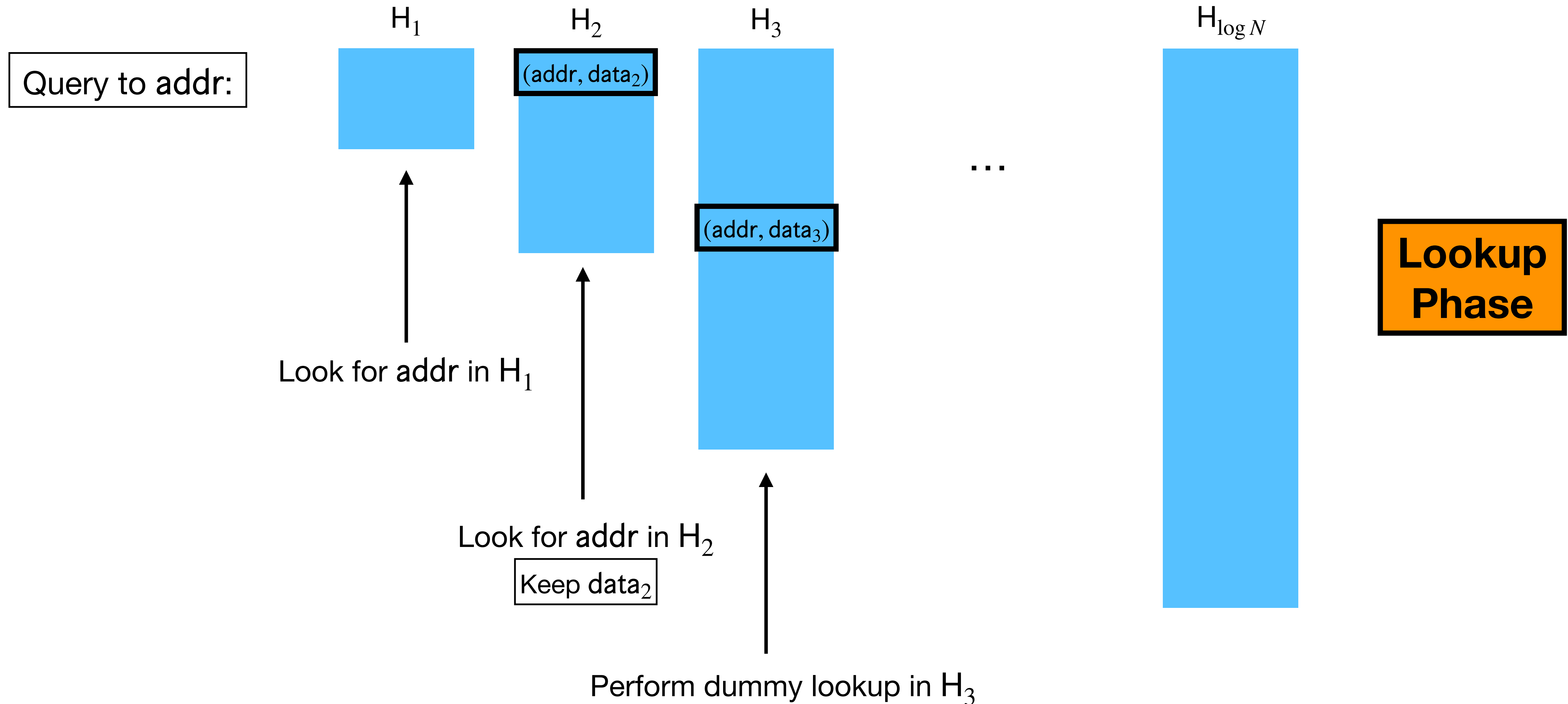
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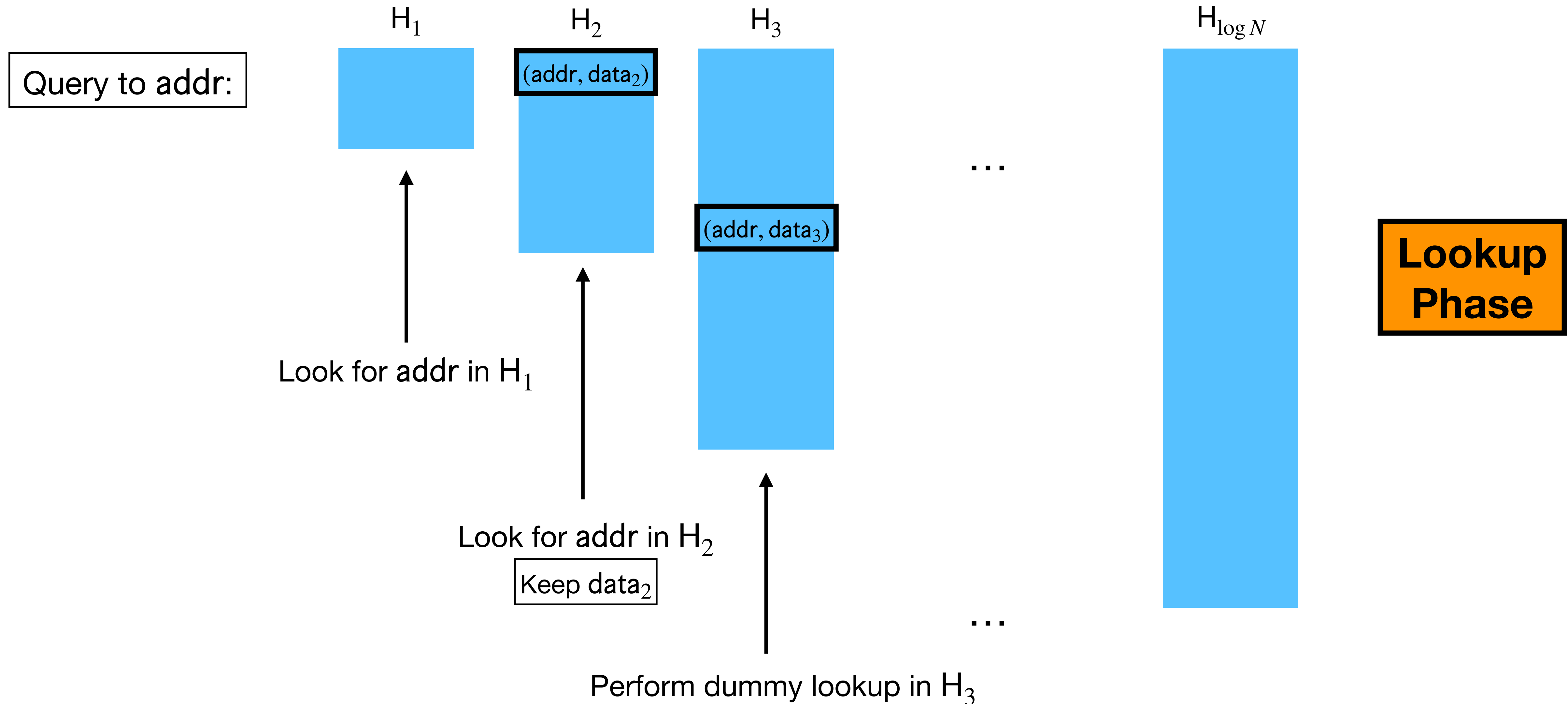
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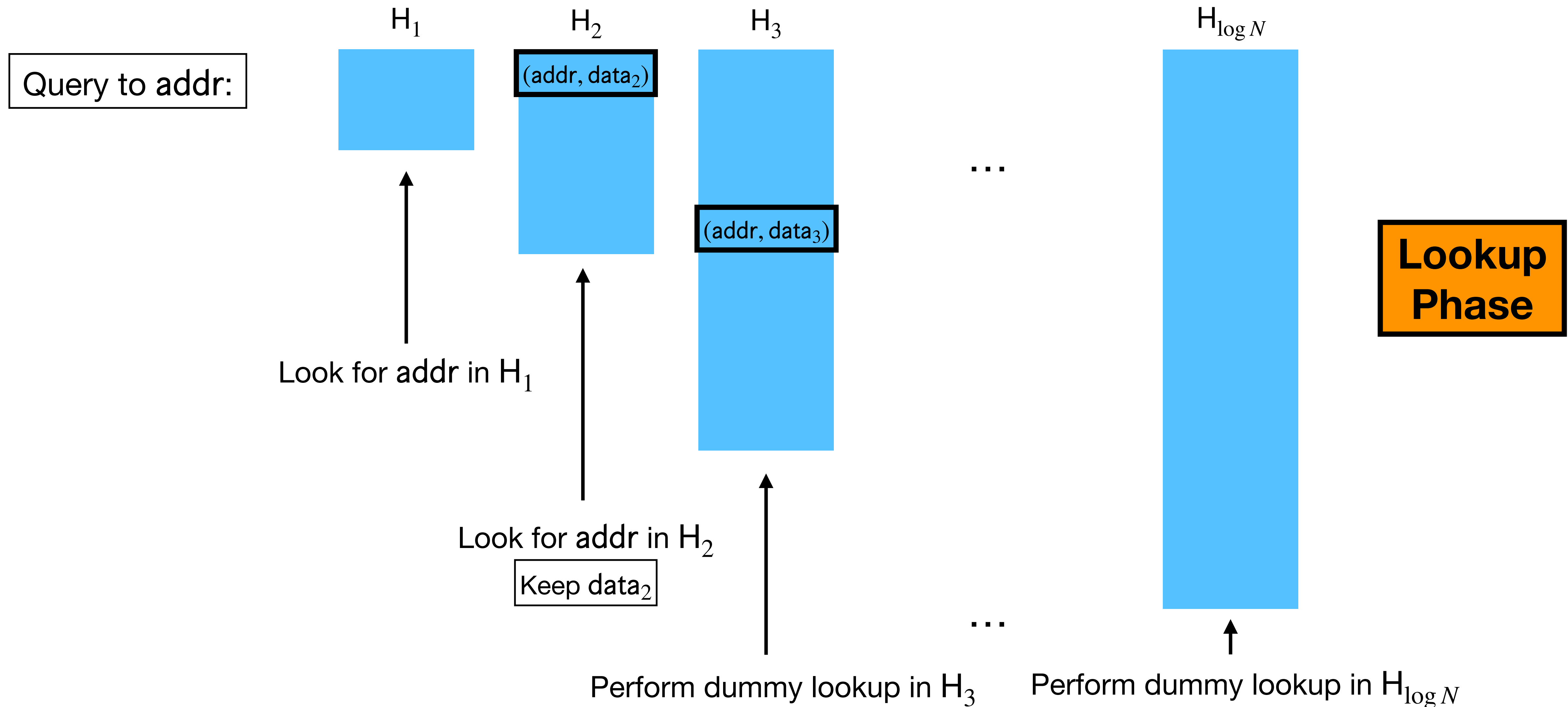
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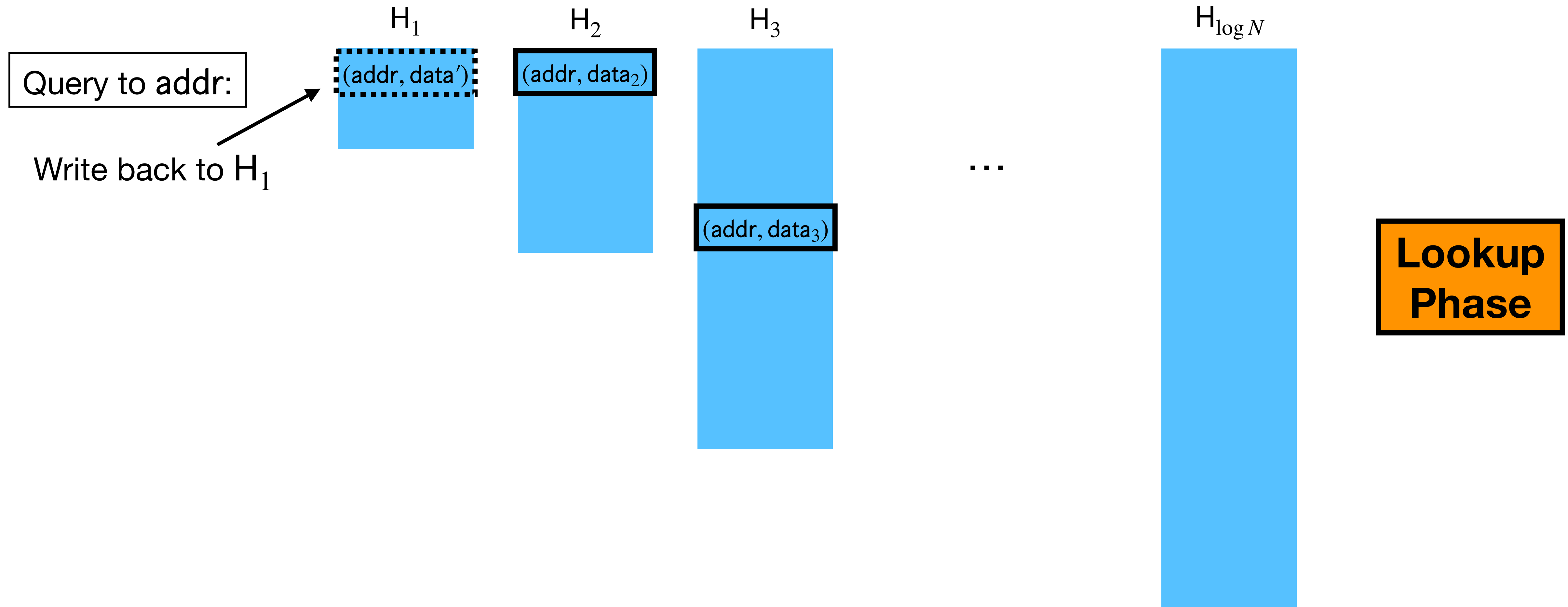
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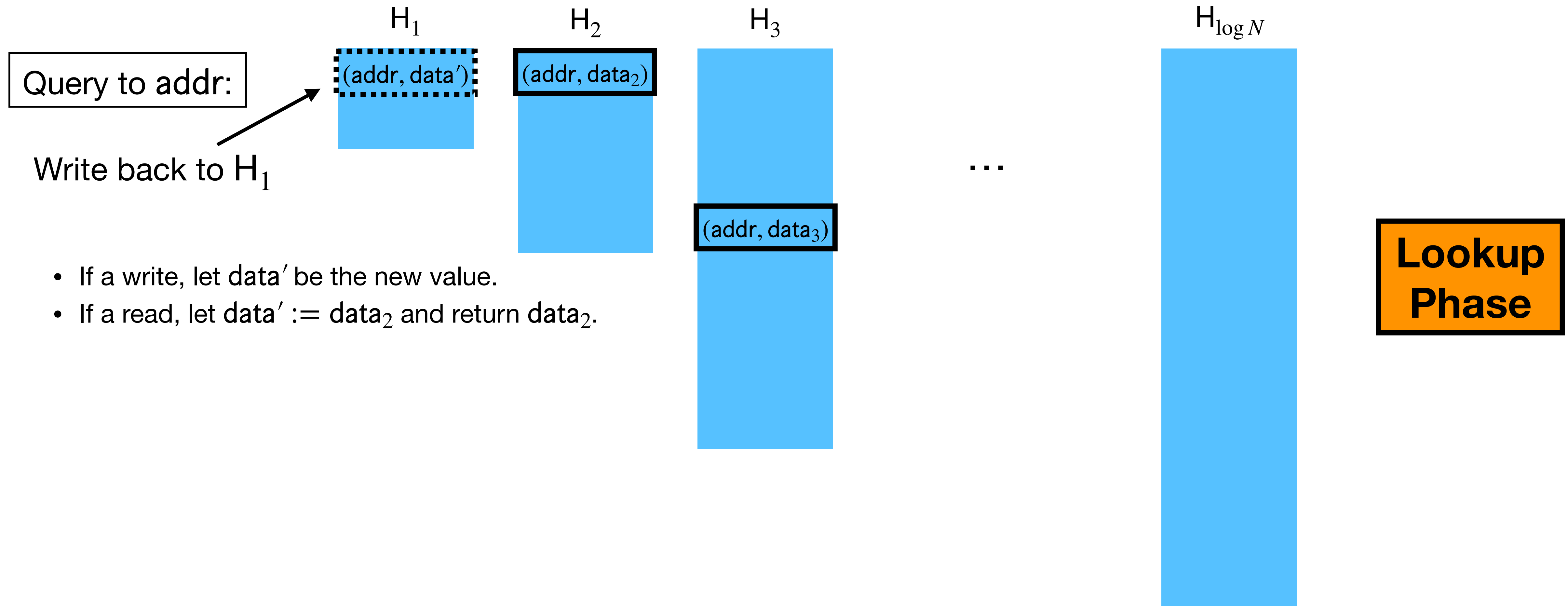


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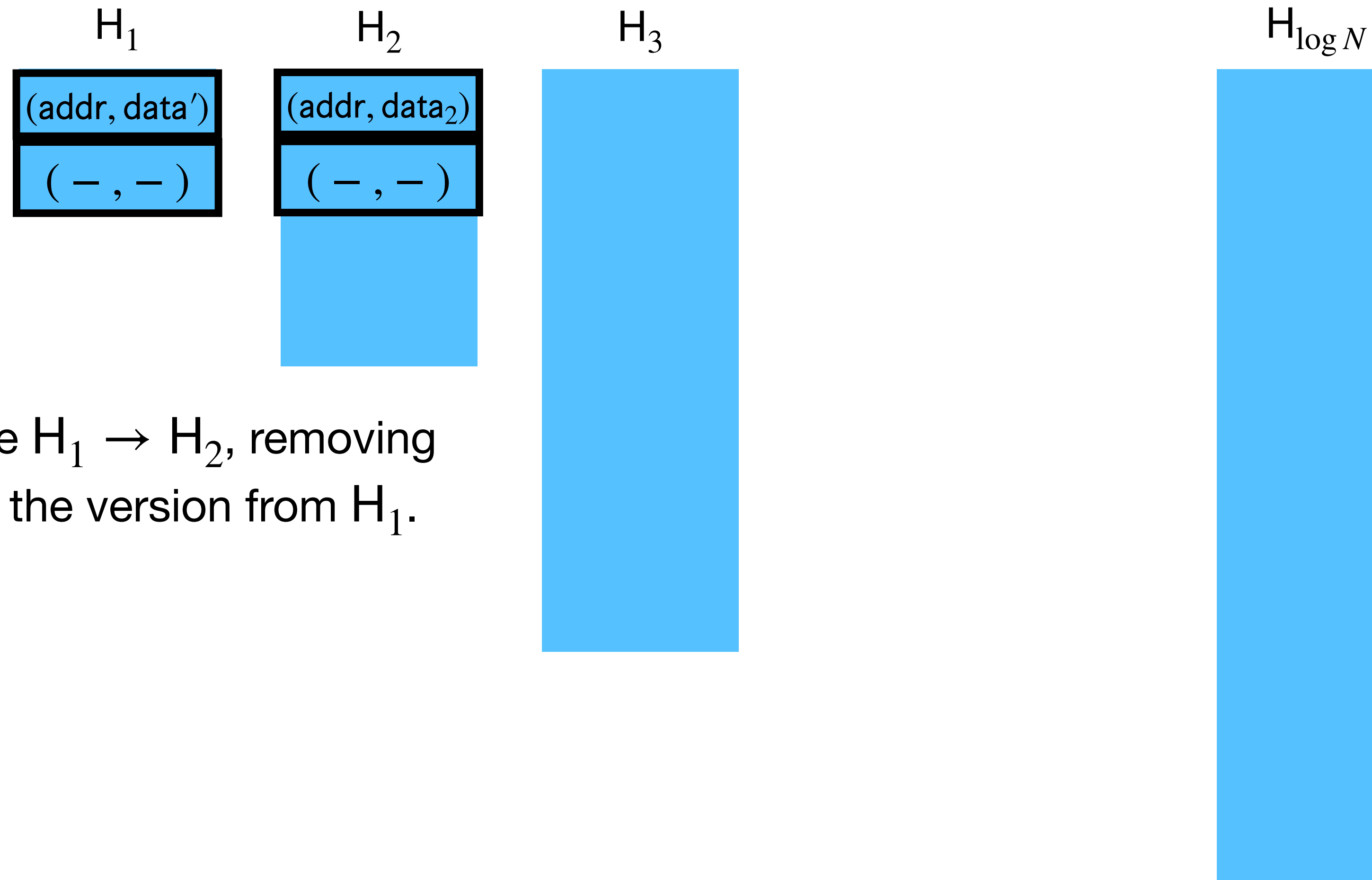




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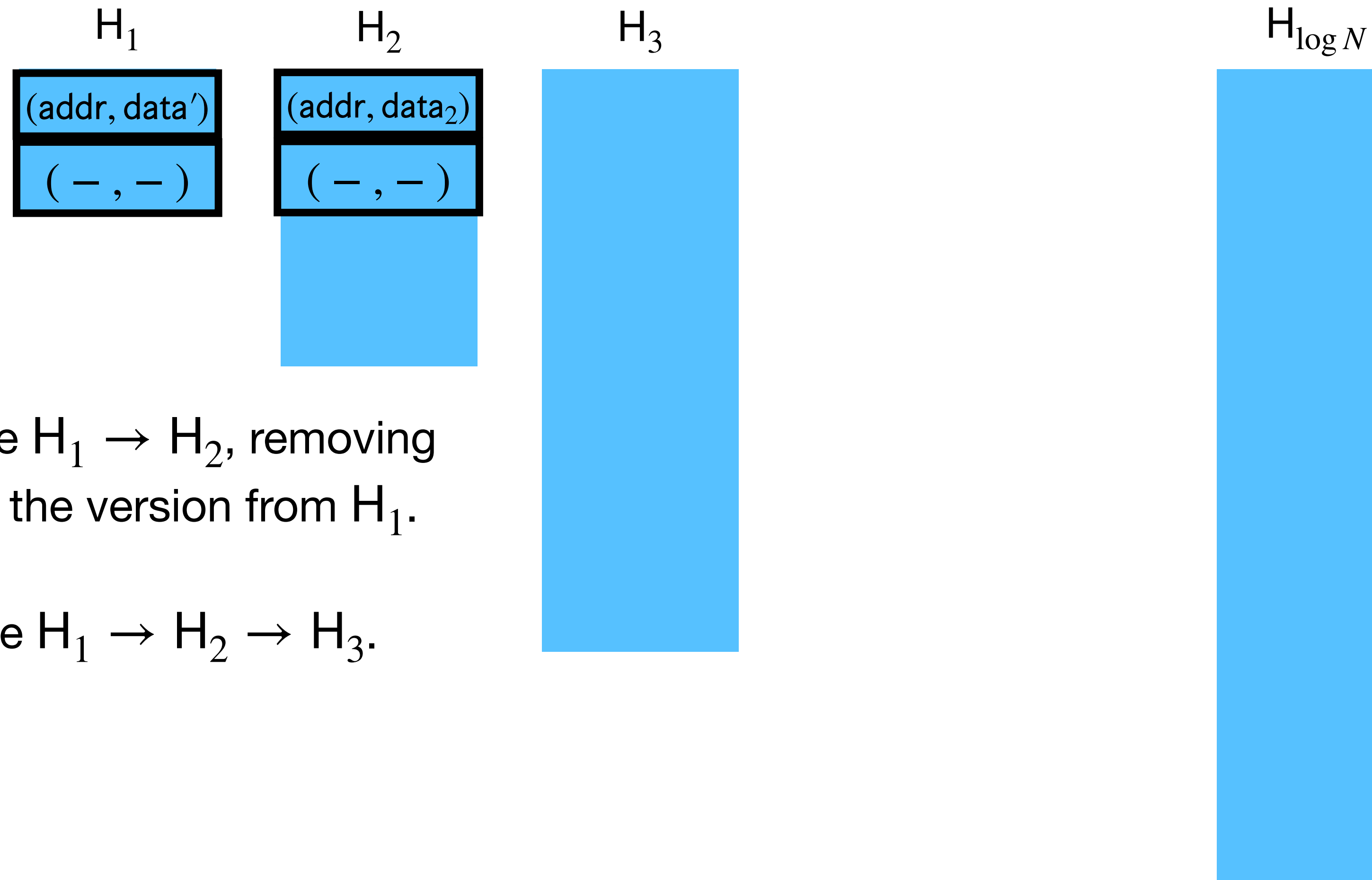
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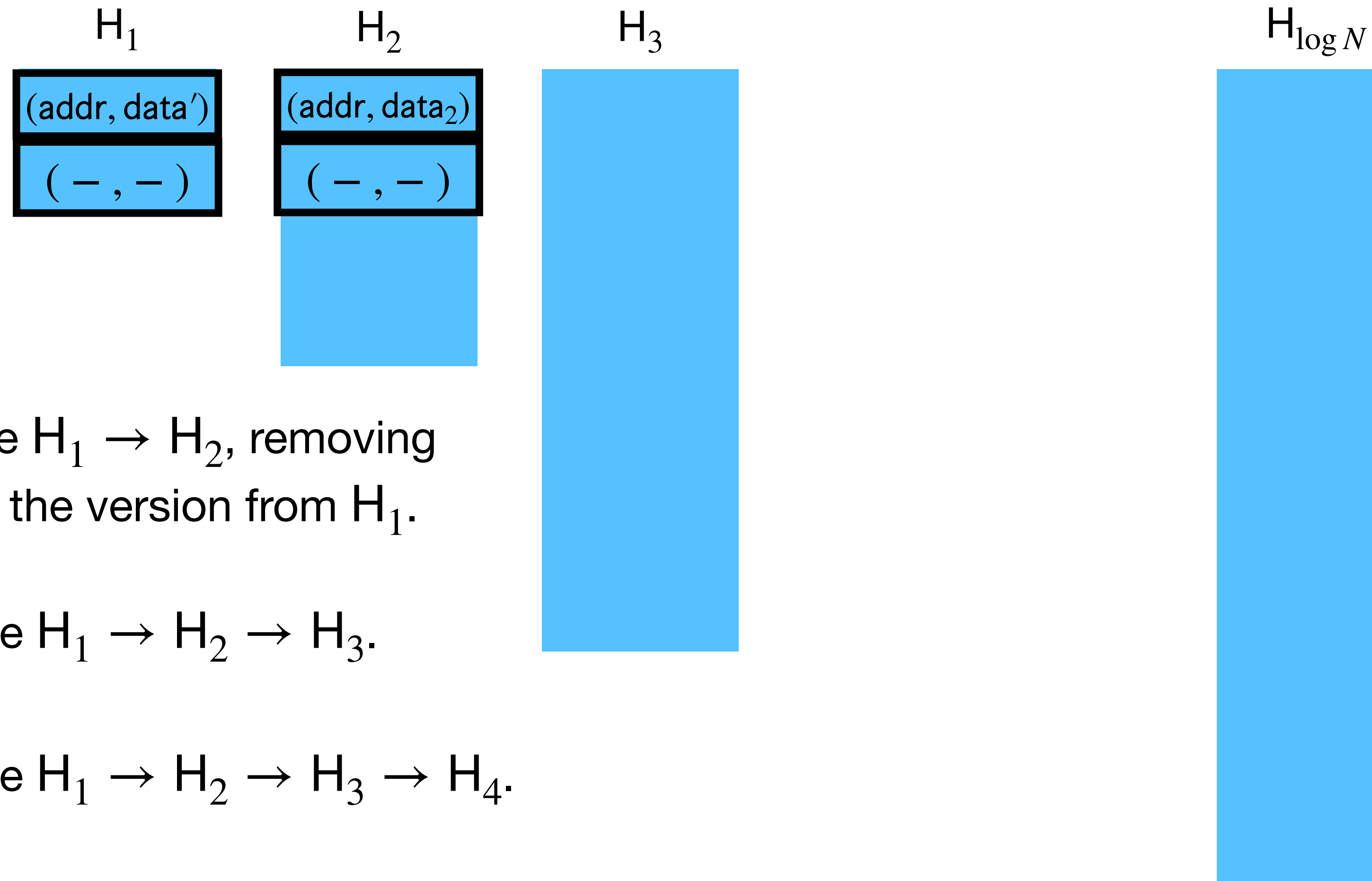
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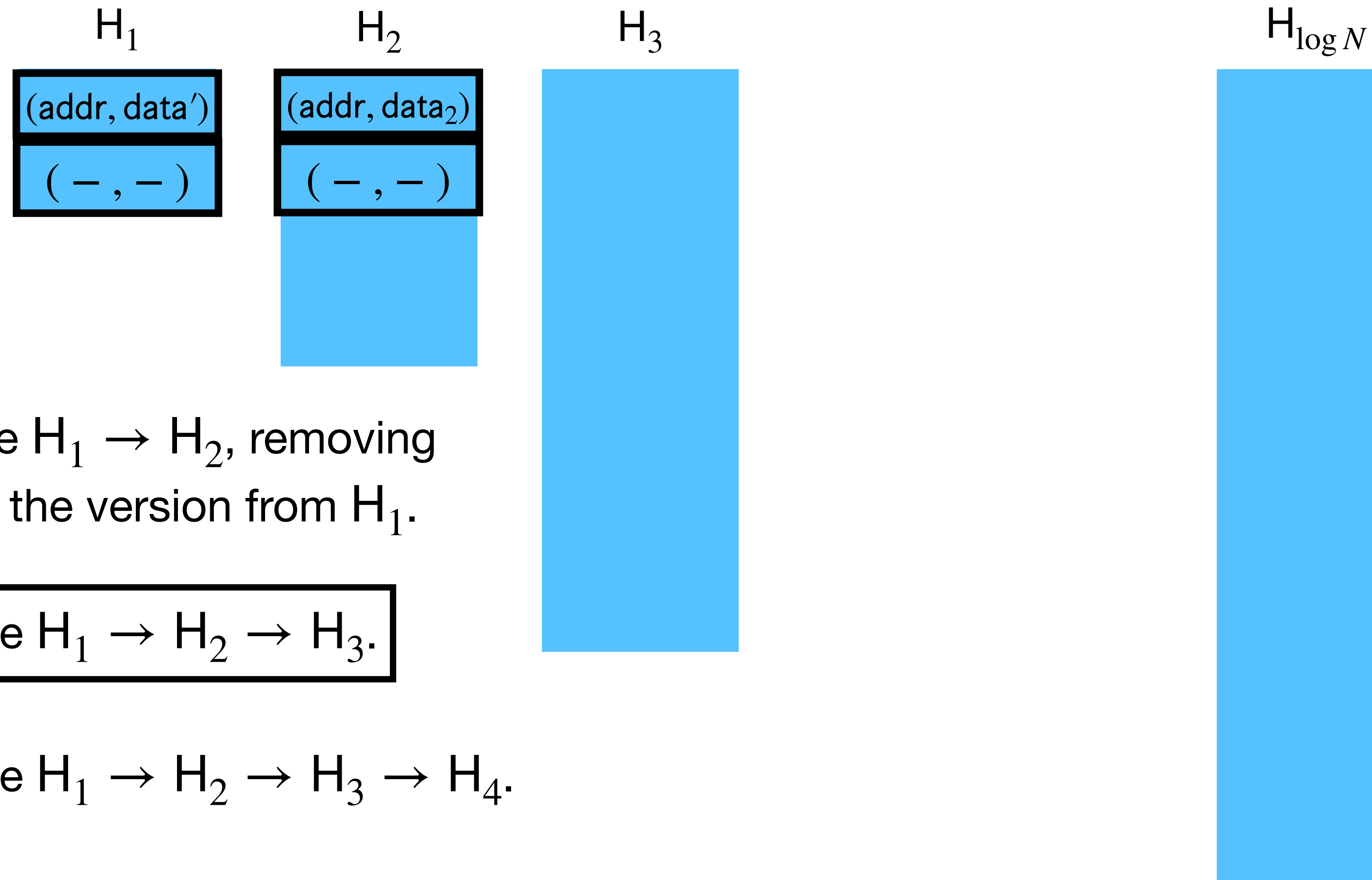
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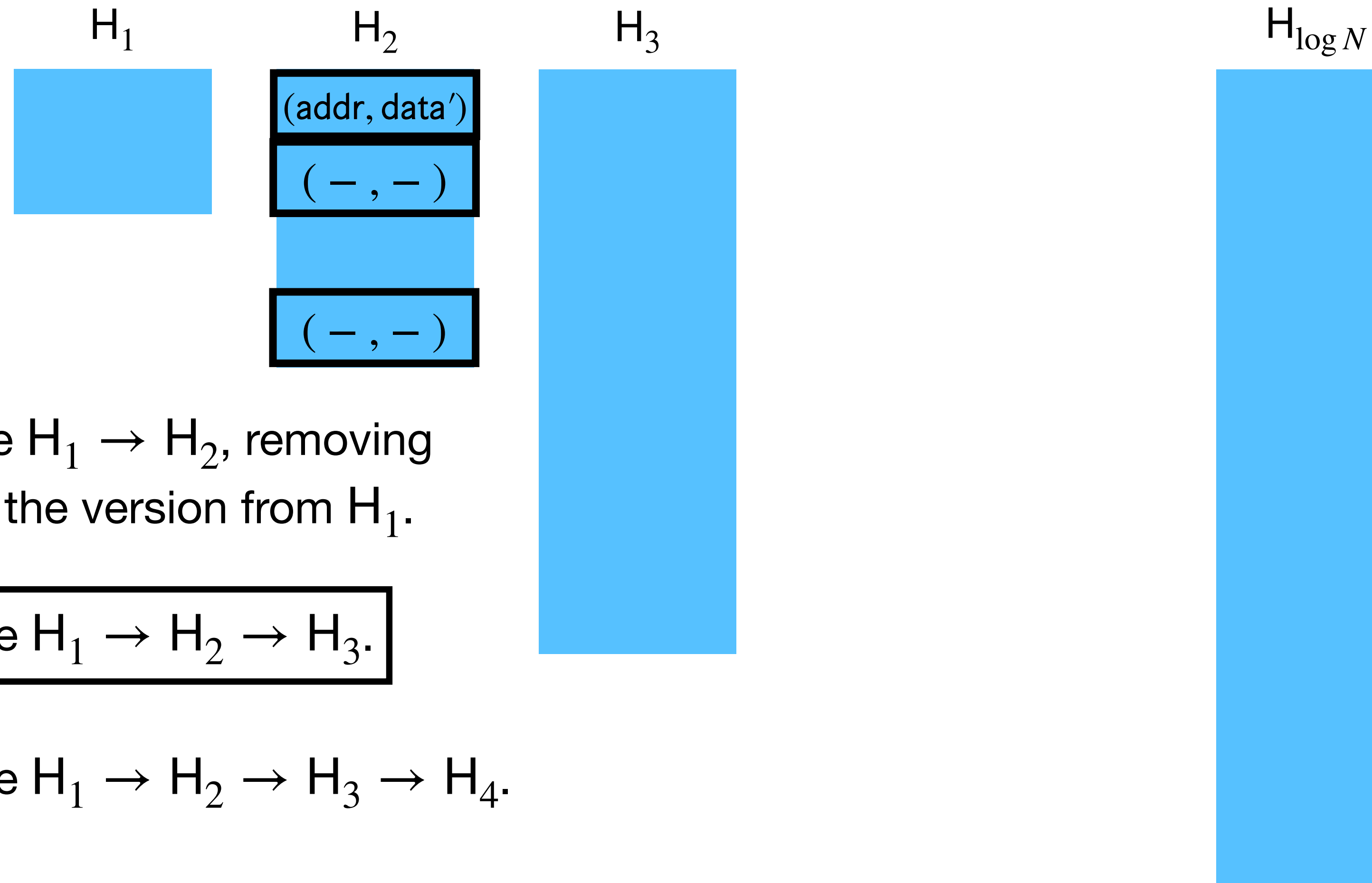
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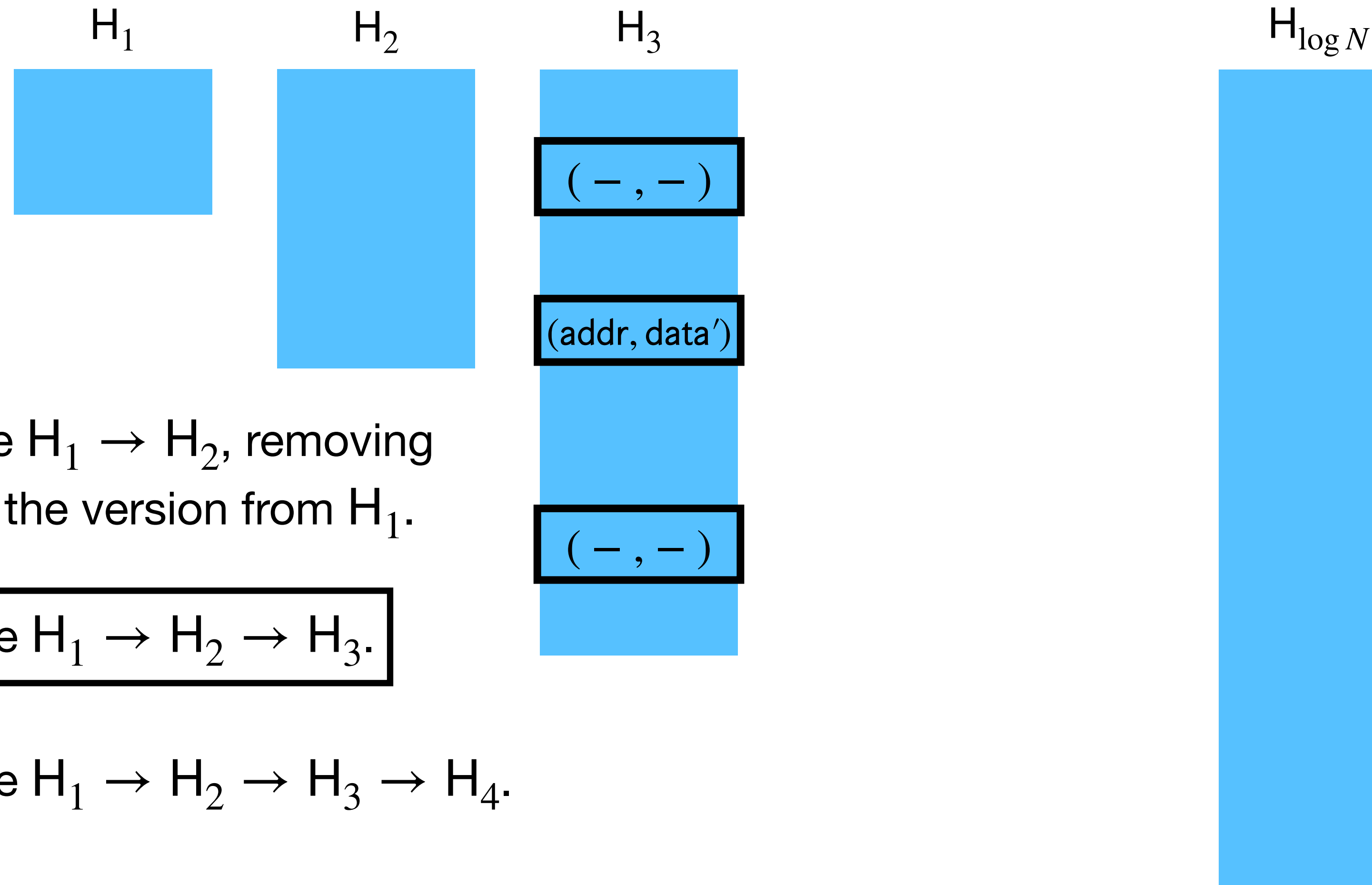
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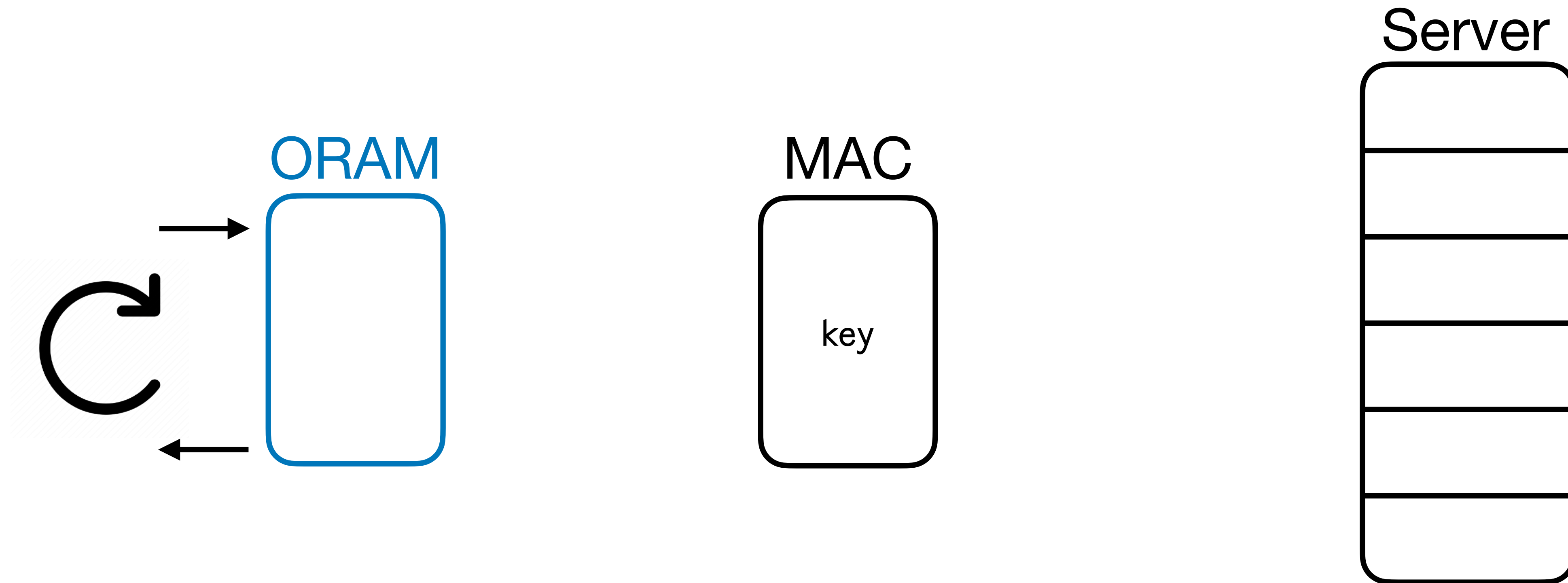
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# Overview of our techniques



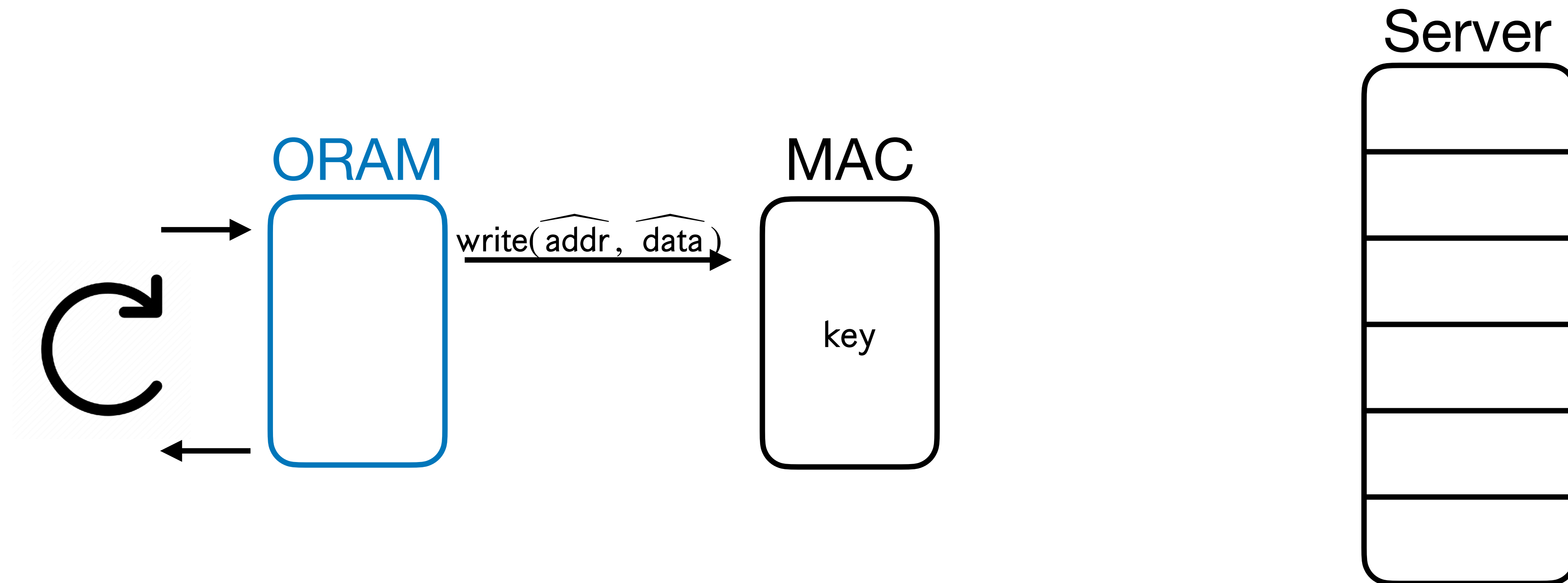
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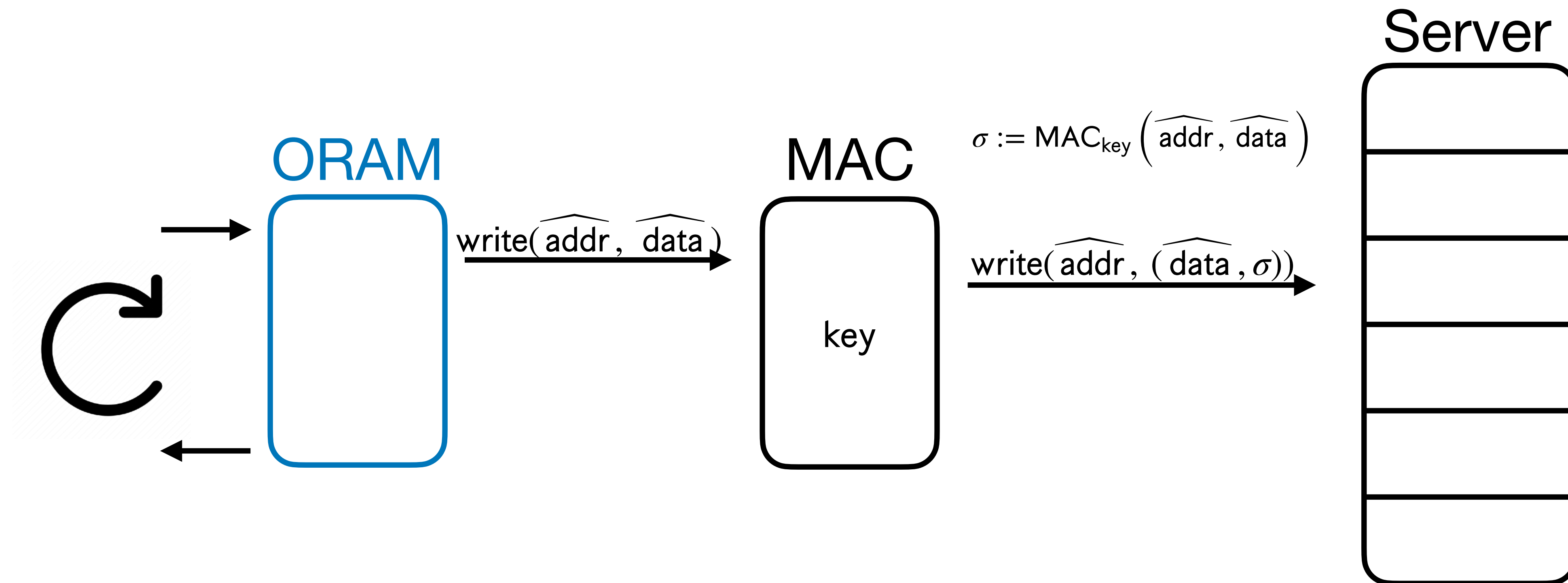
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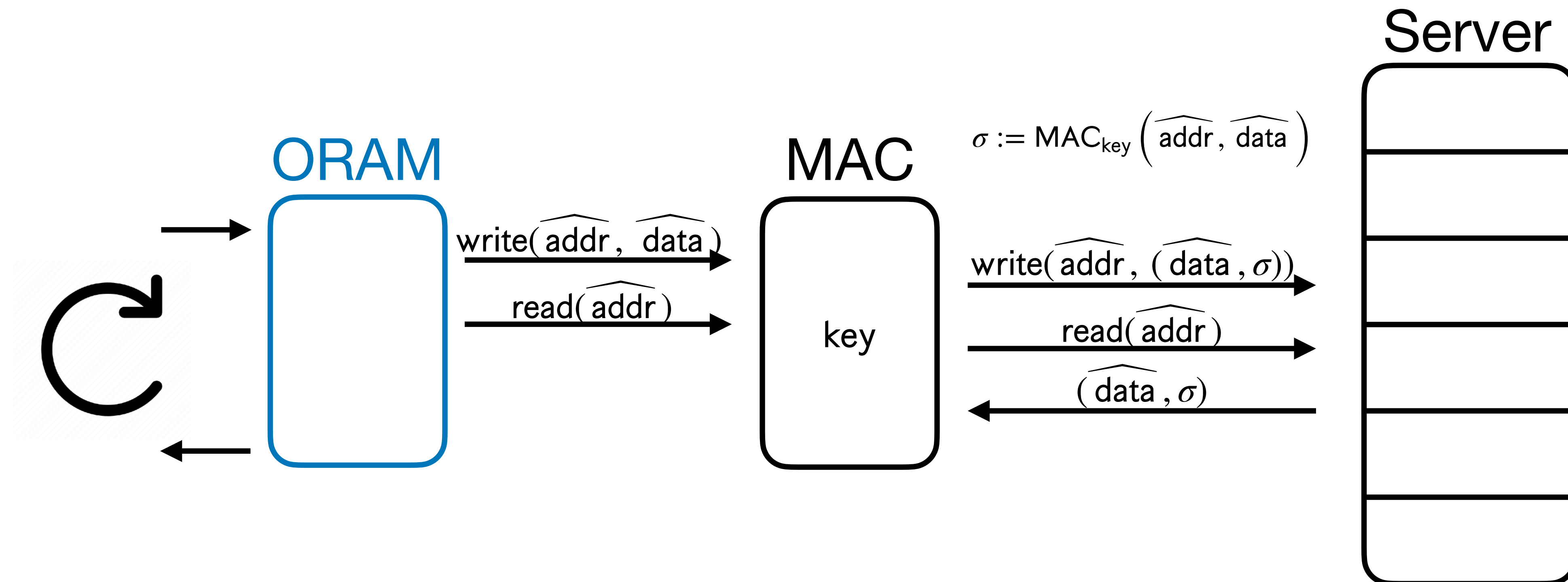
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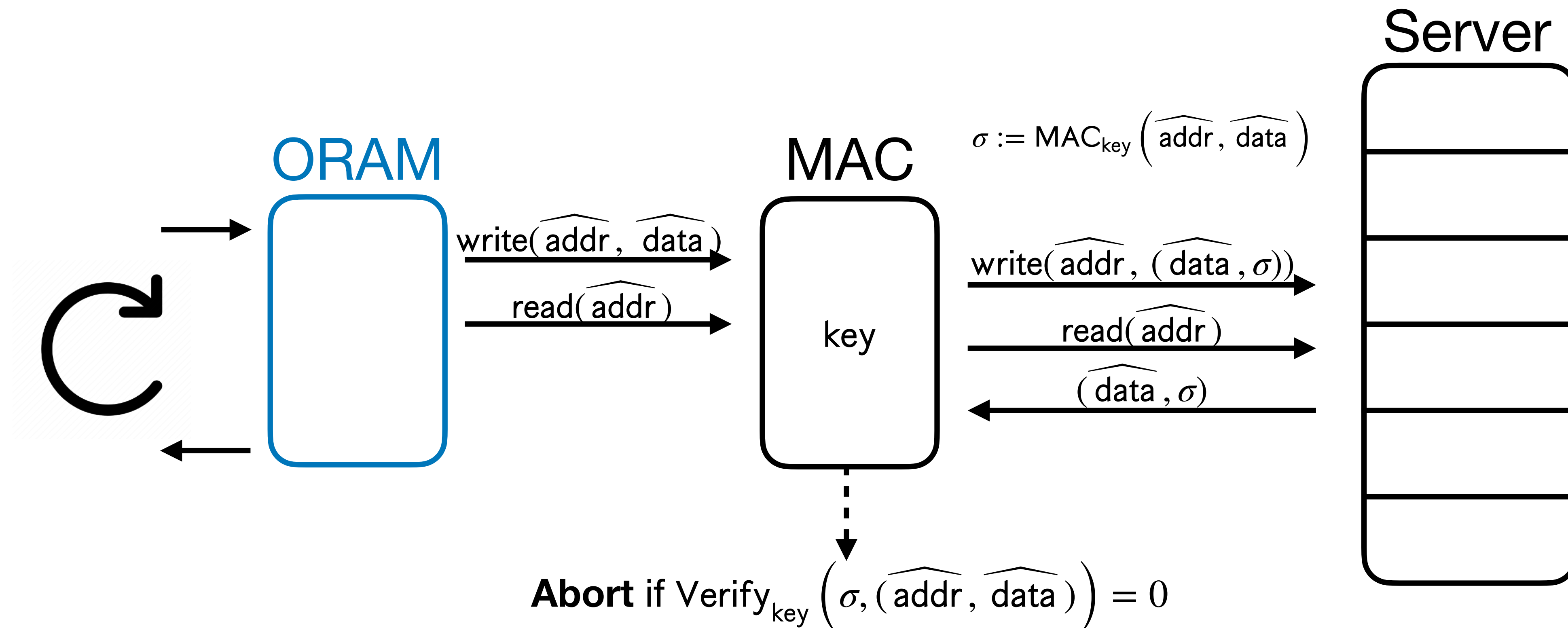
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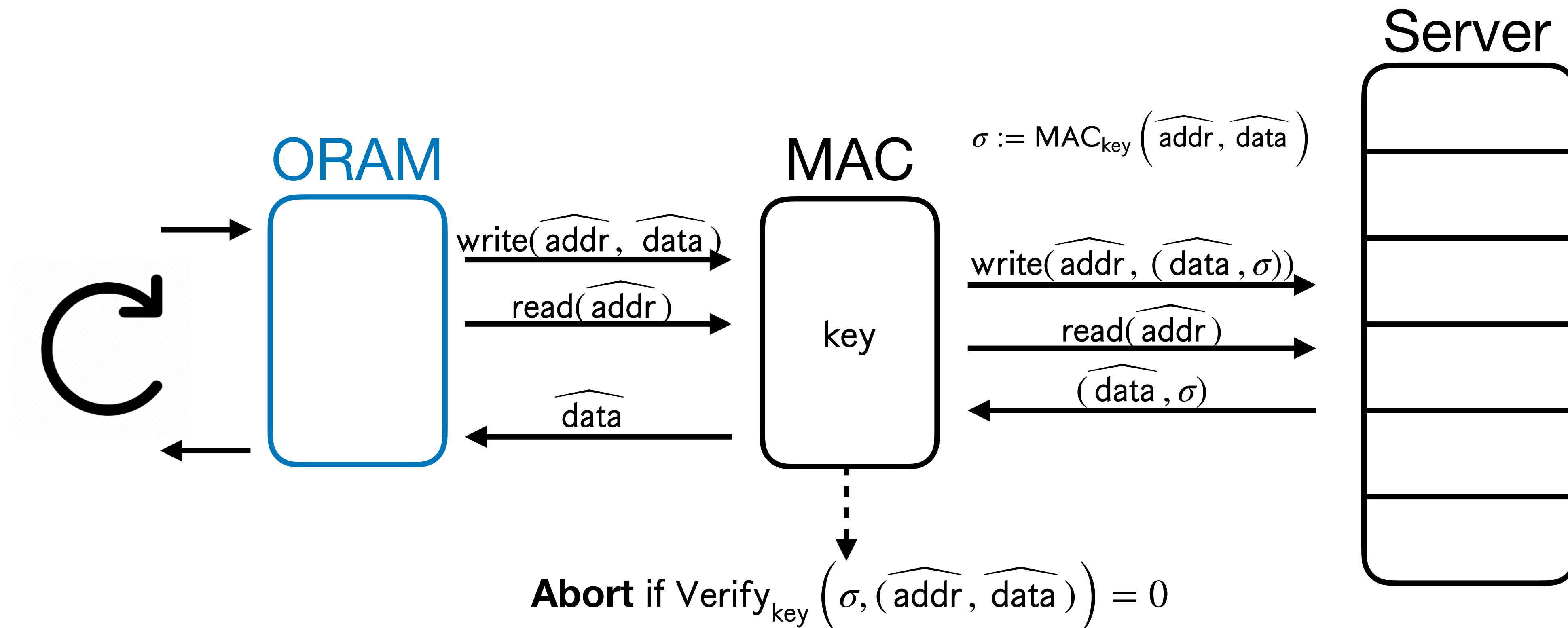
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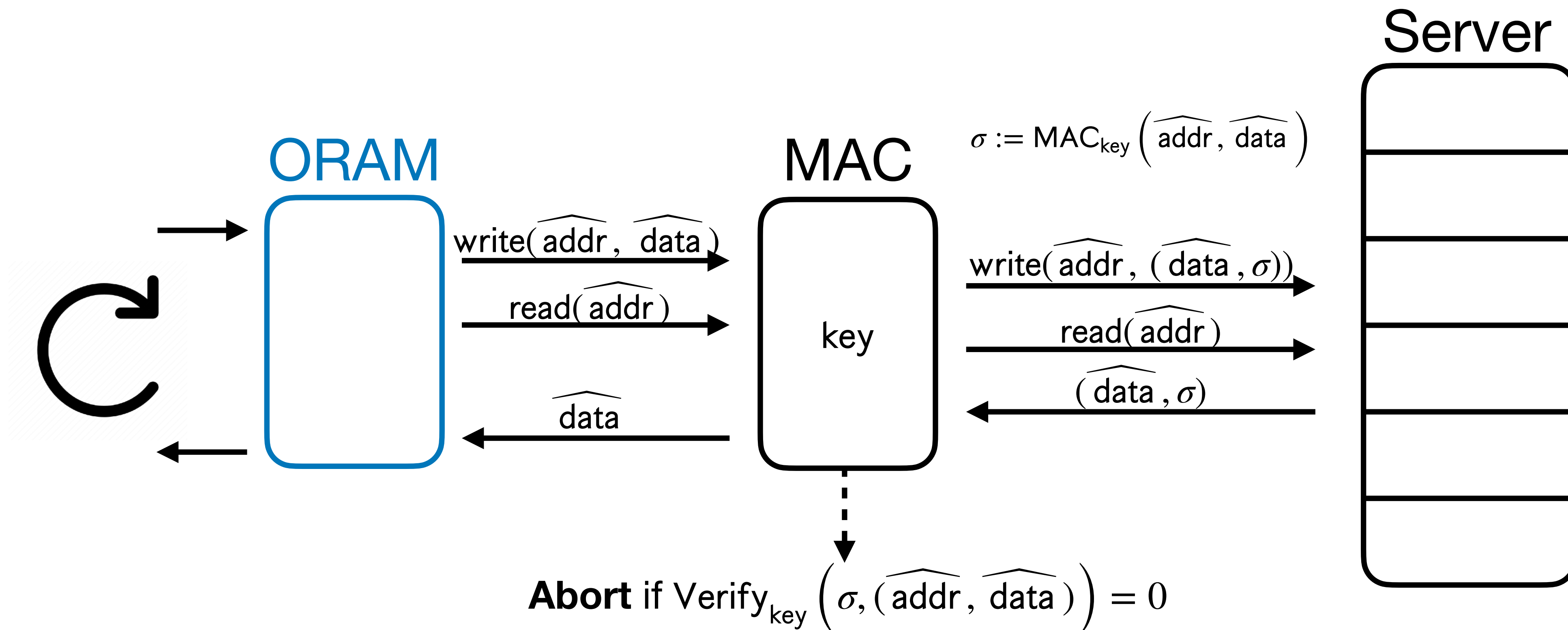
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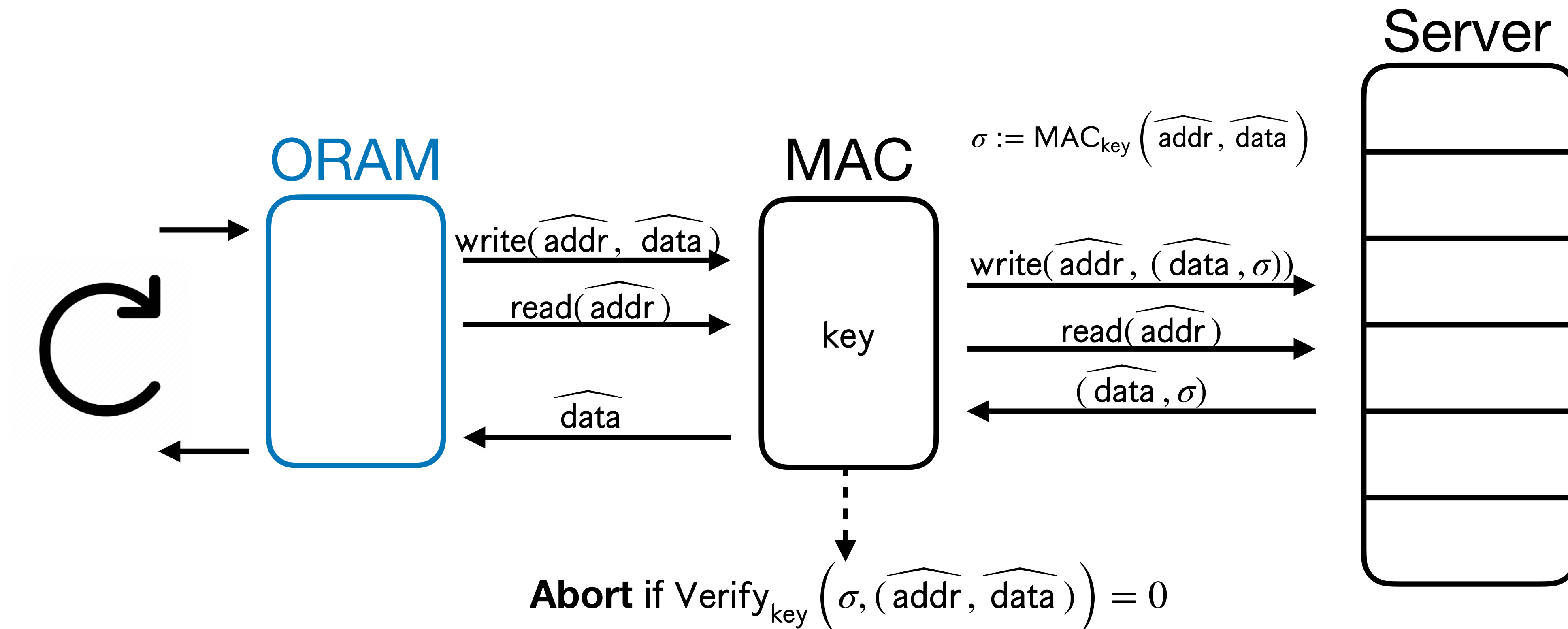
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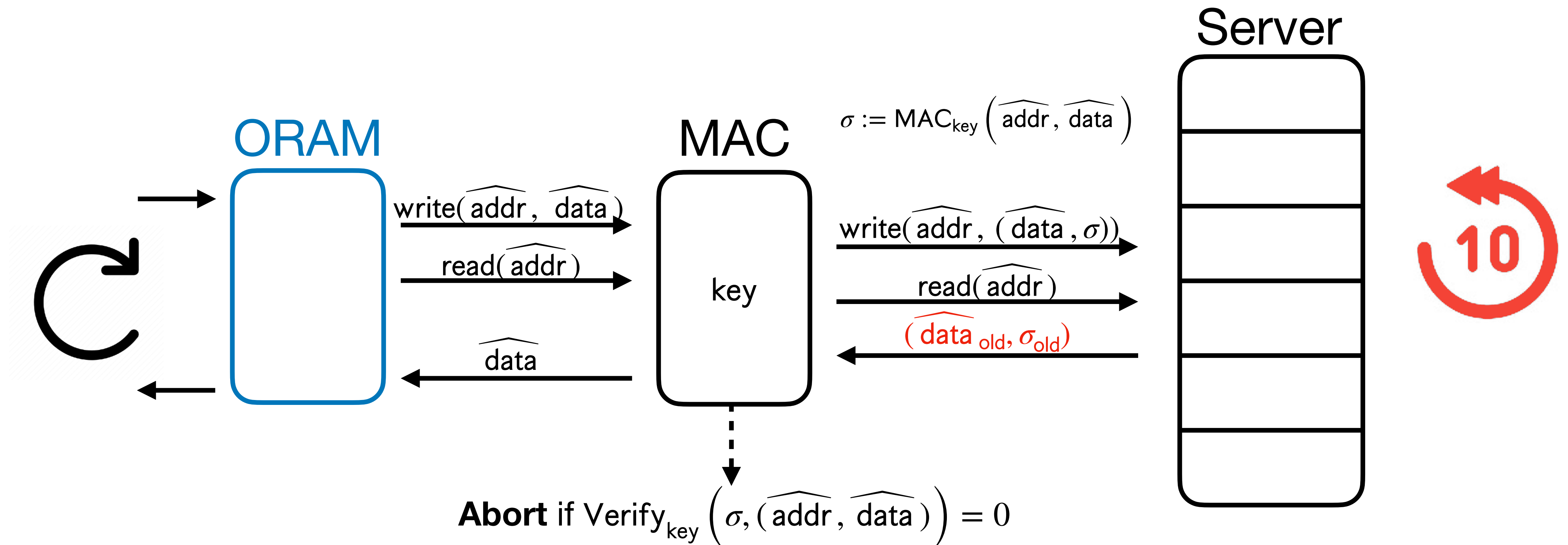
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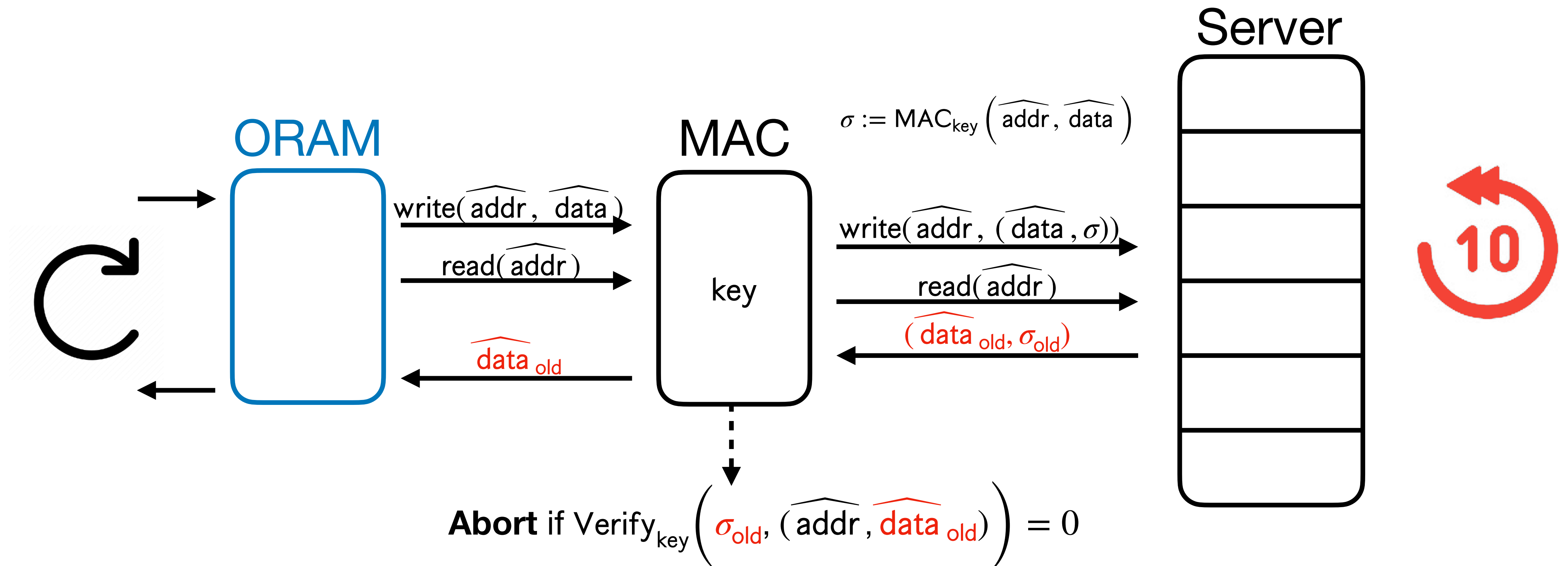
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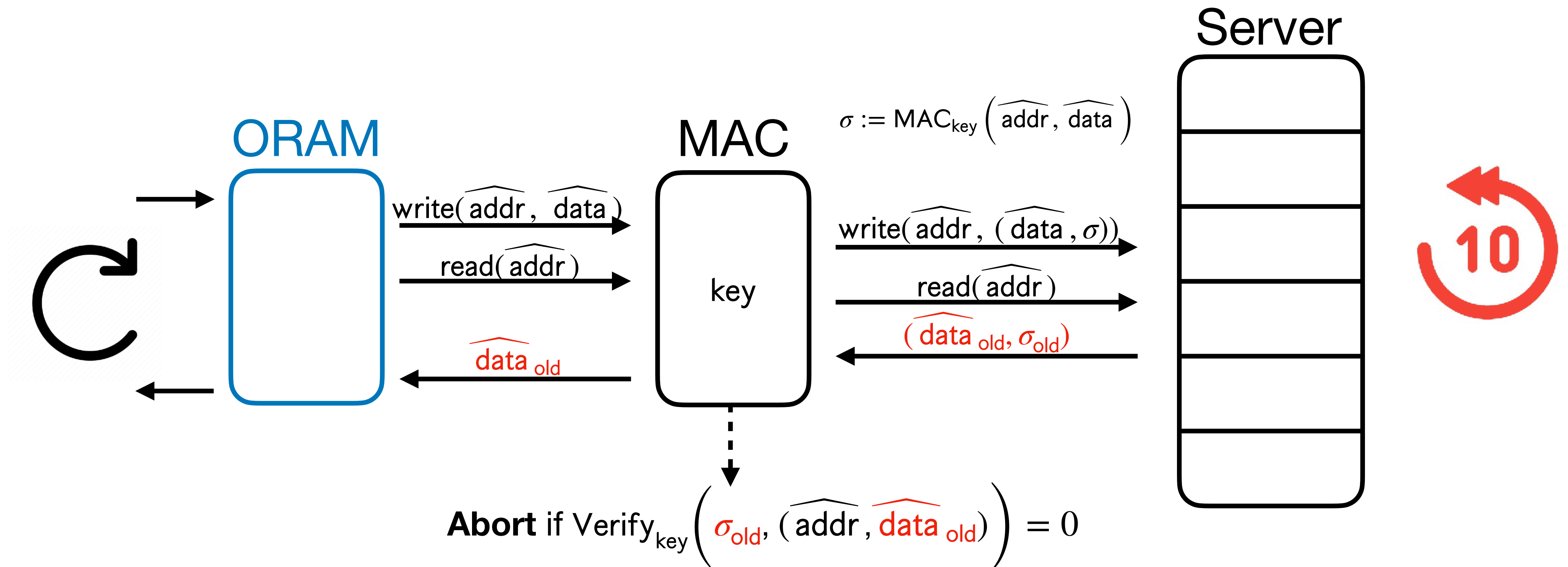
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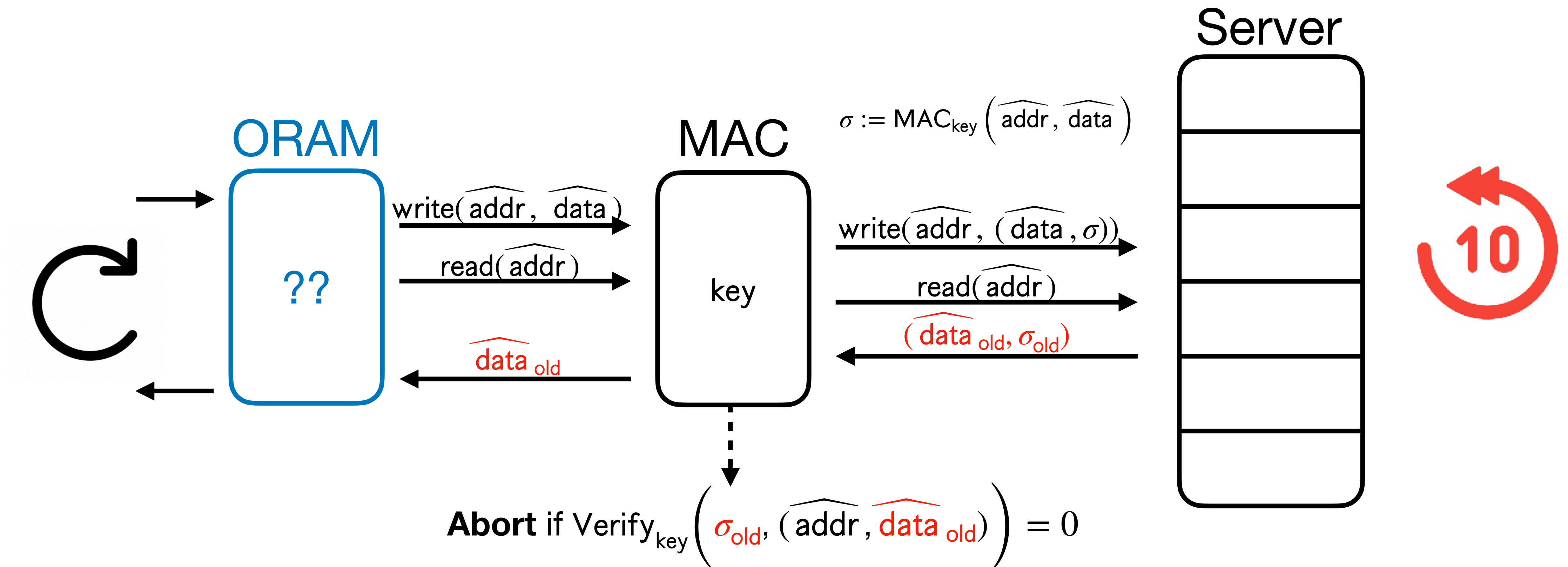
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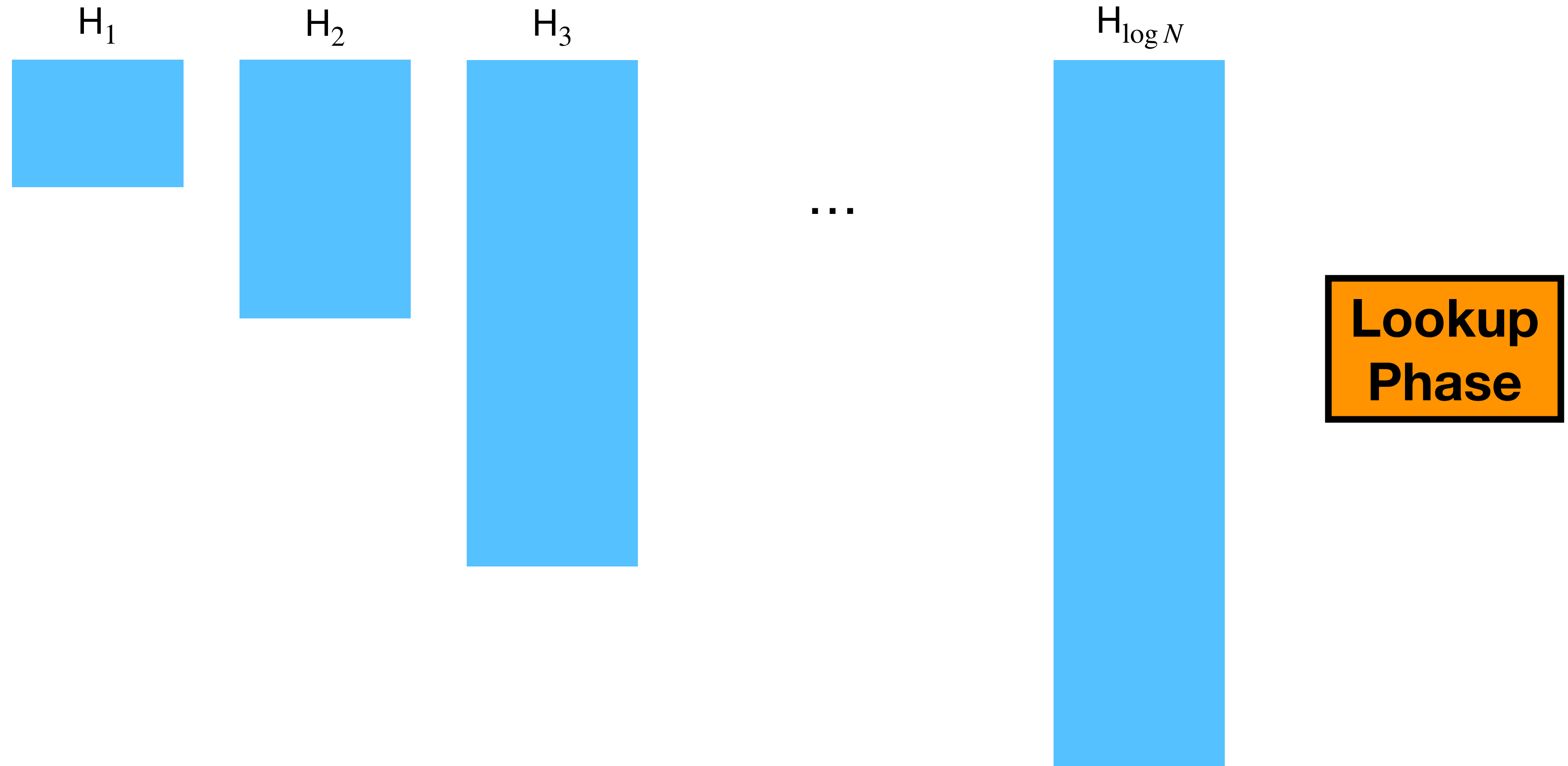
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  - In honest-but-curious setting, looking up **dummies** and rebuilding hash tables ensures reads will be non-recurrent.

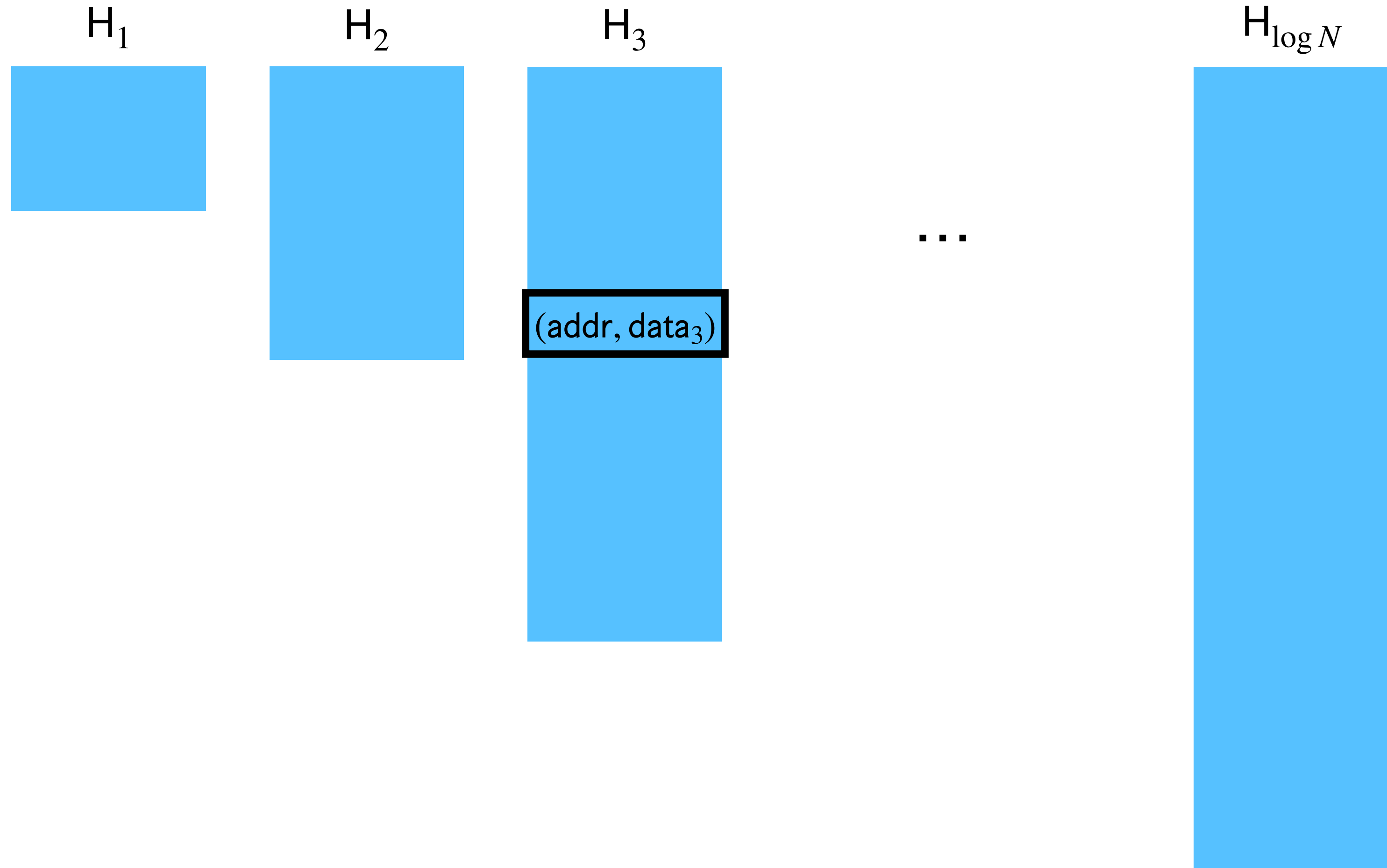


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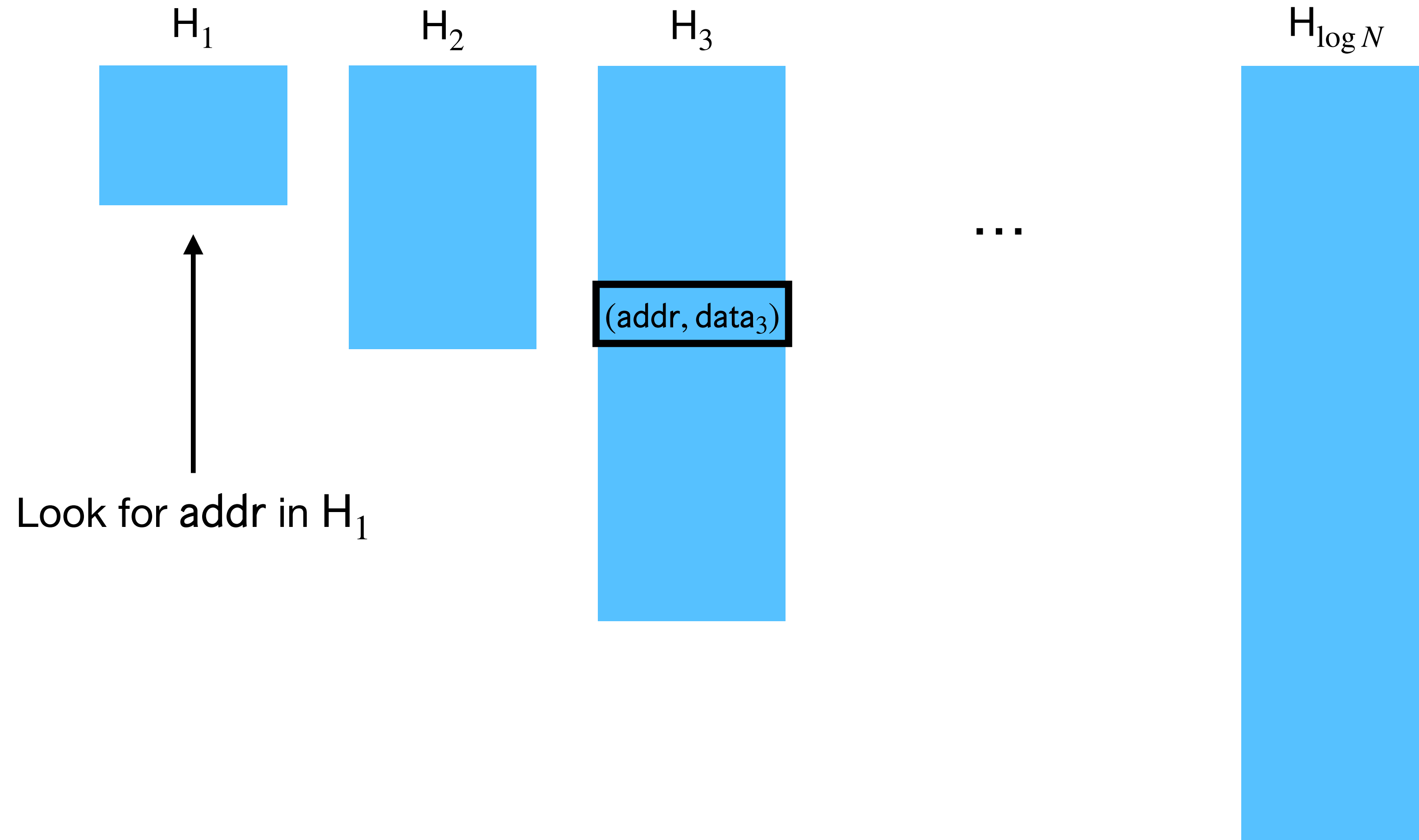
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**Lookup  
Phase**

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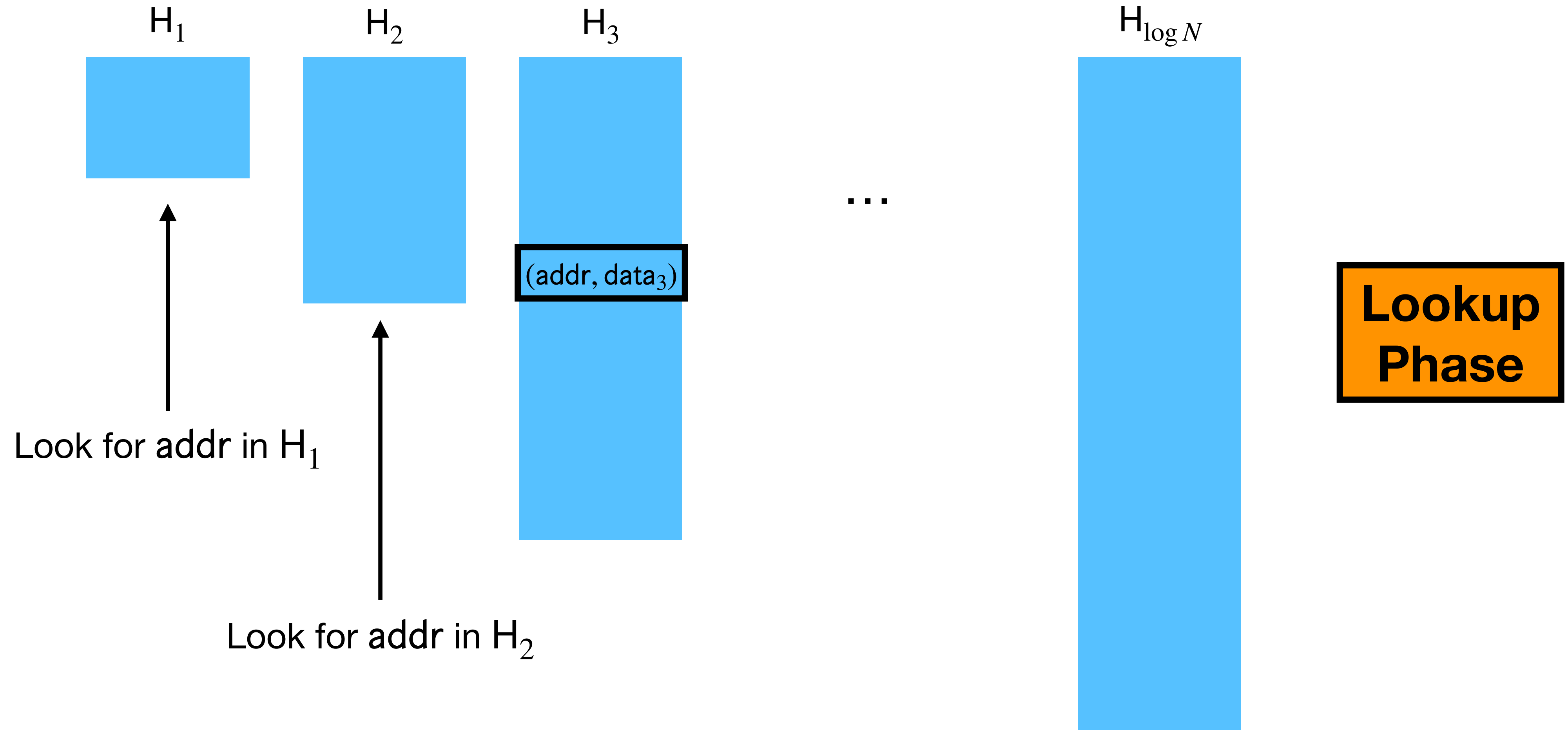
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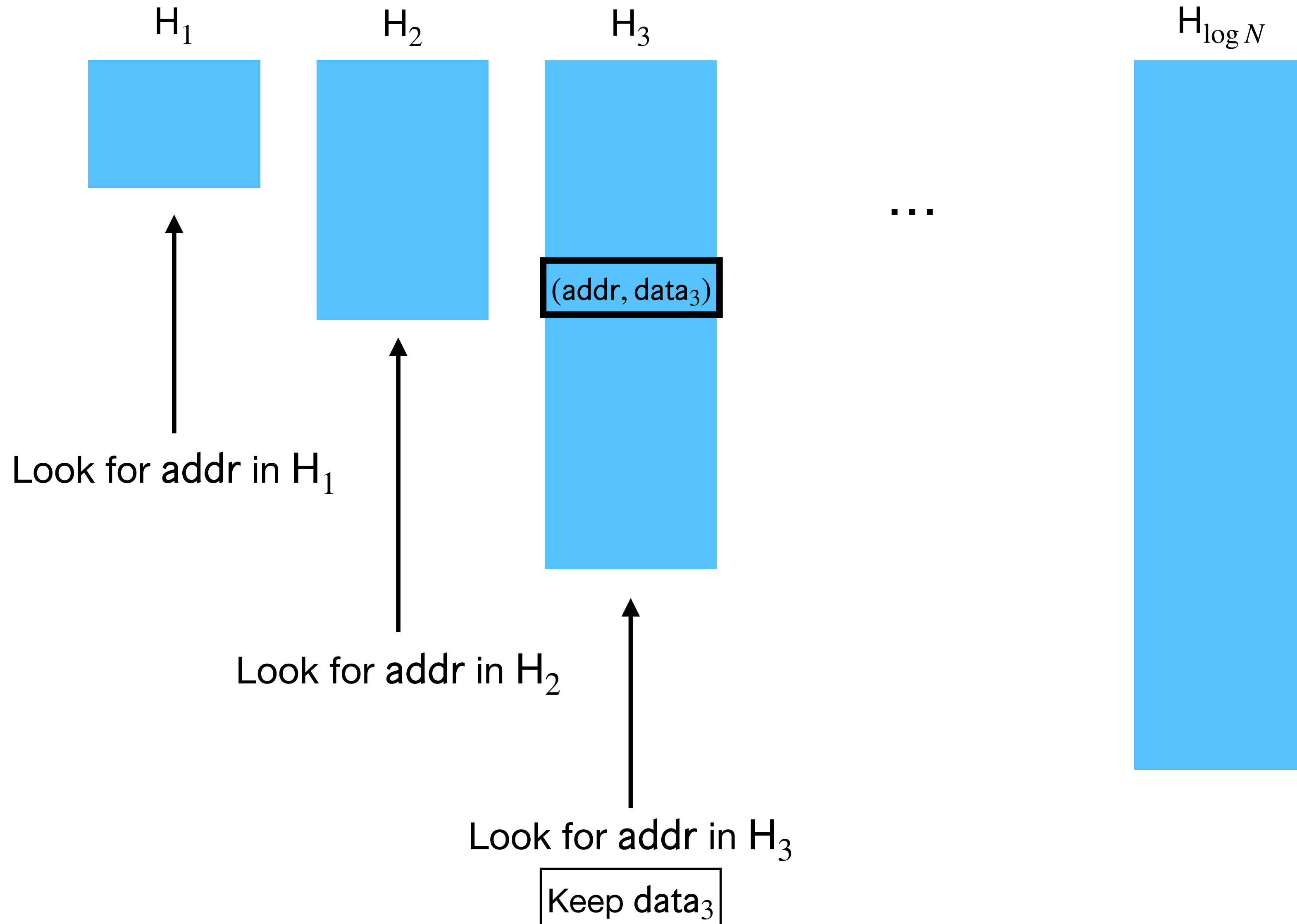
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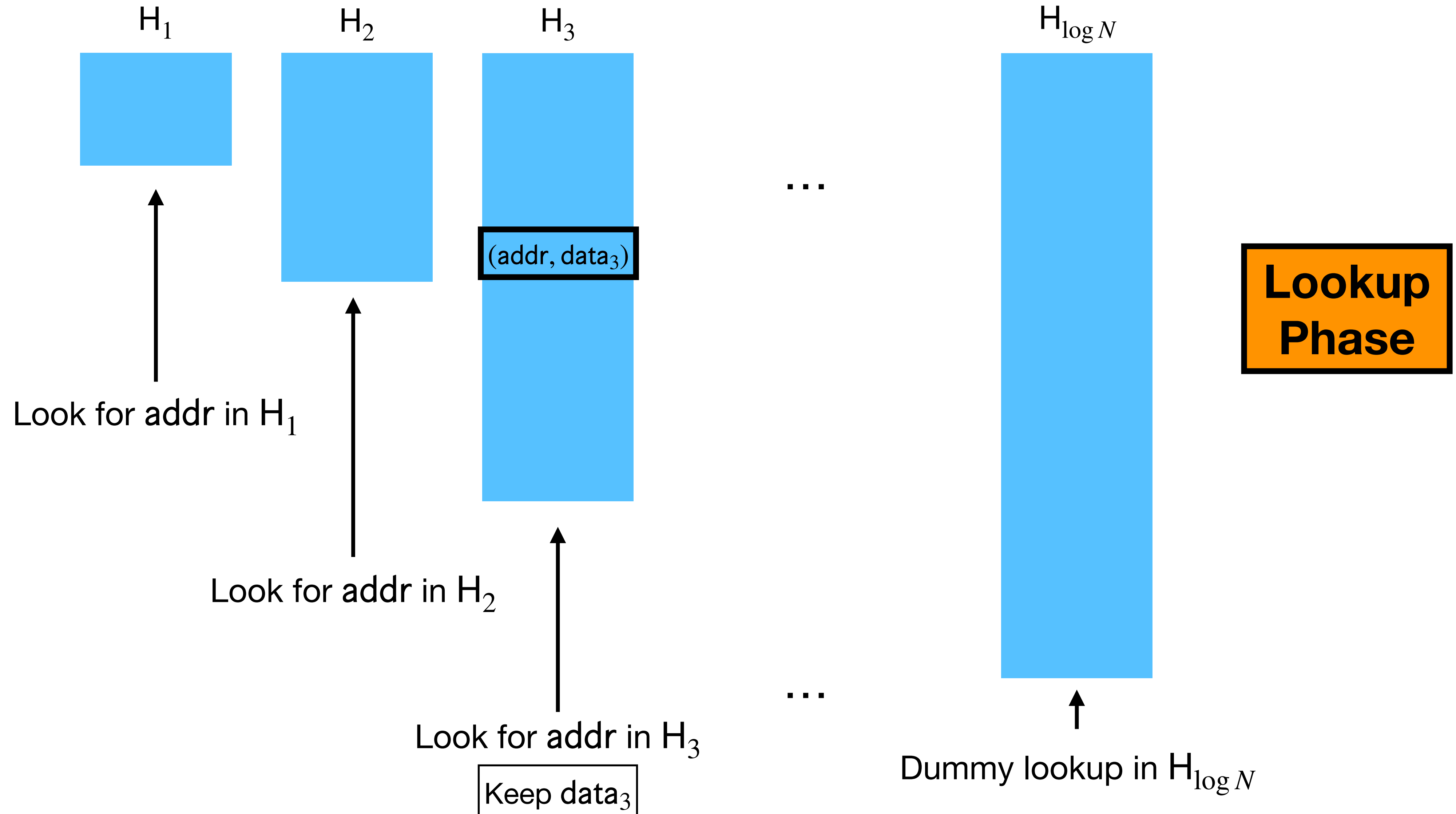
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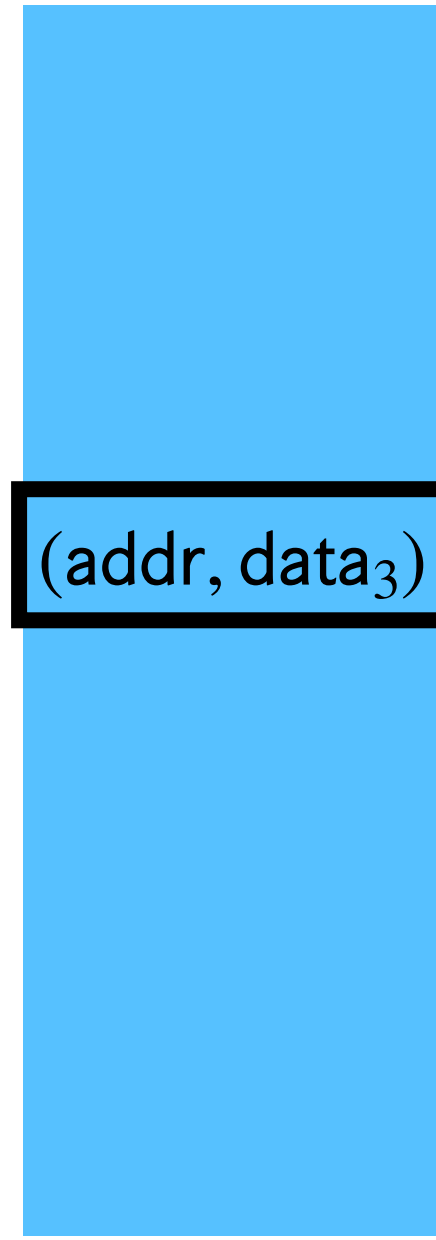
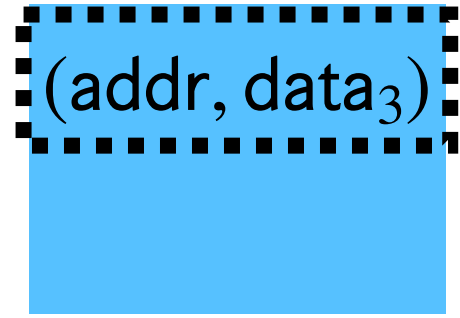
Read addr:

$H_1$

$H_2$

$H_3$

$H_{\log N}$



...



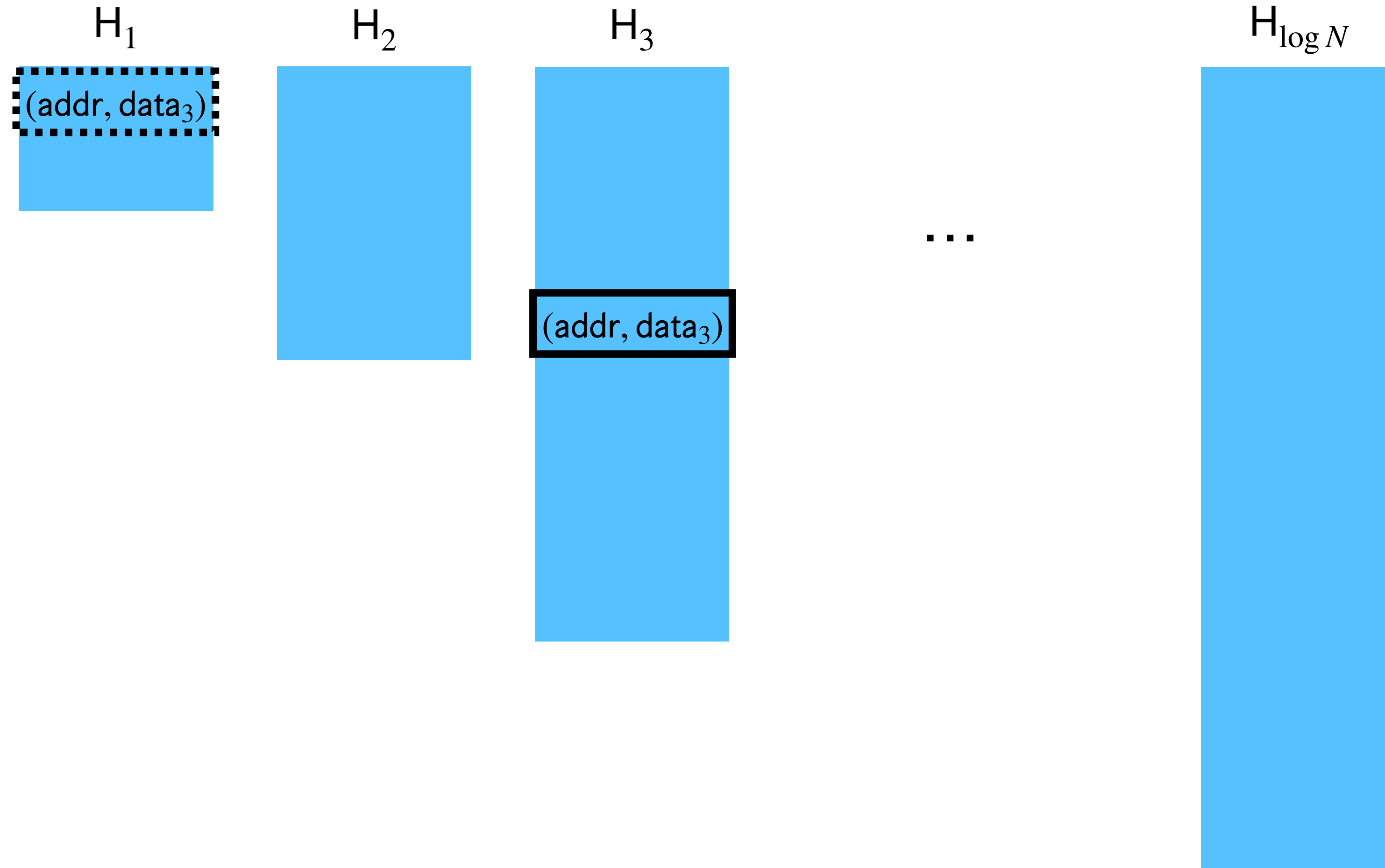
Write back to  $H_1$

**Lookup  
Phase**

# Replay Attack

Read addr:

Write to addr:



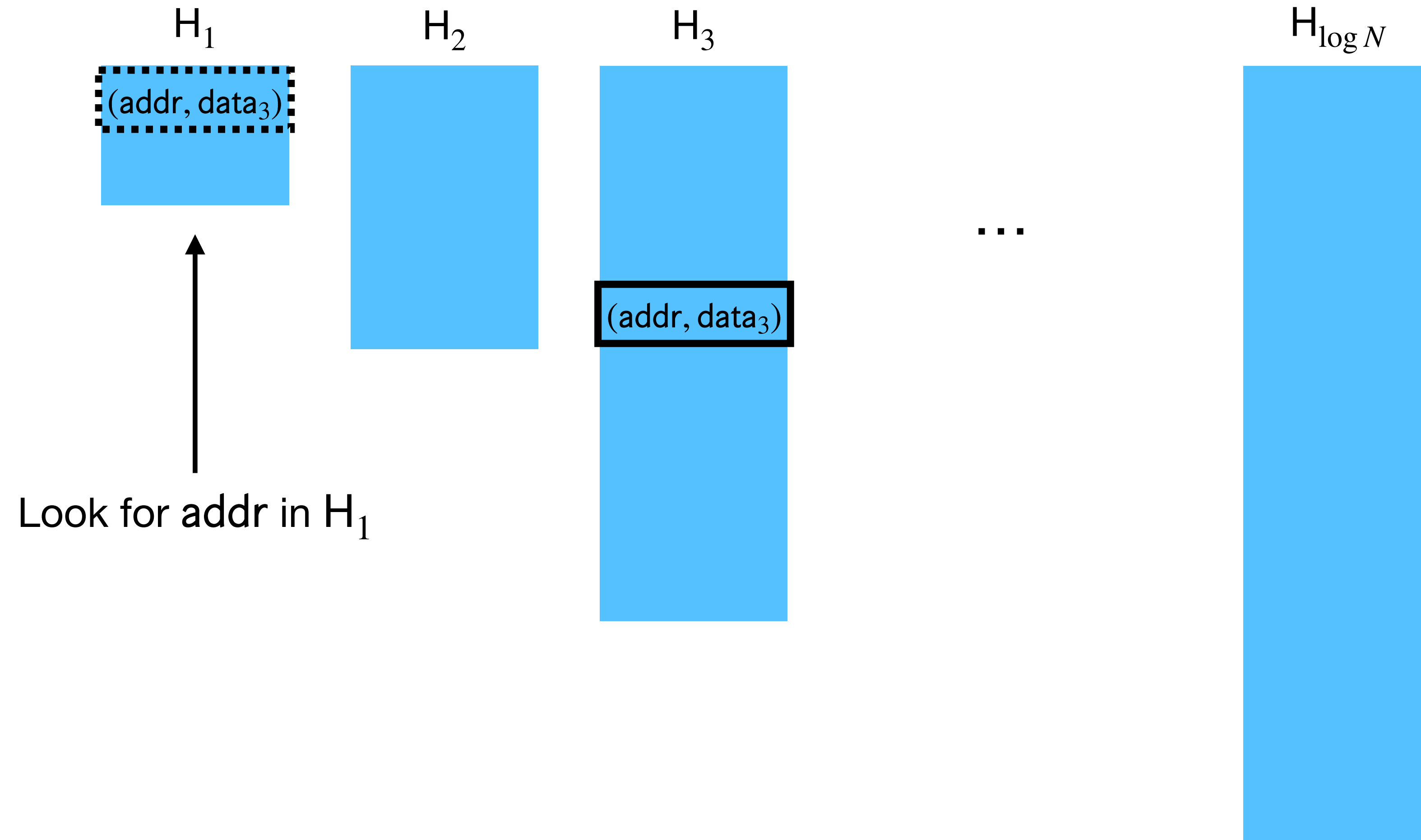
**Lookup  
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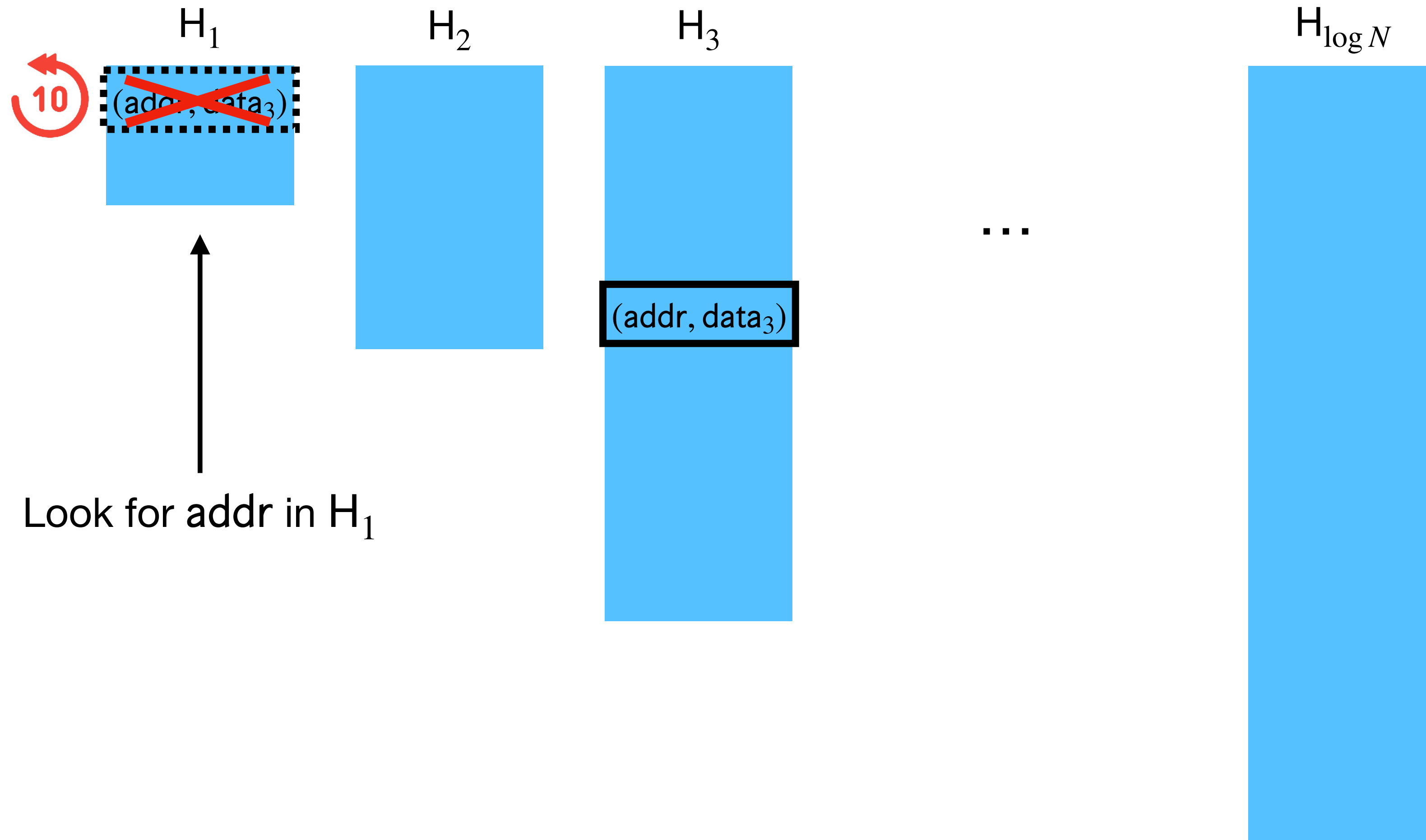


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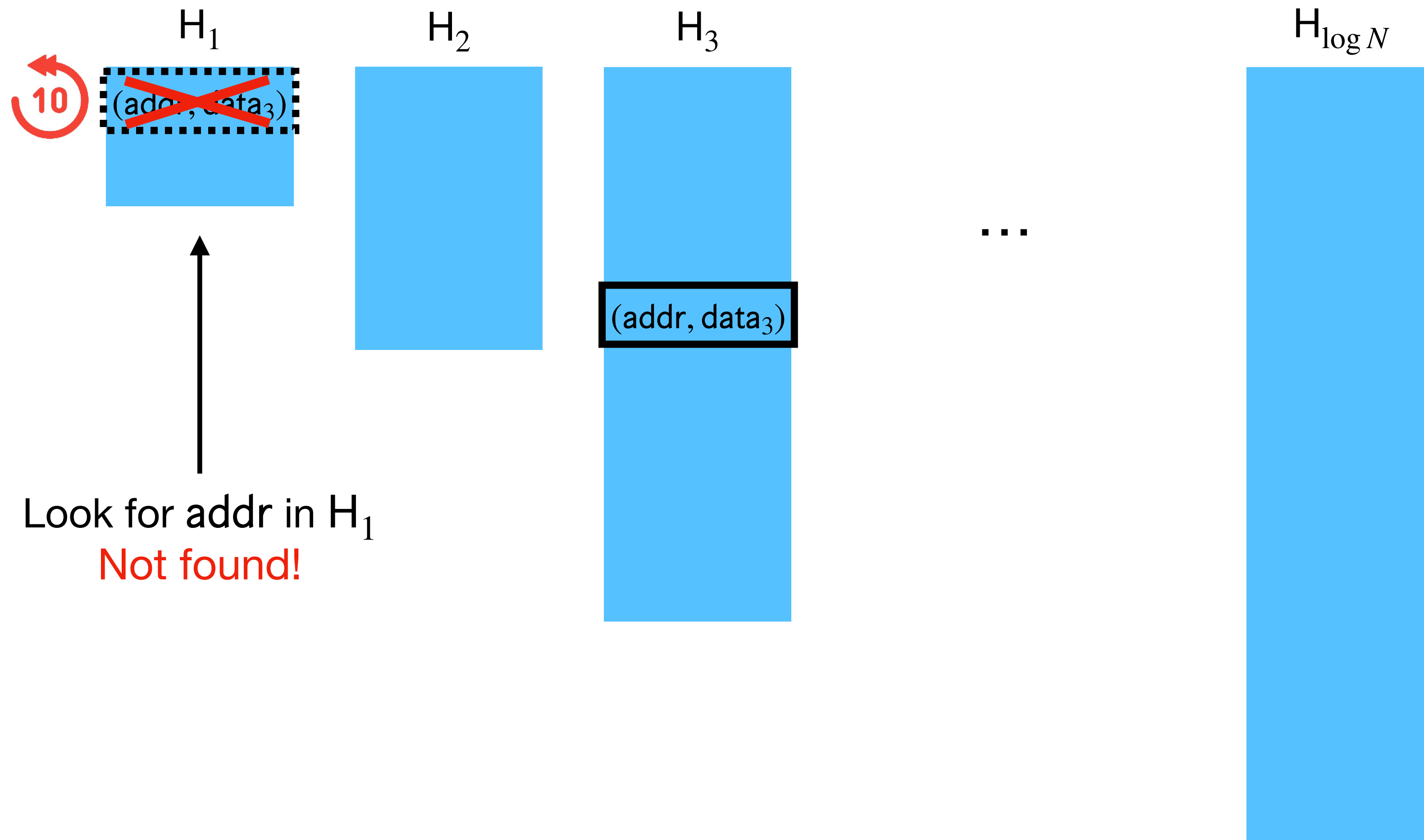


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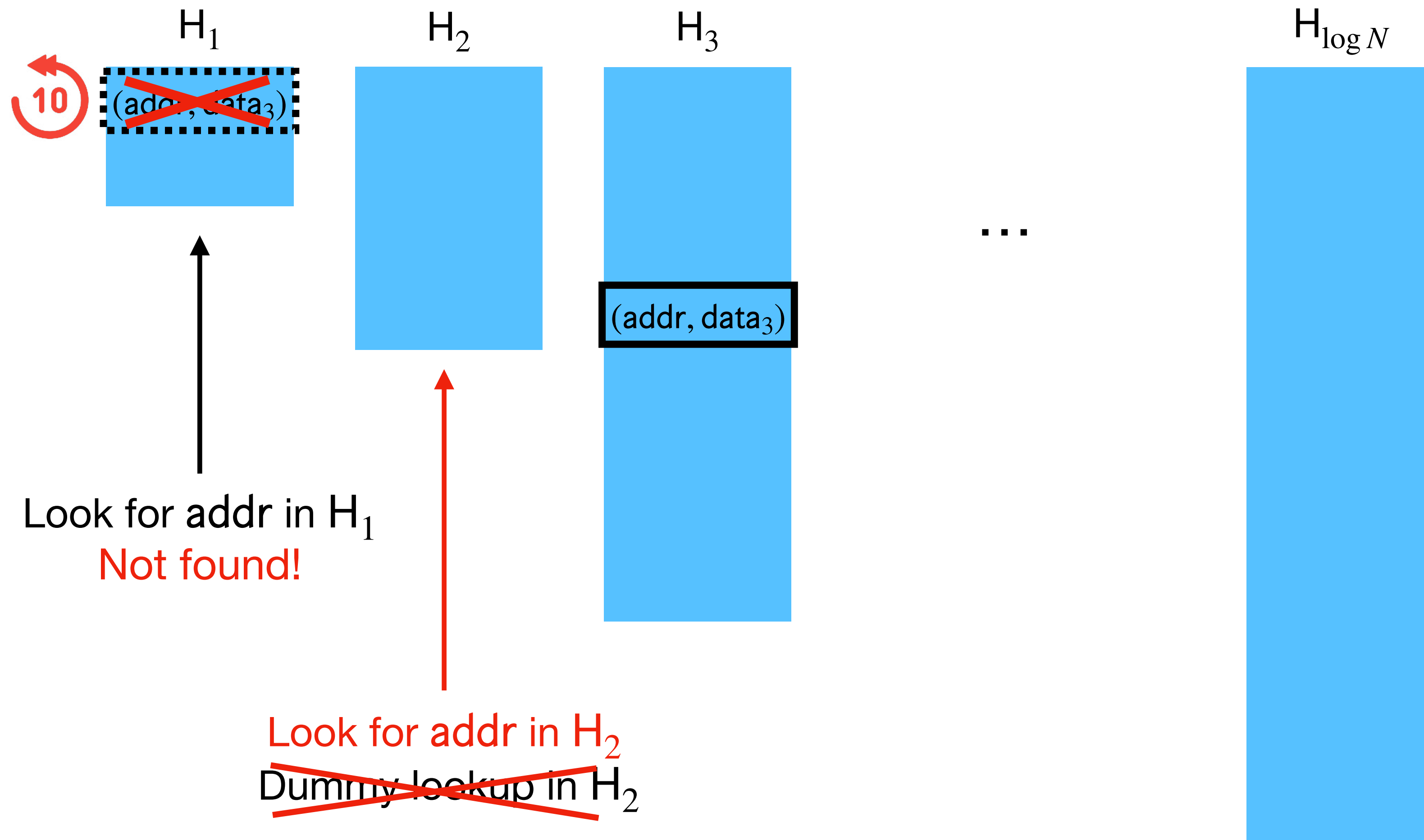


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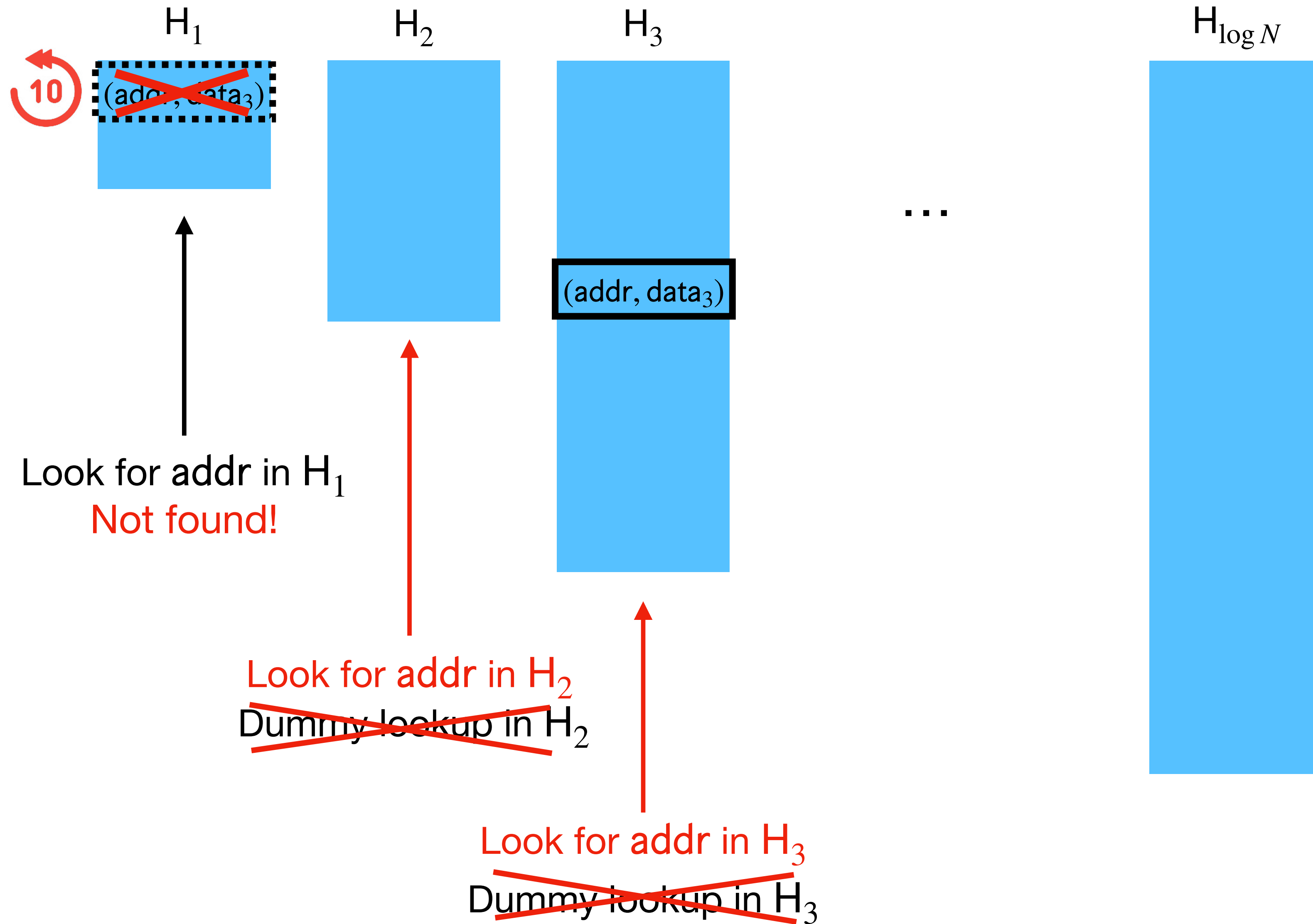
Write to addr:



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Write to addr:



**Lookup Phase**

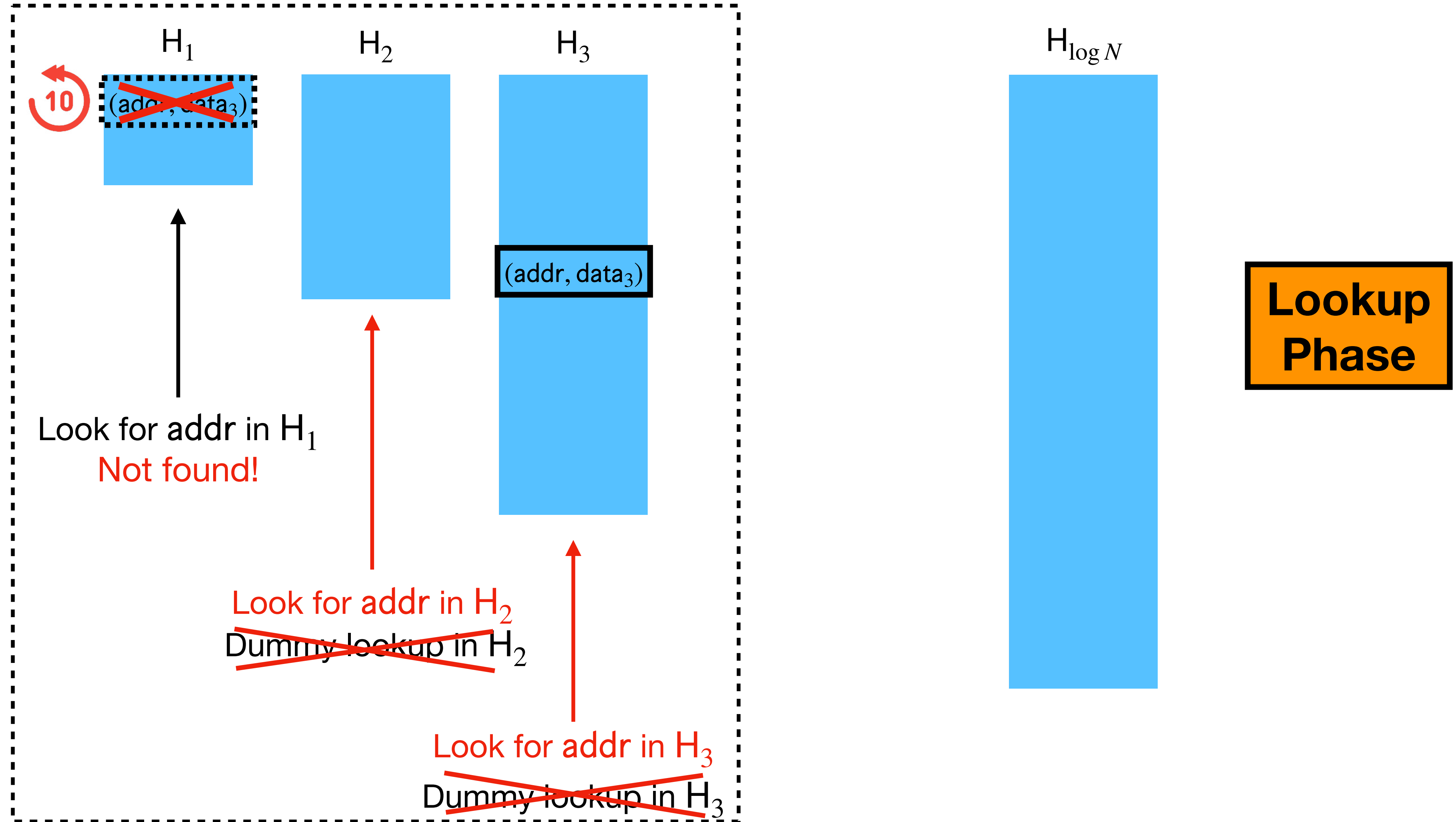
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Exact same  
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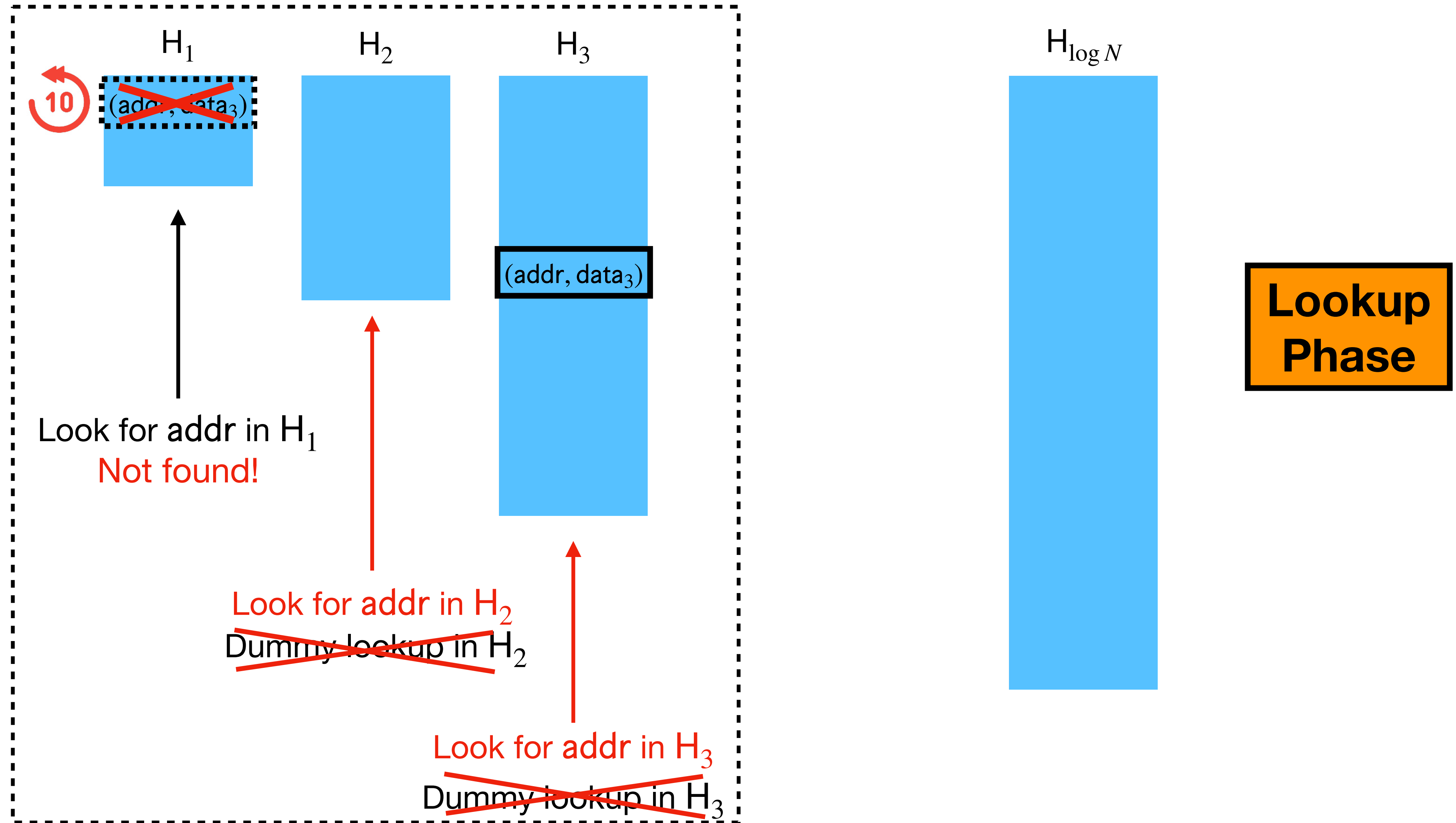
Read addr:

Write to addr:

Exact same  
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**Obliviousness**  
of  $H_i$  lookups  
depends on  
**correctness** of  
 $H_{<i}$  lookups!



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- As is, the hierarchical paradigm with MACs is susceptible to replay attacks, so it's still maliciously insecure.



# Replay Attack for Hierarchical

- As is, the hierarchical paradigm with MACs is susceptible to replay attacks, so it's still maliciously insecure.
- Is there a fix?

# General Fix For Replay Attacks: Time-Stamping

- [Ostrovsky '90, Goldreich-Ostrovsky '96] noticed that *time-stamping* is sufficient to prevent replay attacks with MACs (in their  $O(\log^3 N)$  ORAM).

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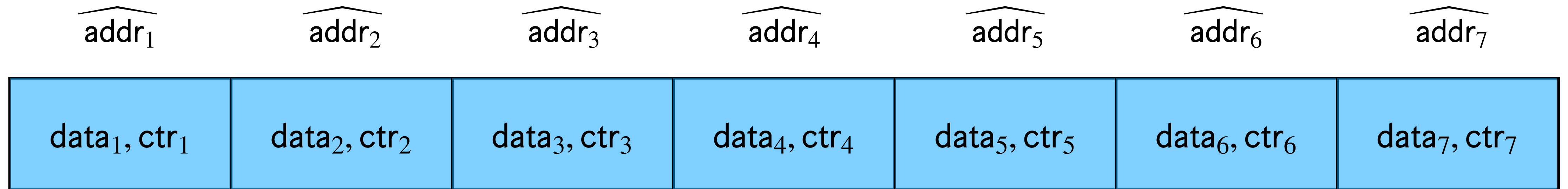
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- **Theorem** [GO '96]: If ORAM has **local, low-space** computable  $\text{PrevTime}$ , then MACs + time-stamping converts honest-but-curious ORAM to maliciously secure ORAM with the same asymptotic overhead.

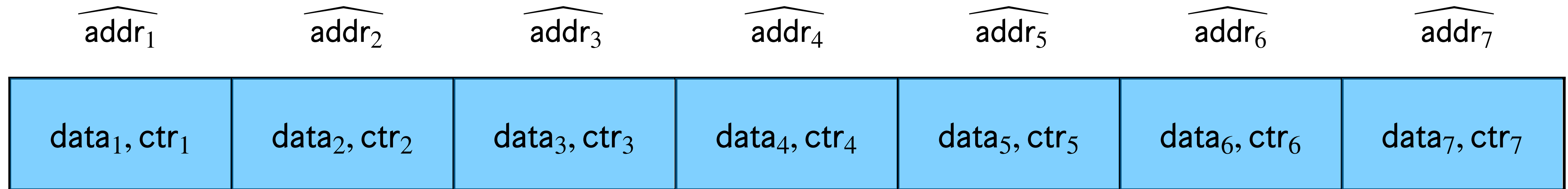
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Current time: ctr

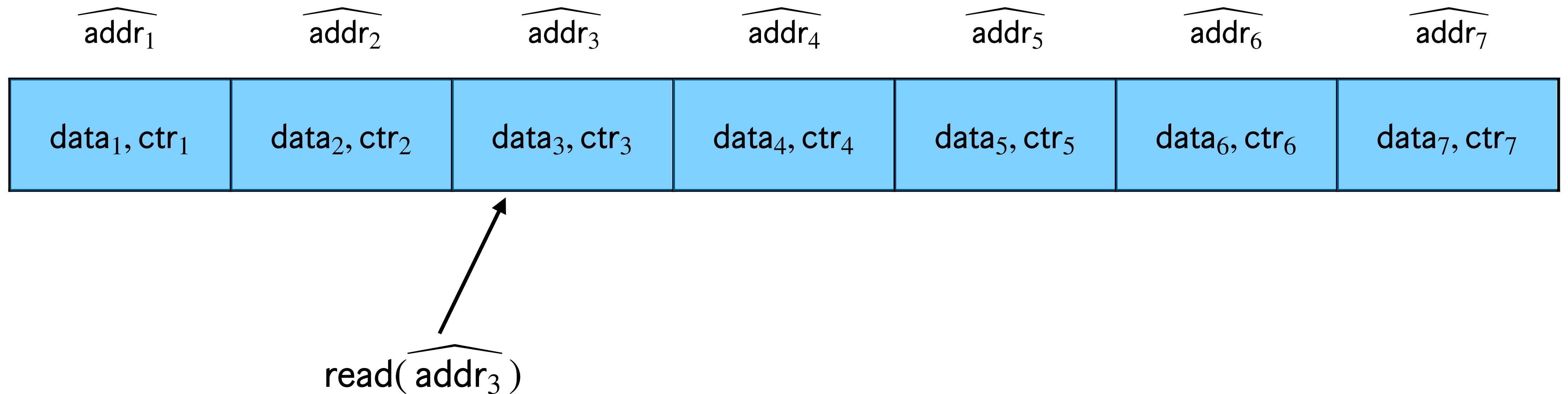


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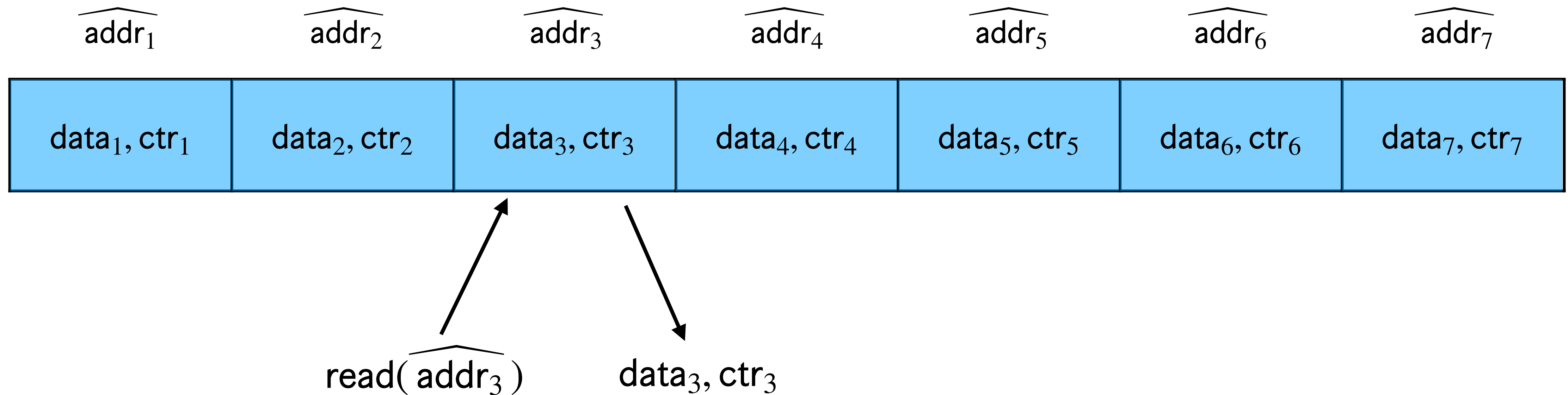
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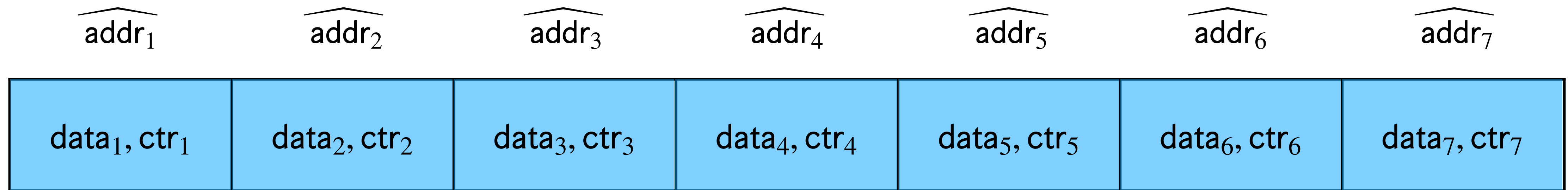
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read(addr<sub>3</sub>)

data<sub>3</sub>, ctr<sub>3</sub>

PrevTime(ctr, addr<sub>3</sub>) = ctr<sub>3</sub> ✓

PrevTime(ctr, addr) := most recent time (up until ctr)  
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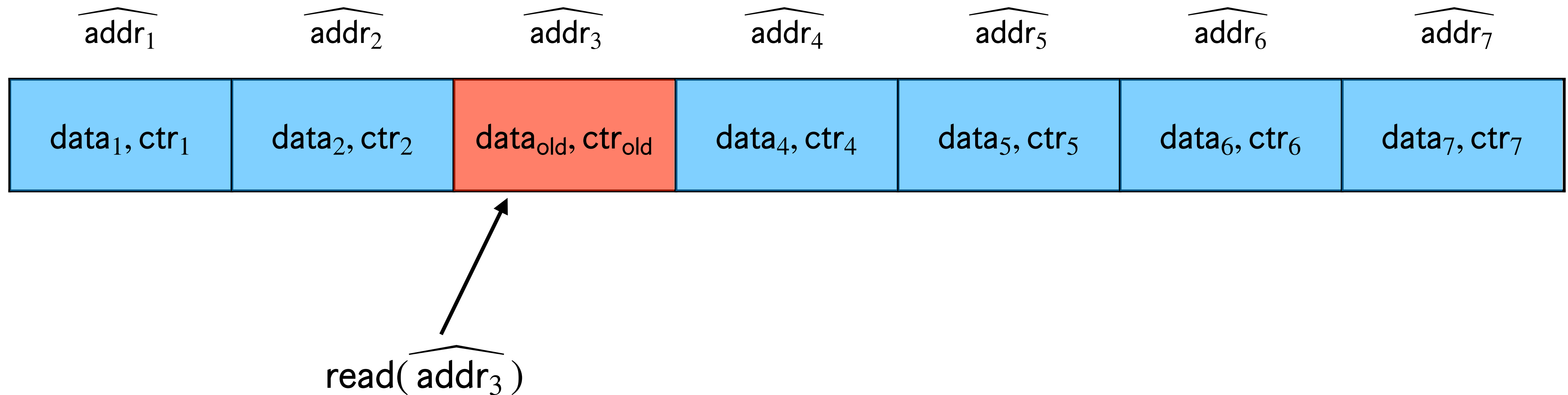
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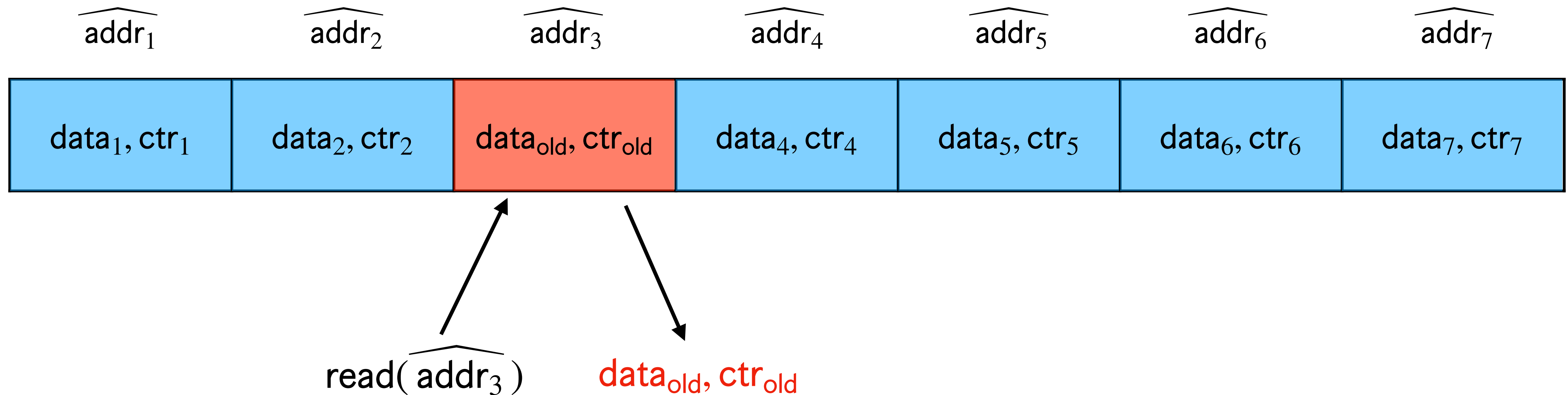
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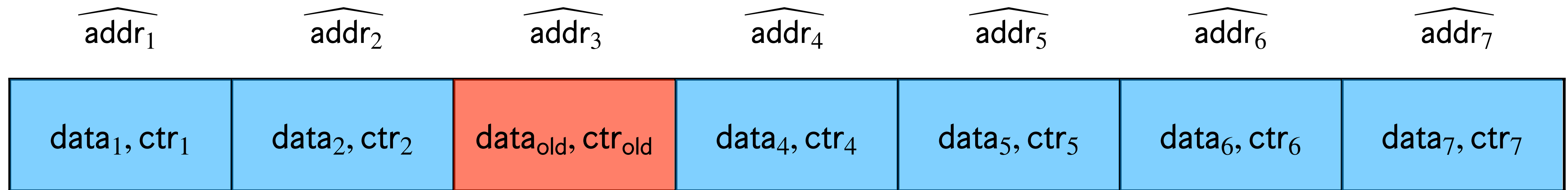
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$read(\widehat{addr}_3)$

$data_{old}, ctr_{old}$

Since  $ctr_{old} < ctr_3 = \text{PrevTime}(ctr, \widehat{addr}_3)$ ,

**replay attack detected!**

$\text{PrevTime}(ctr, \widehat{addr}) :=$  most recent time (up until ctr)  
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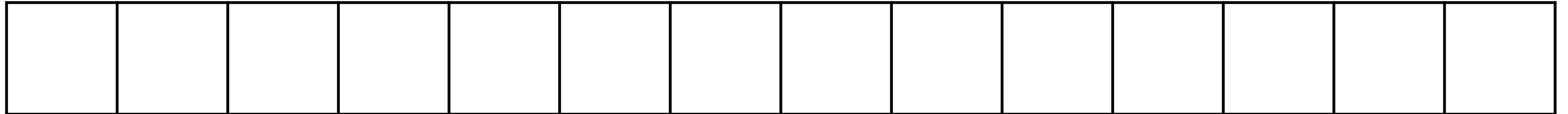
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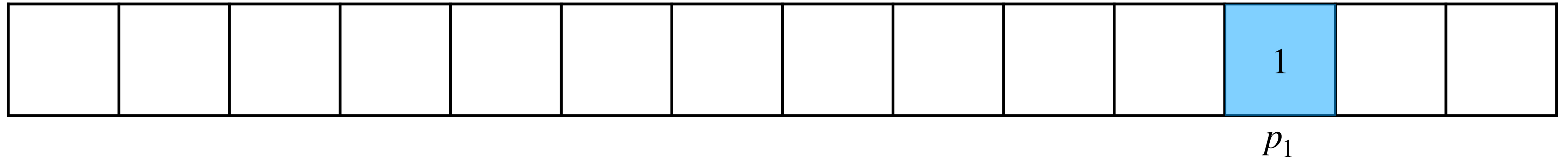
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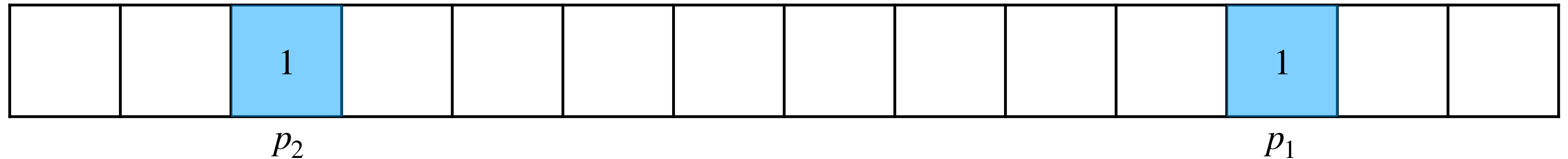
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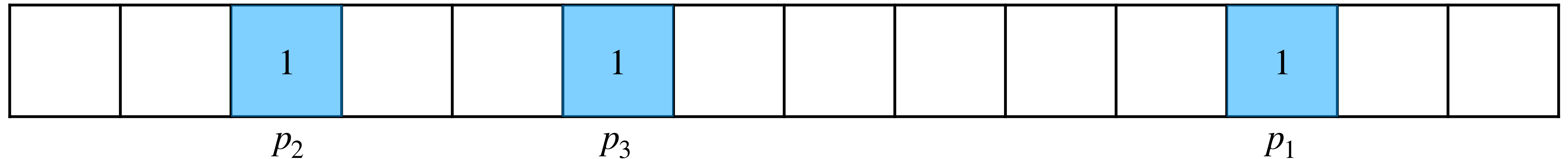
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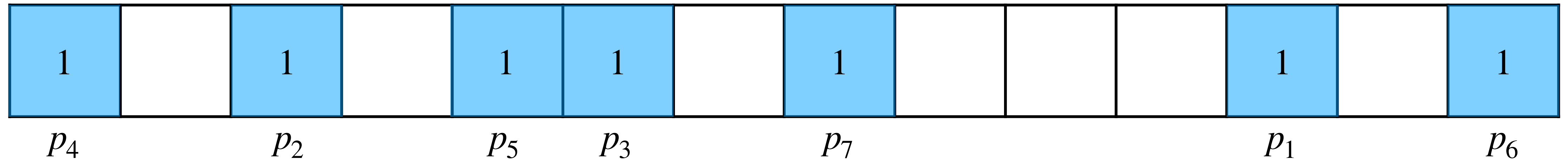
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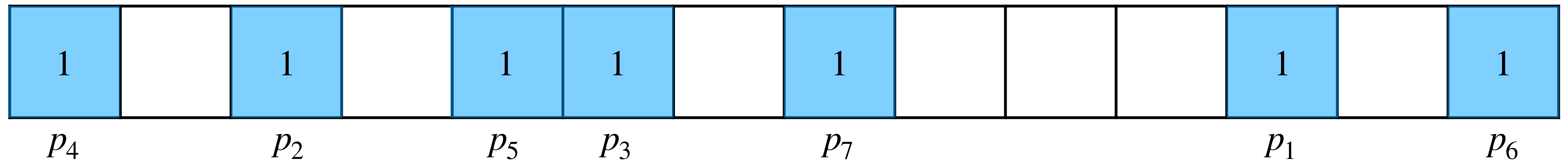
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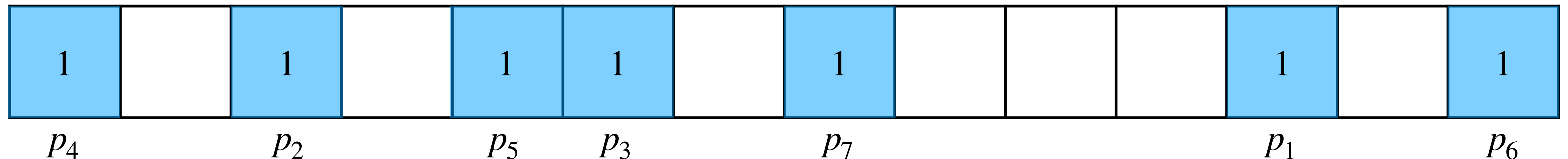
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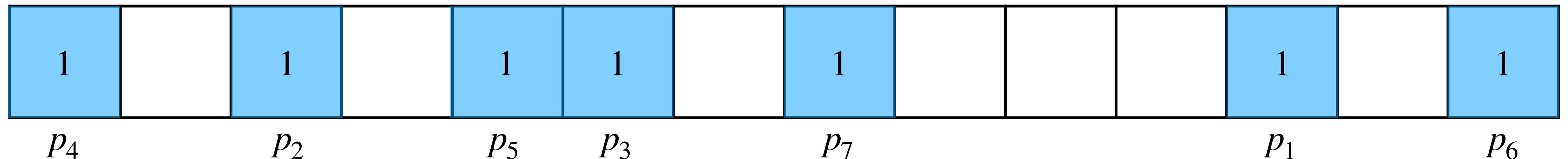
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- Random sequence of  $p_i$  has entropy  $\Theta(N \log N)$ , so no way to time-stamp with even  $O(N)$  bits of space, let alone  $O(\log N)$  bits.

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- We need another technique for malicious security!

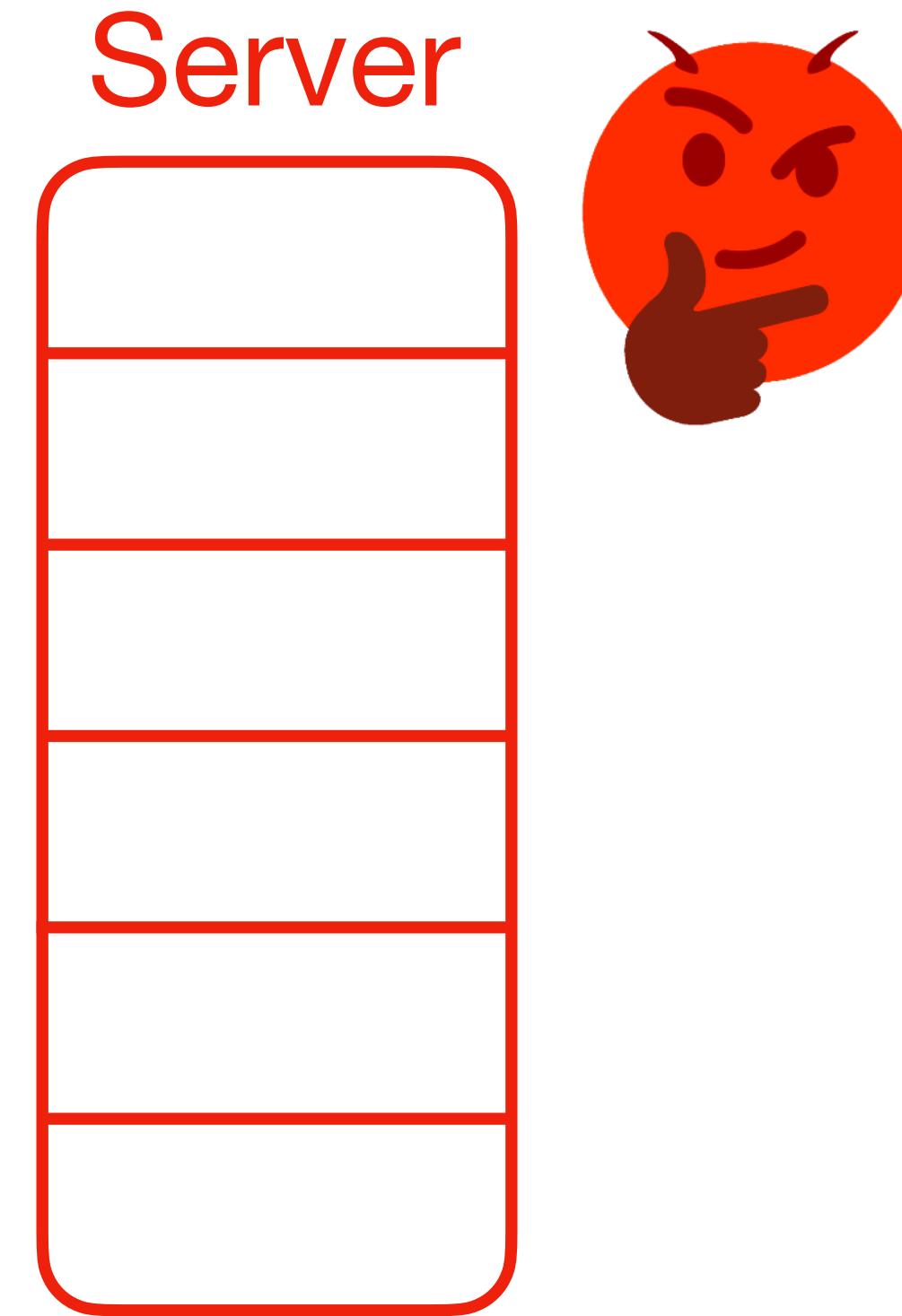
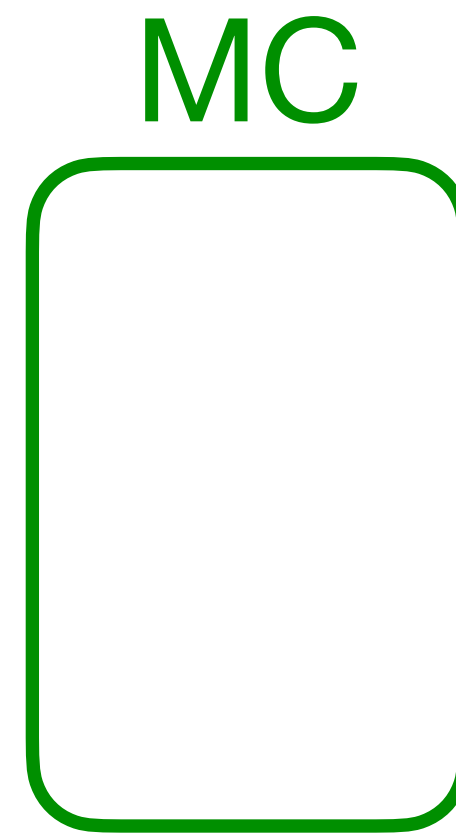
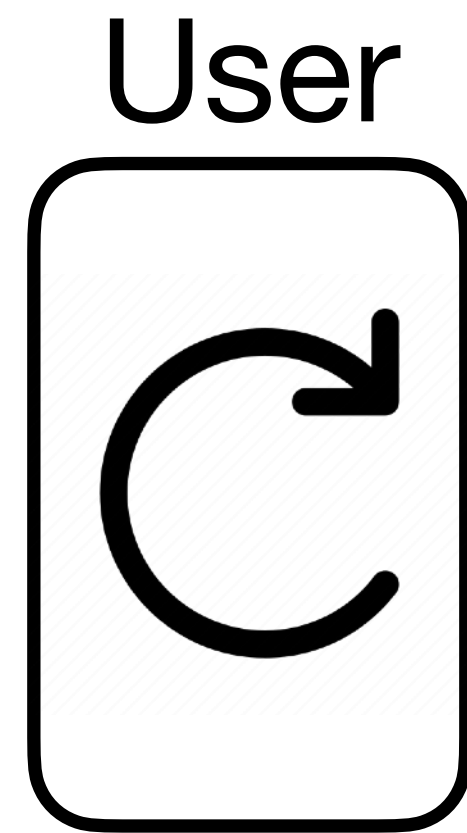
# Technique #2: Memory Checking



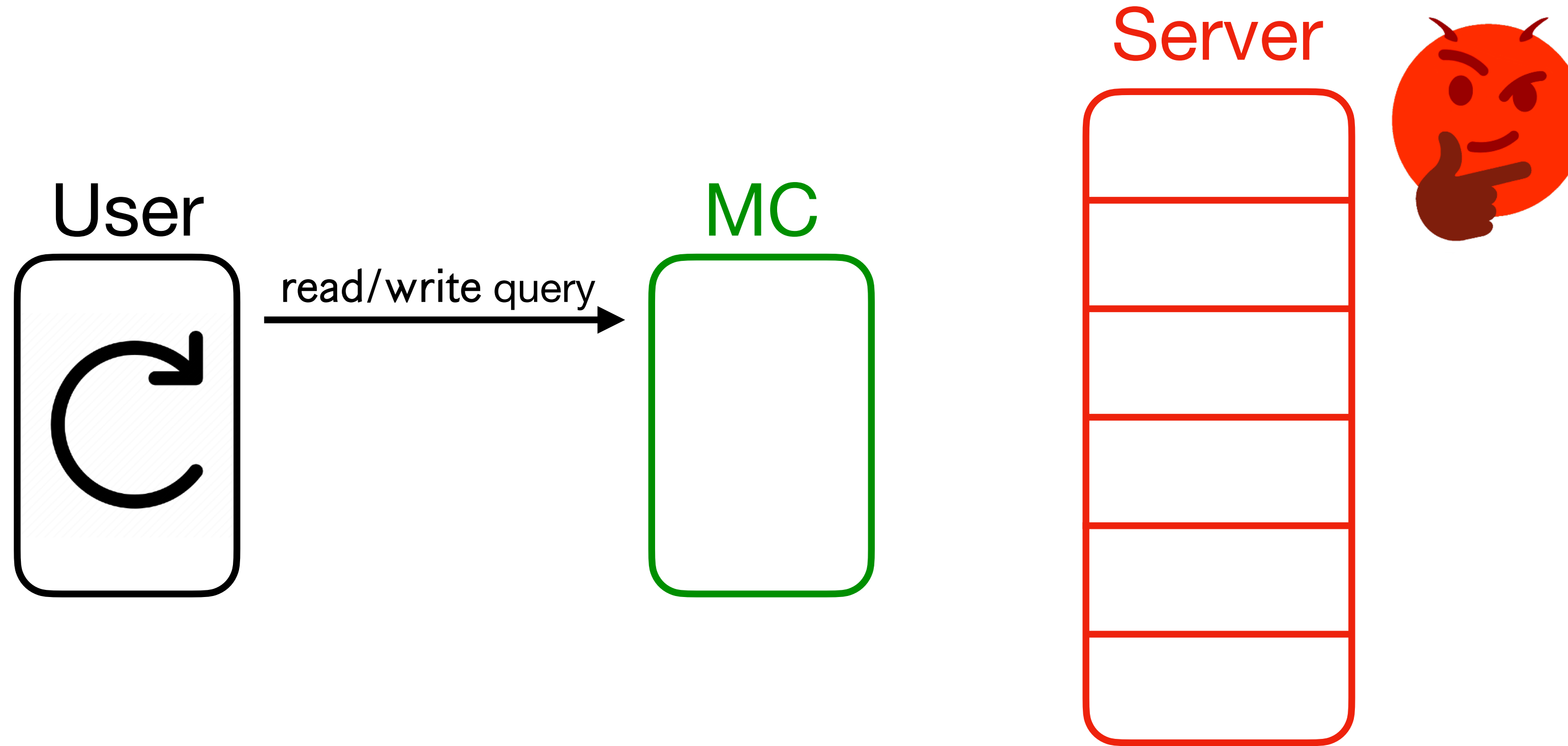
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- A **Memory Checker** (MC) is a protocol that detects whether a malicious server tampered with RAM. [Blum, Evans, Gemmell, Kannan, Naor '94]

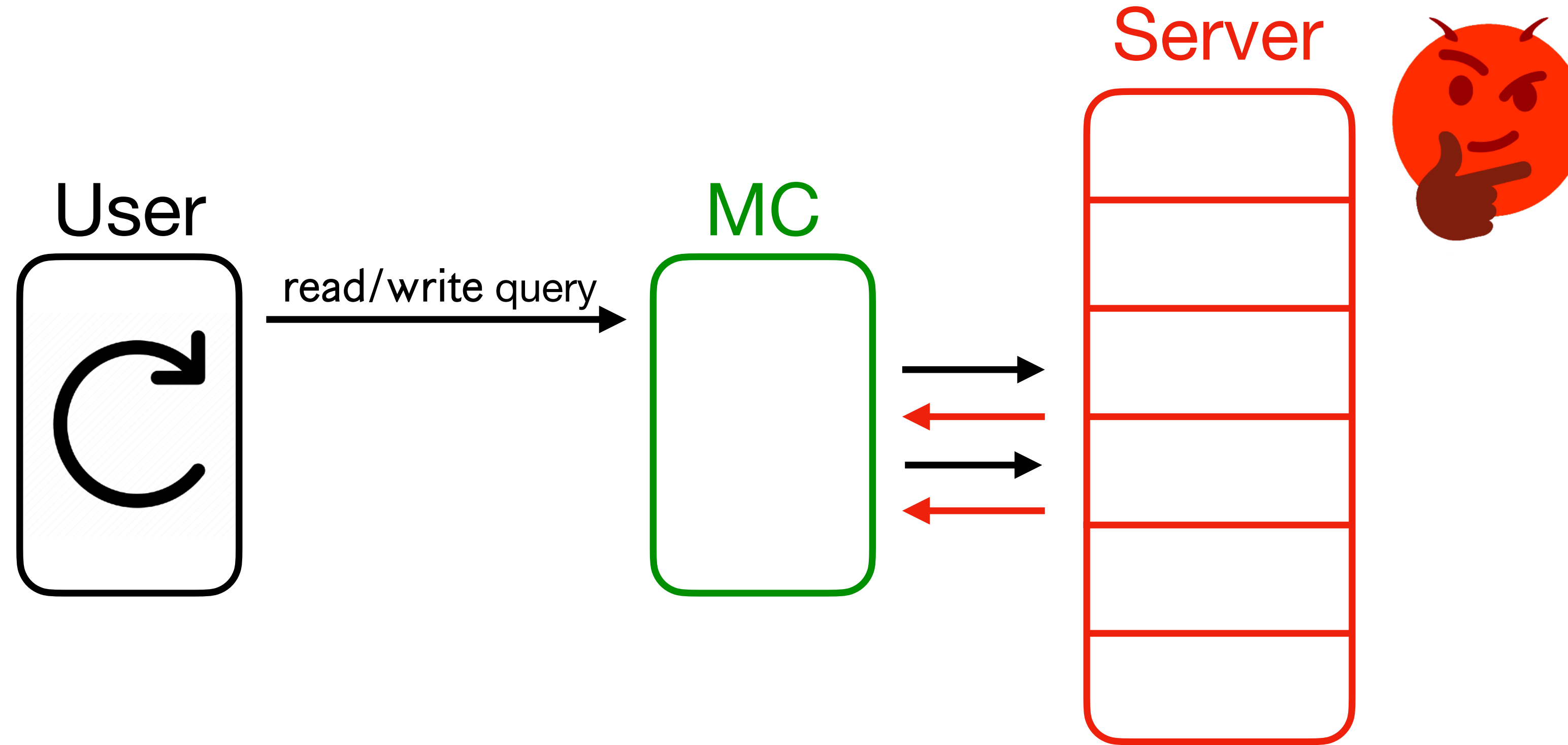
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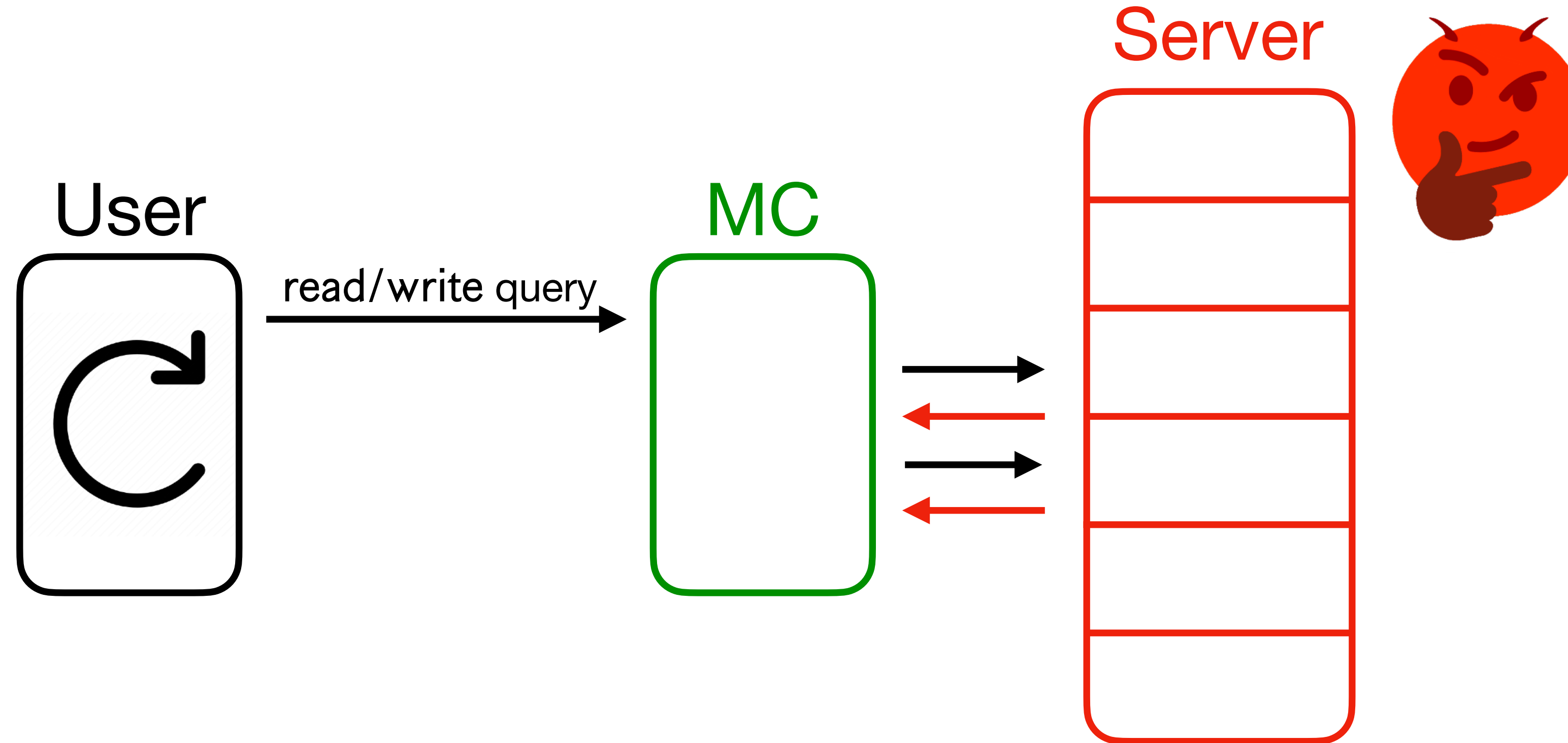
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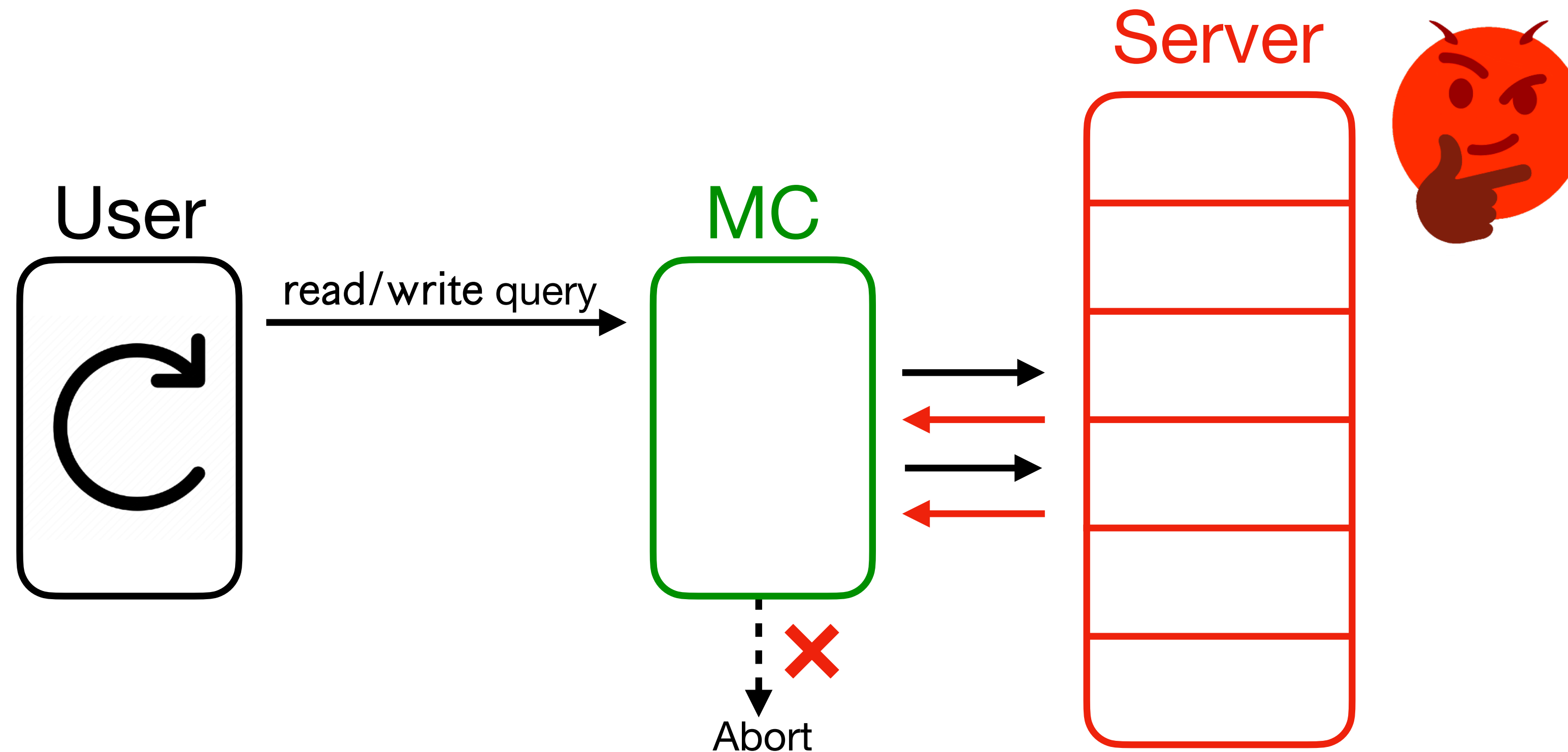


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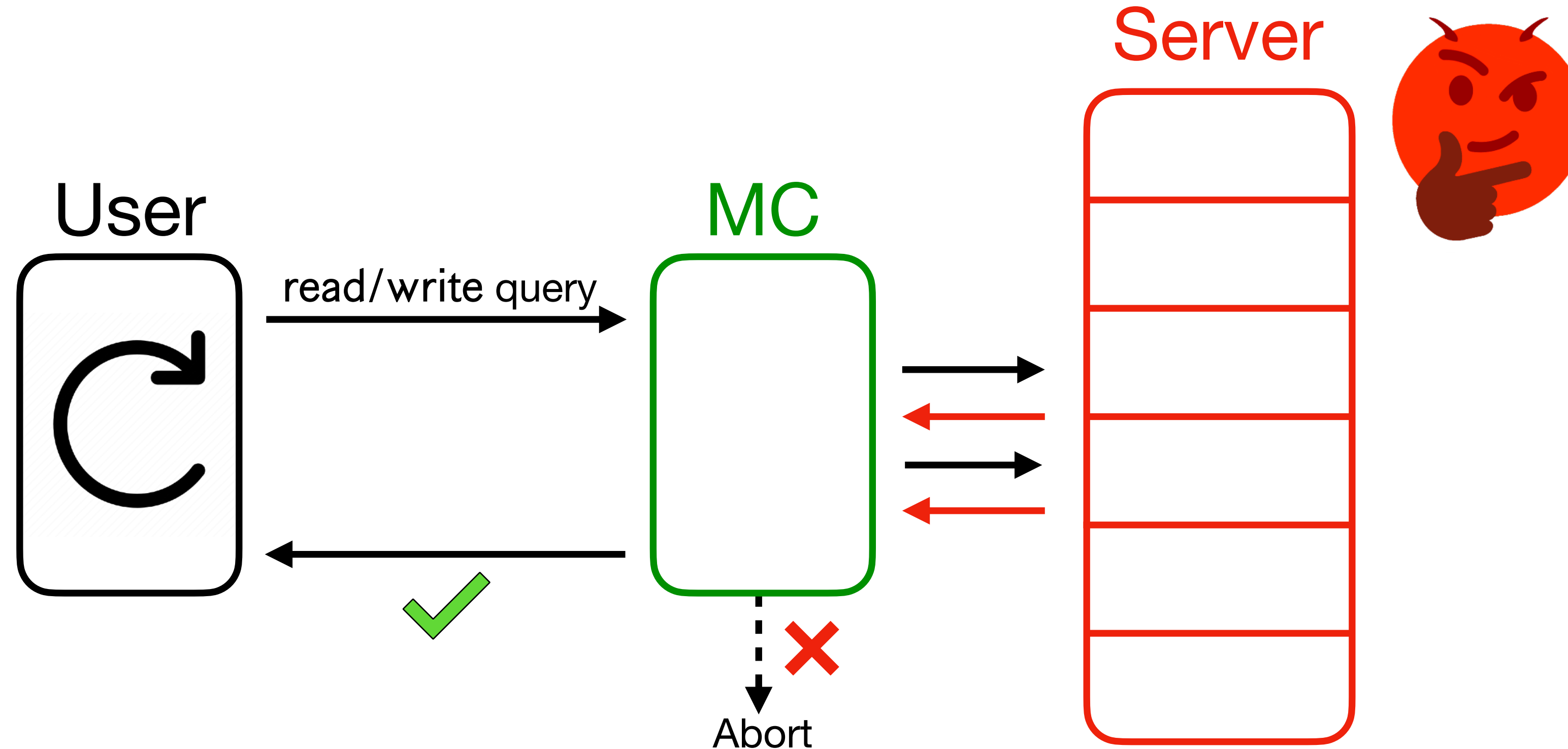
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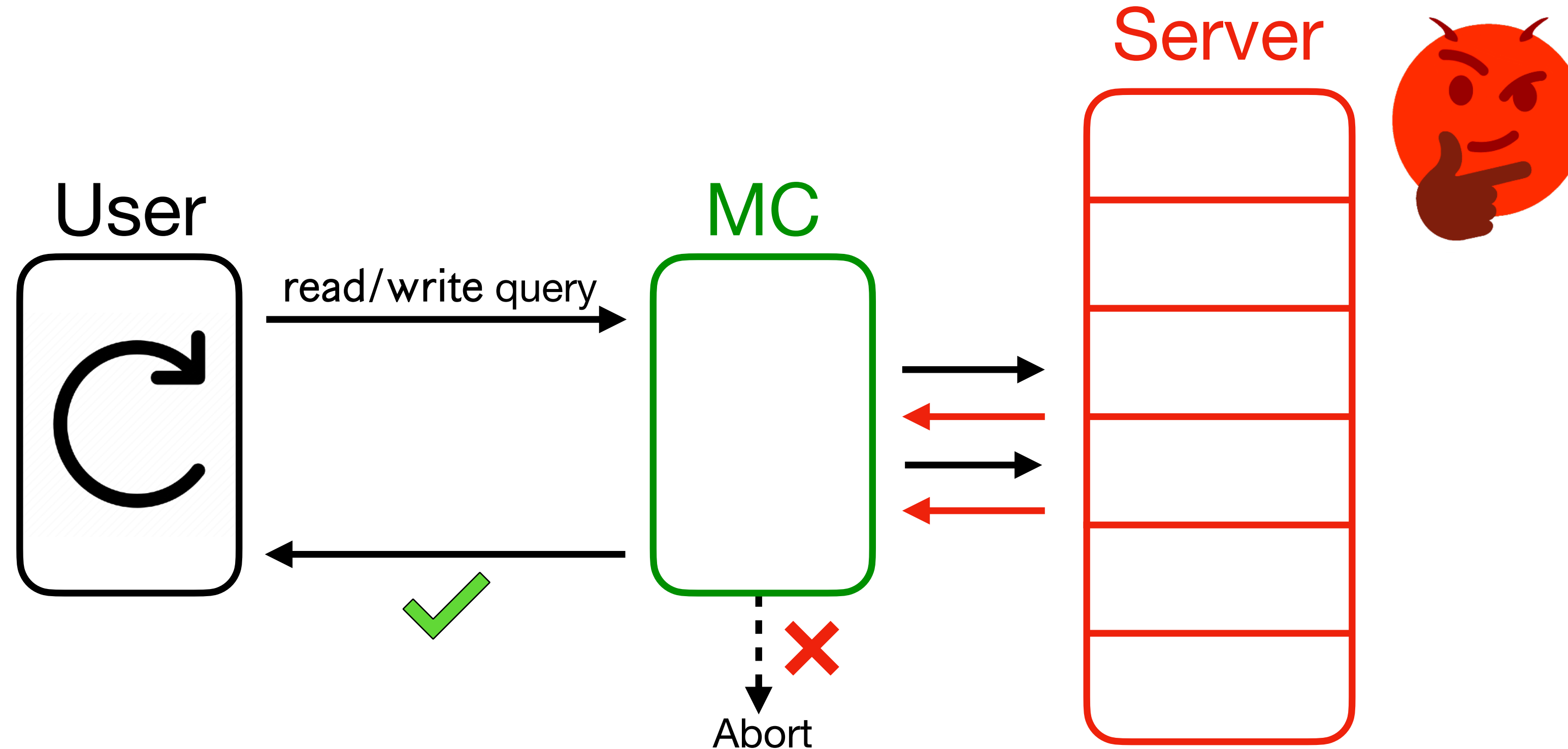
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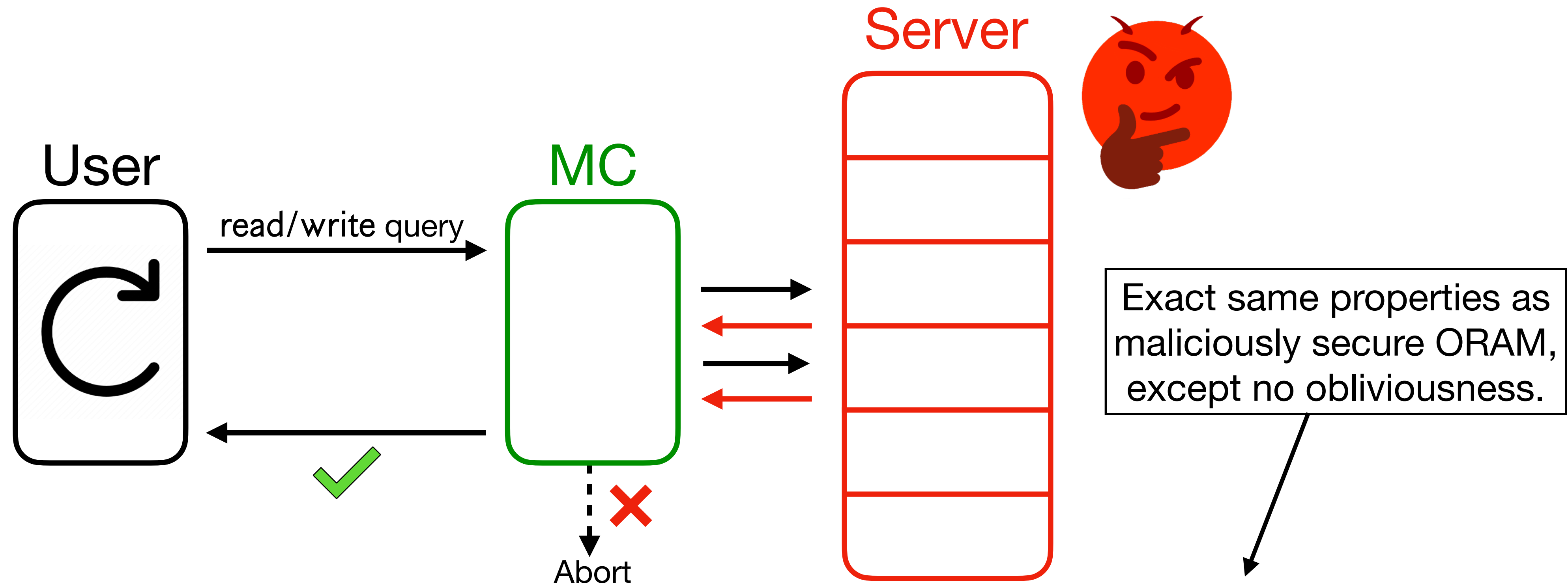
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  - **Lower bound** of  $\Omega(\log N / \log \log N)$  overhead for deterministic, non-adaptive memory checkers (which the existing constructions are).

[Dwork-Naor-Rothblum-Vaikuntanathan '09]

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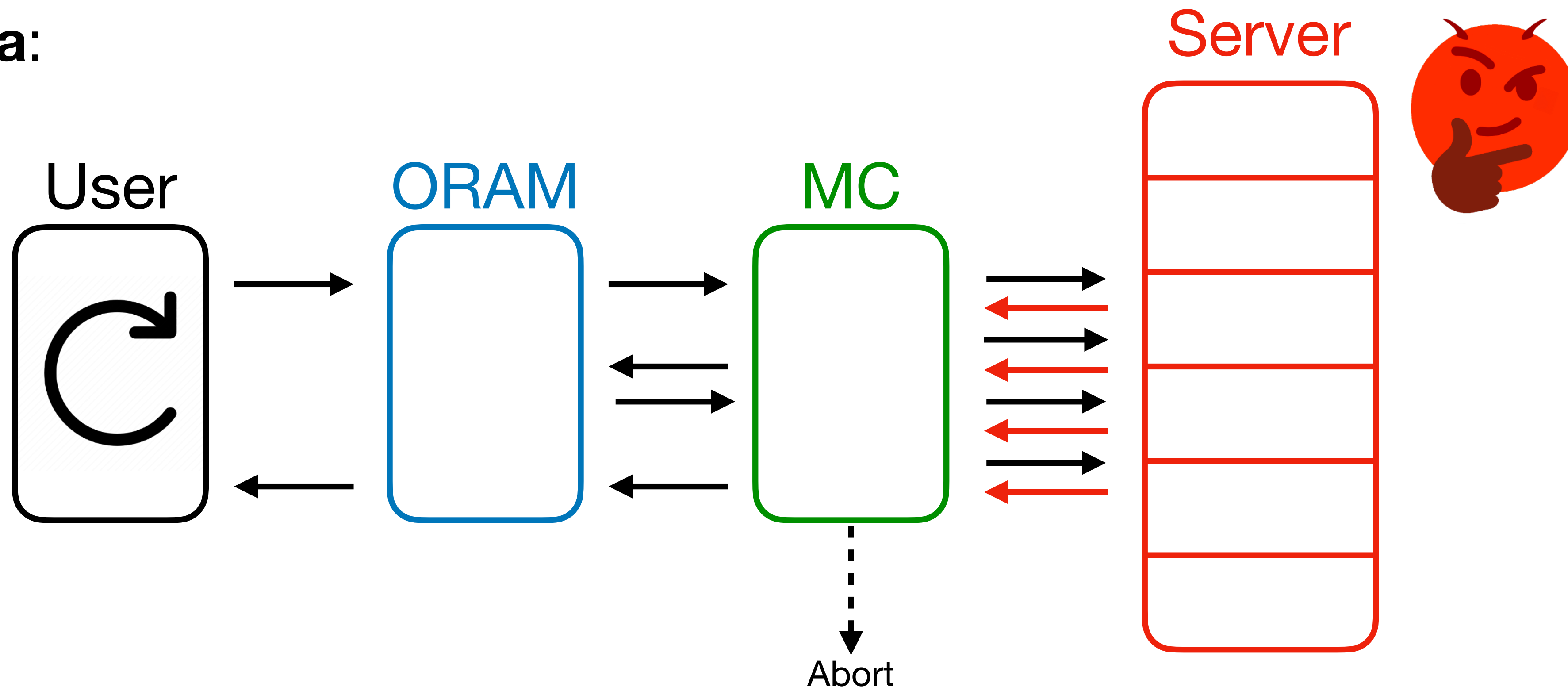
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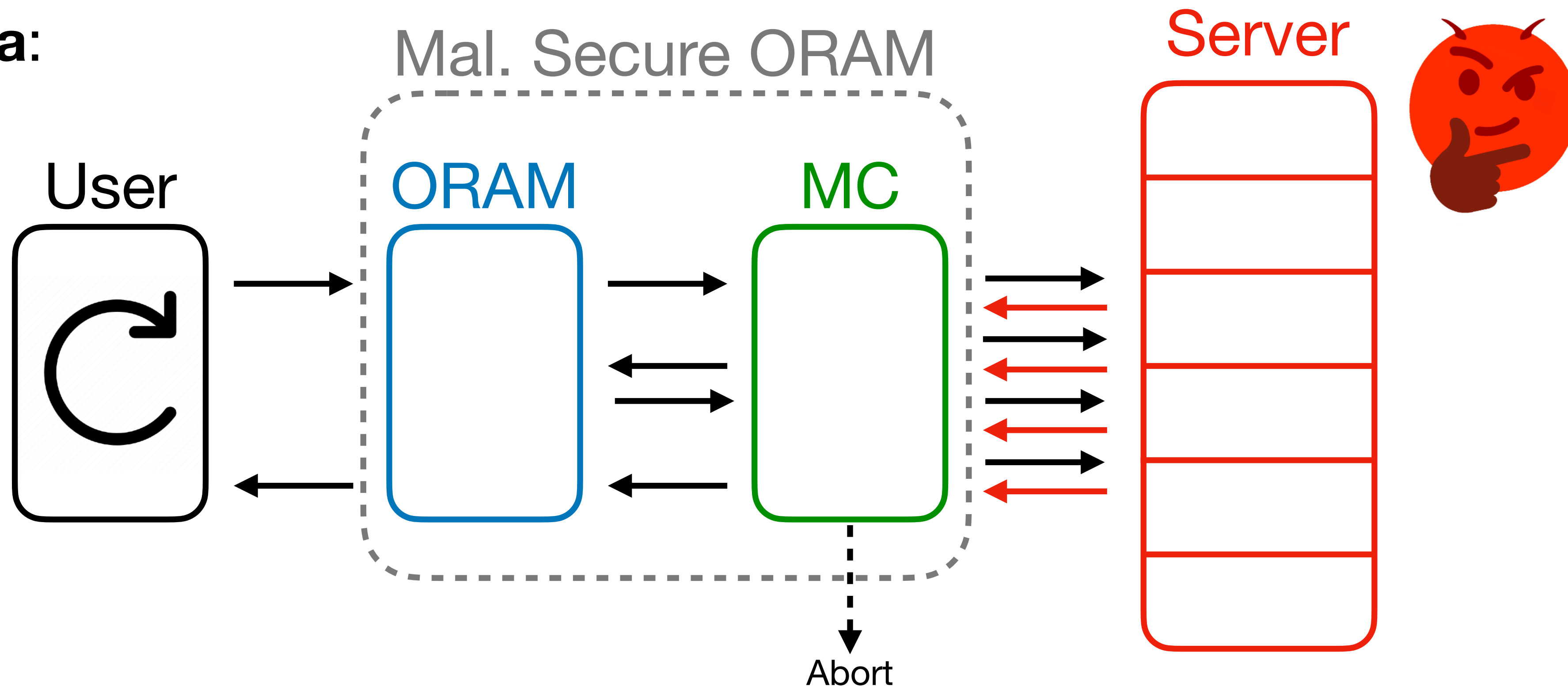
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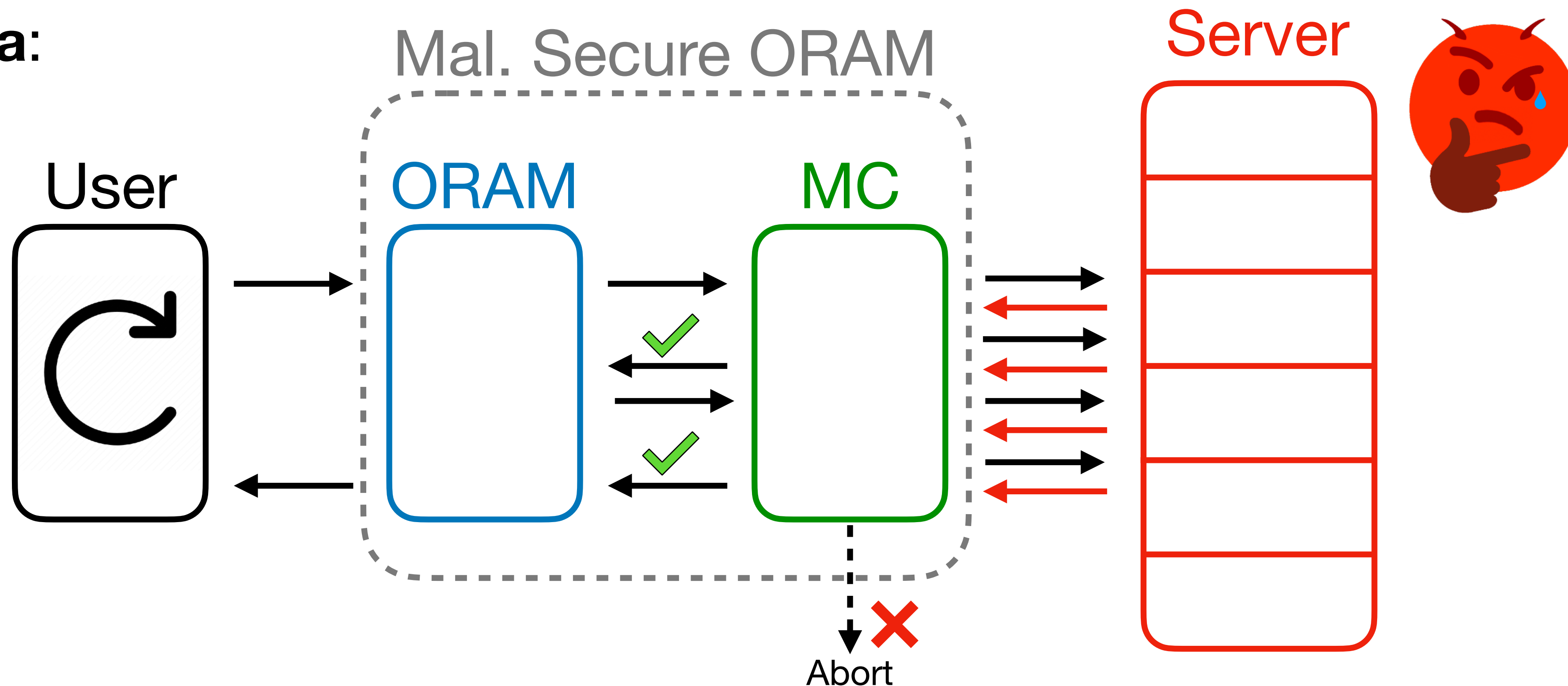
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$$\log^2(N)$$



$$\log N$$



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$$\text{Overhead}(\text{ORAM}_{\text{Mal}}) = \text{Overhead}(\text{ORAM}_{\text{HBC}}) \cdot \text{Overhead}(\text{MC})$$

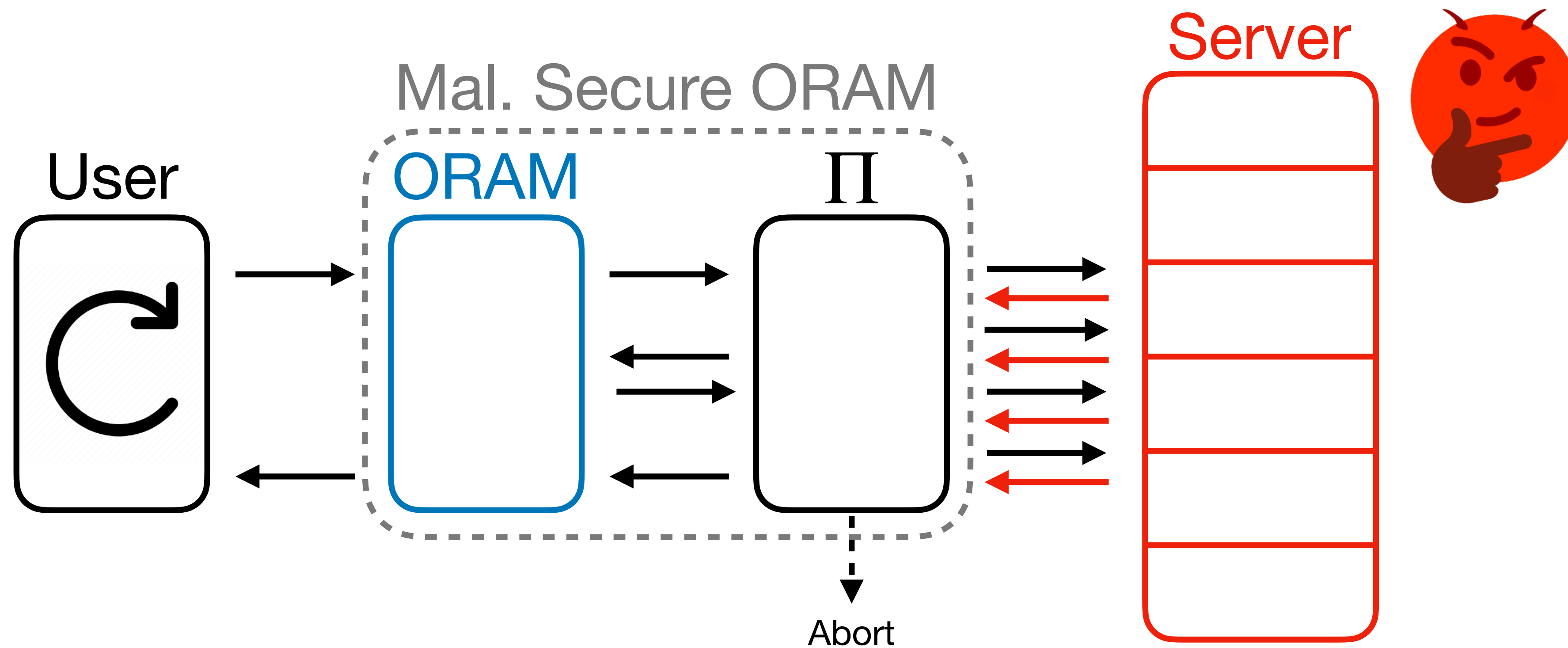
$$\log^2(N)$$


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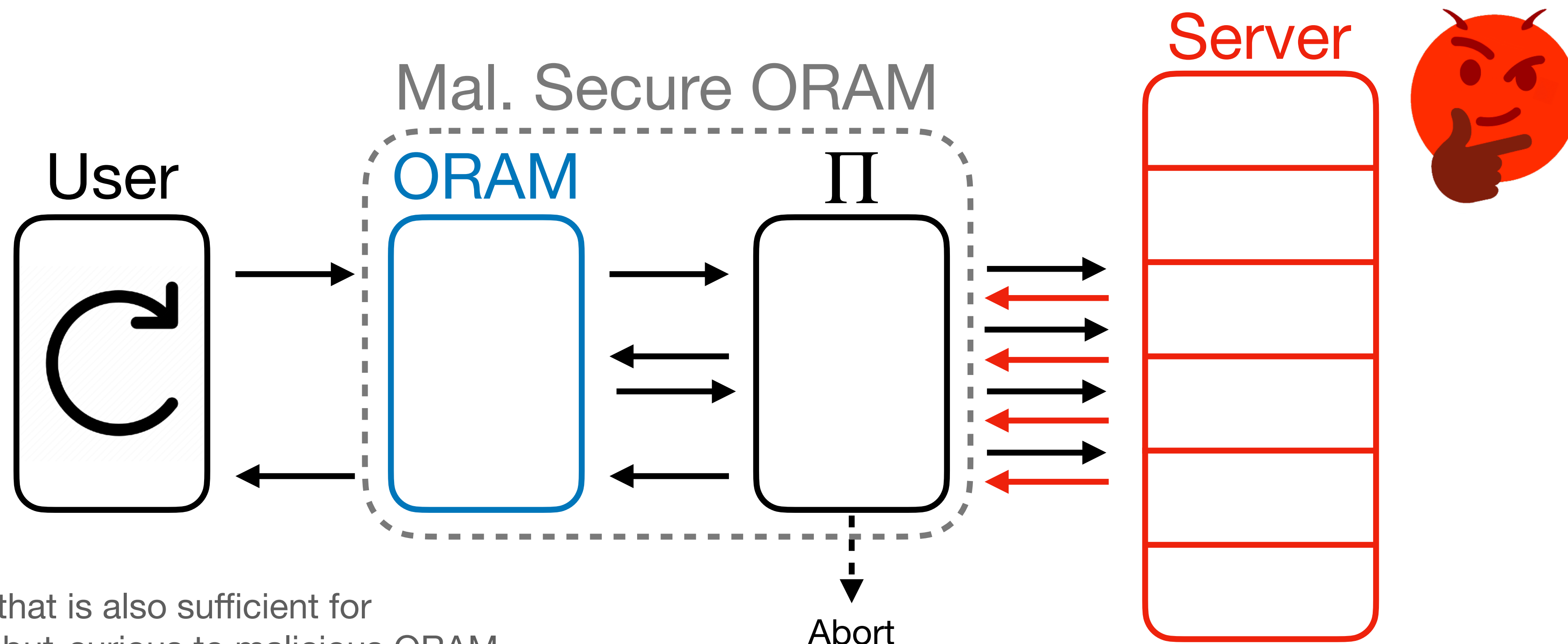
- Do we really need a memory checker? Does a weaker compiler suffice?

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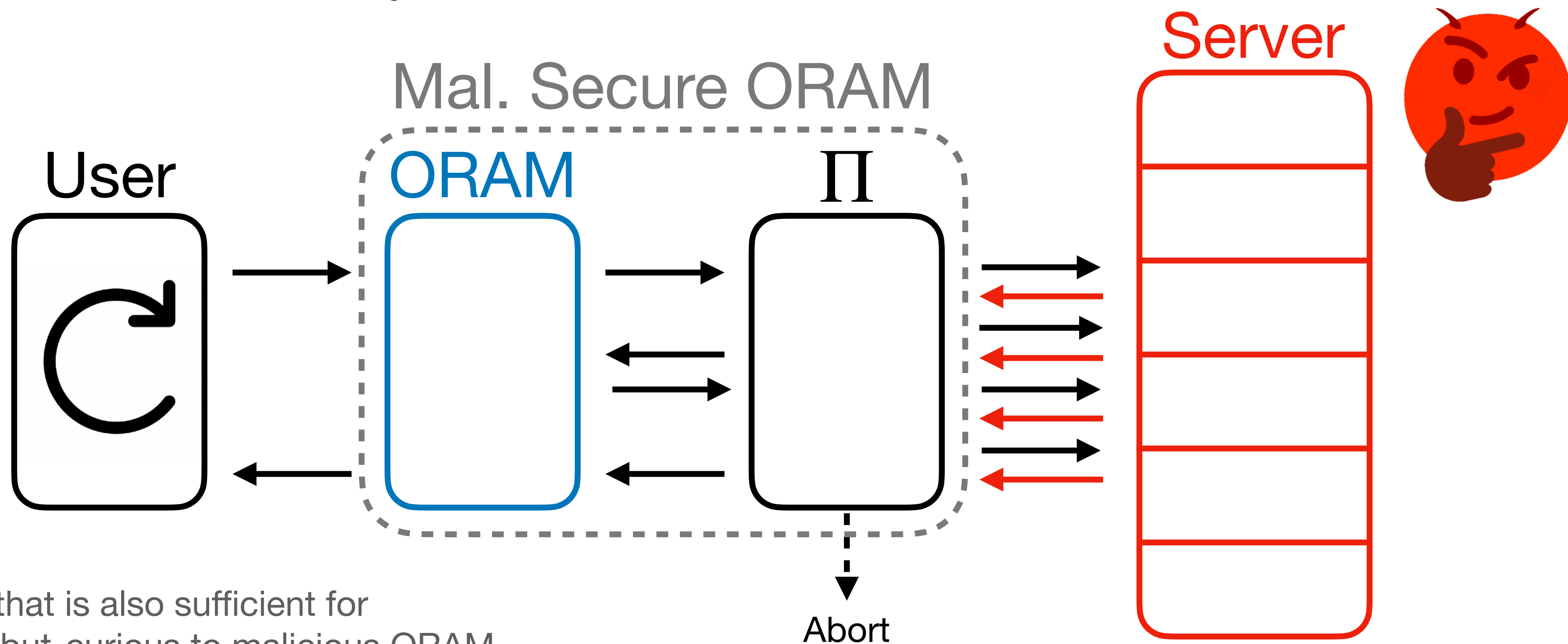


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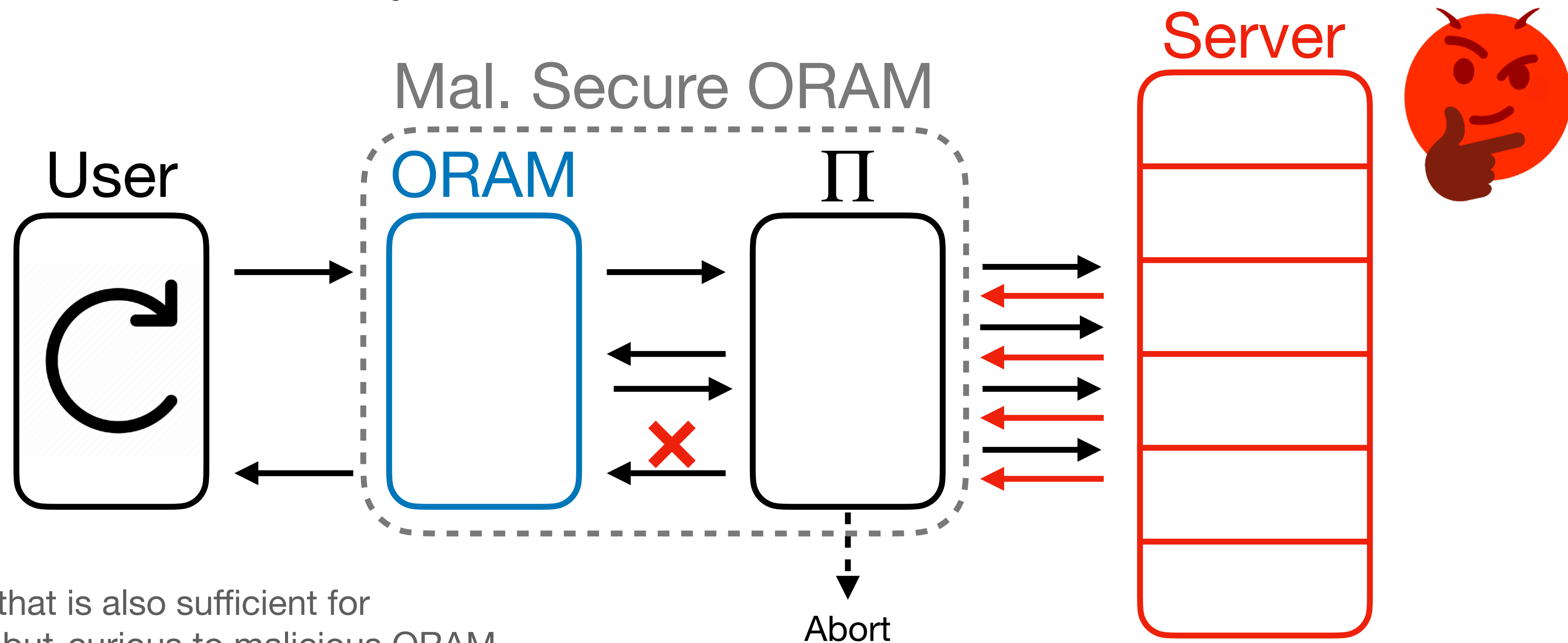
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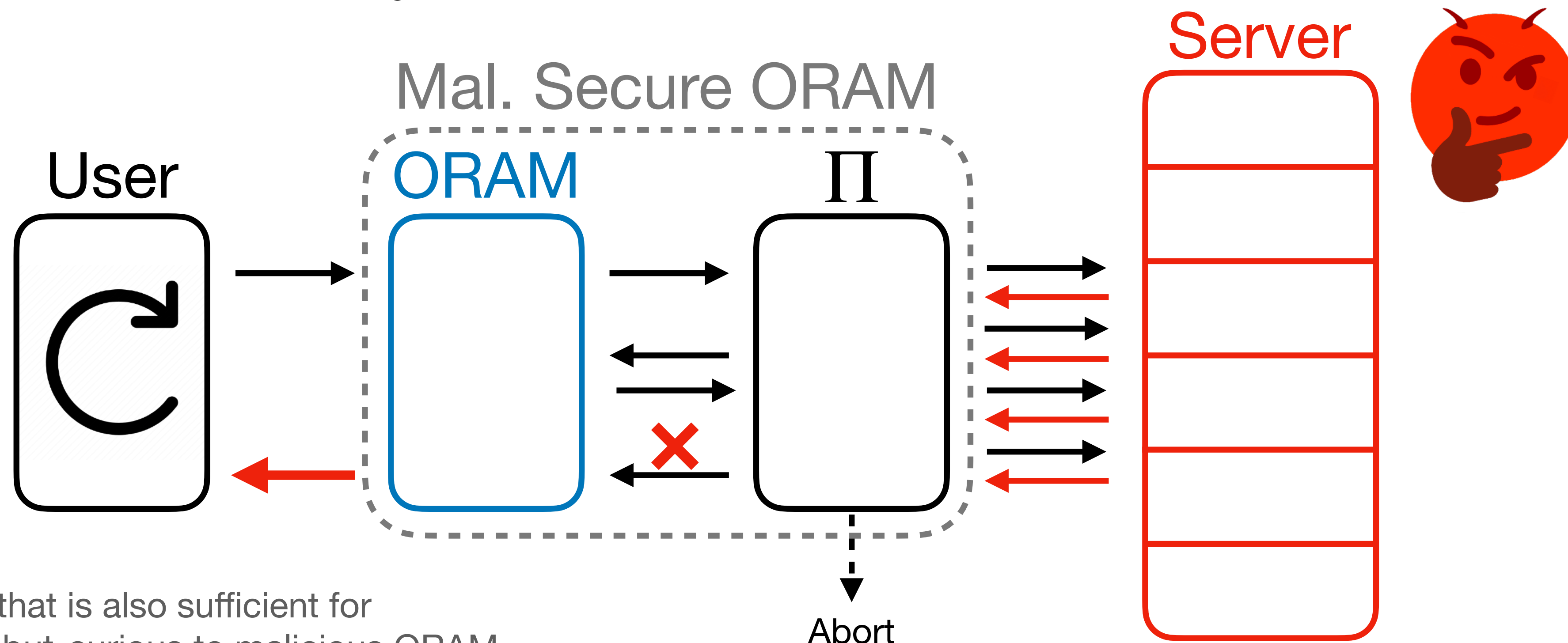


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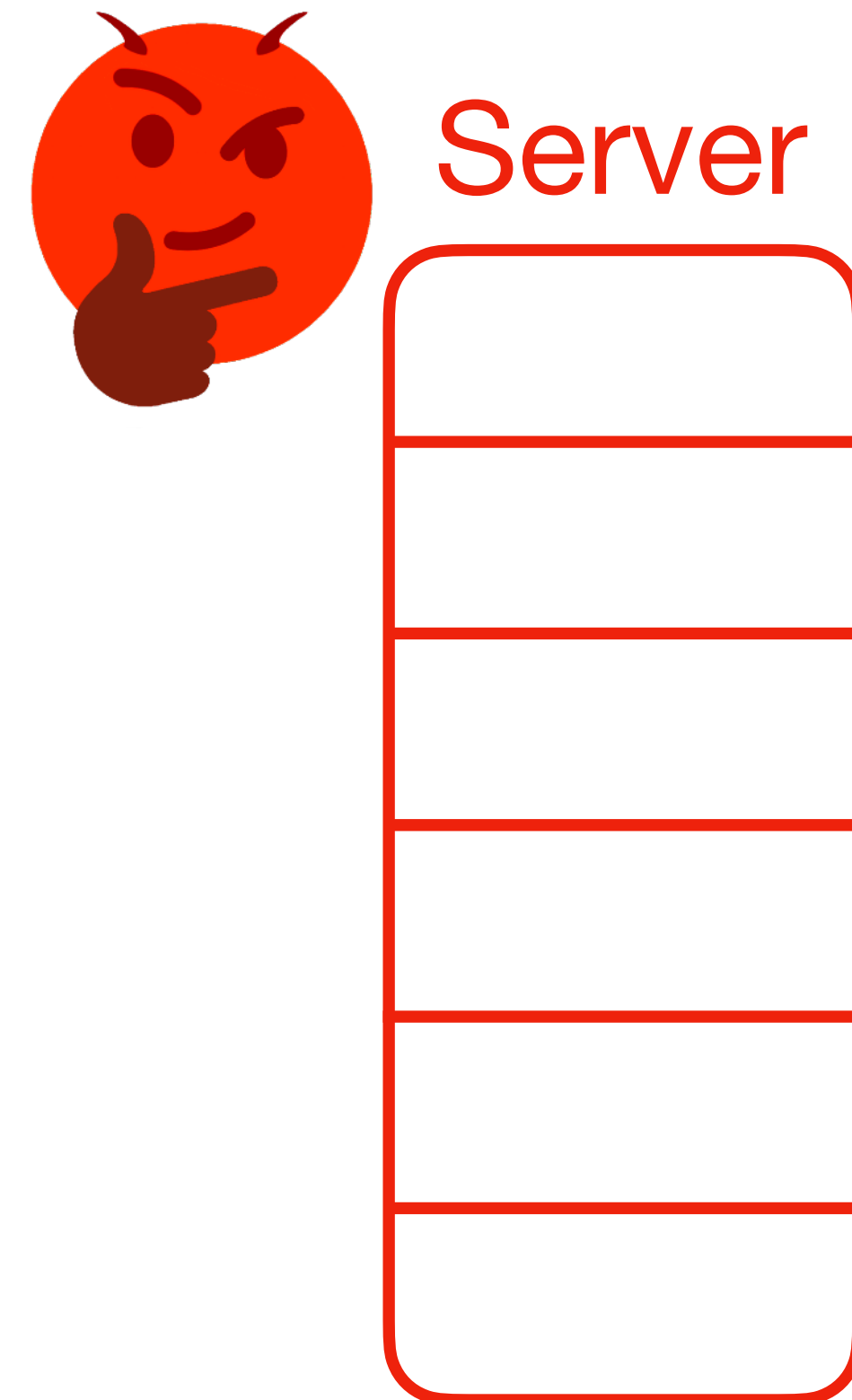
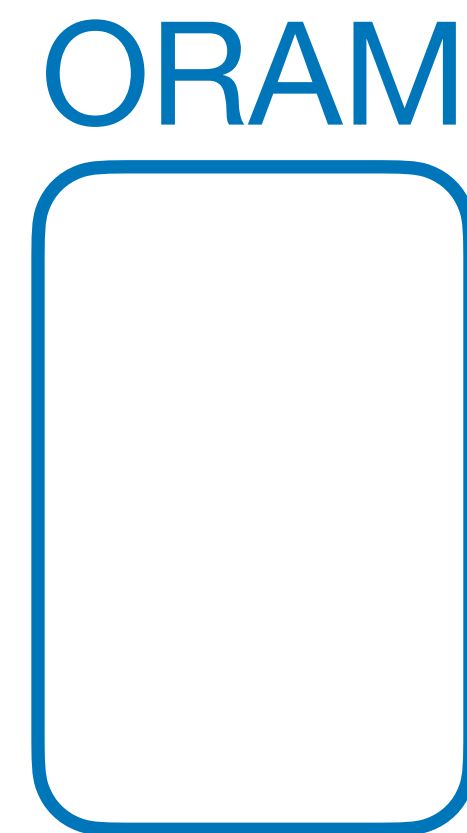
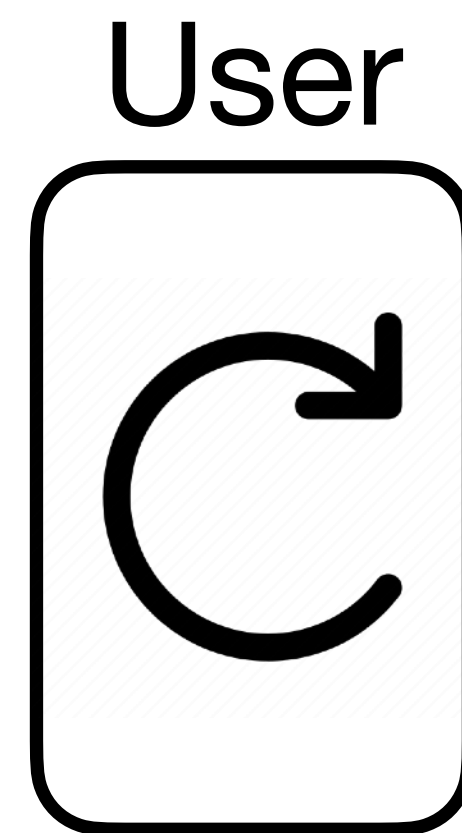
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**We have to handle OptORAMa in a white-box way!**

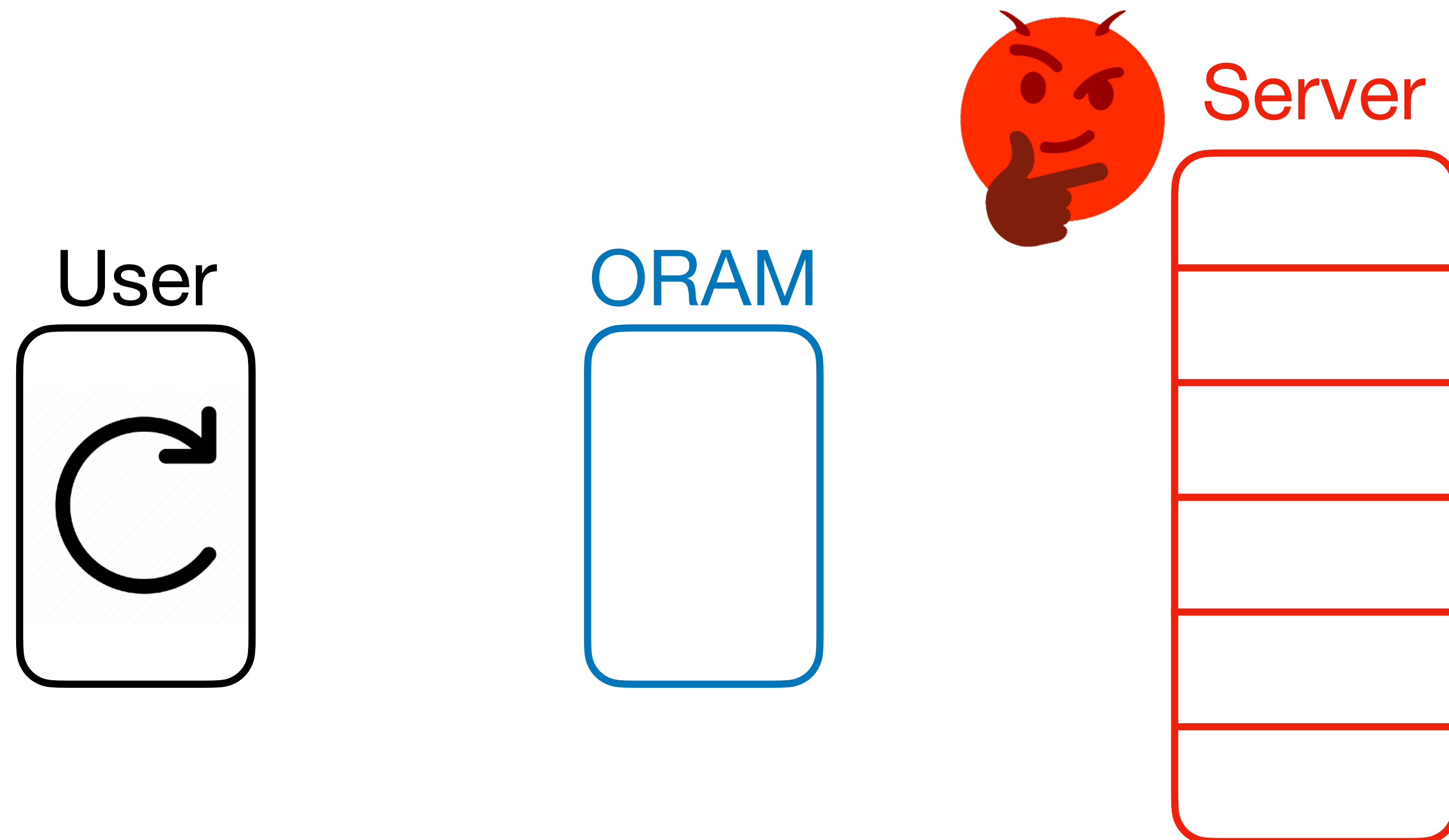
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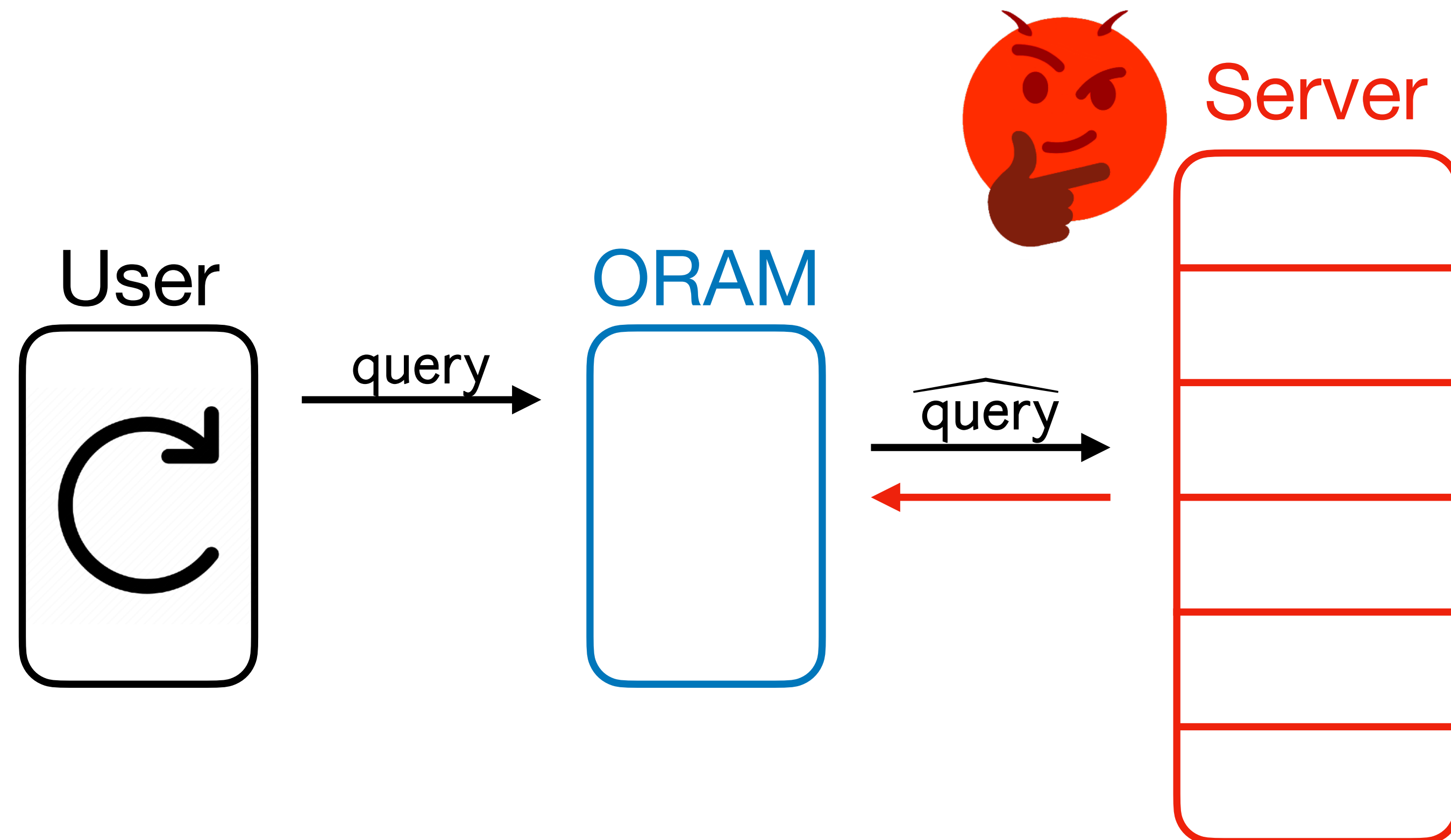
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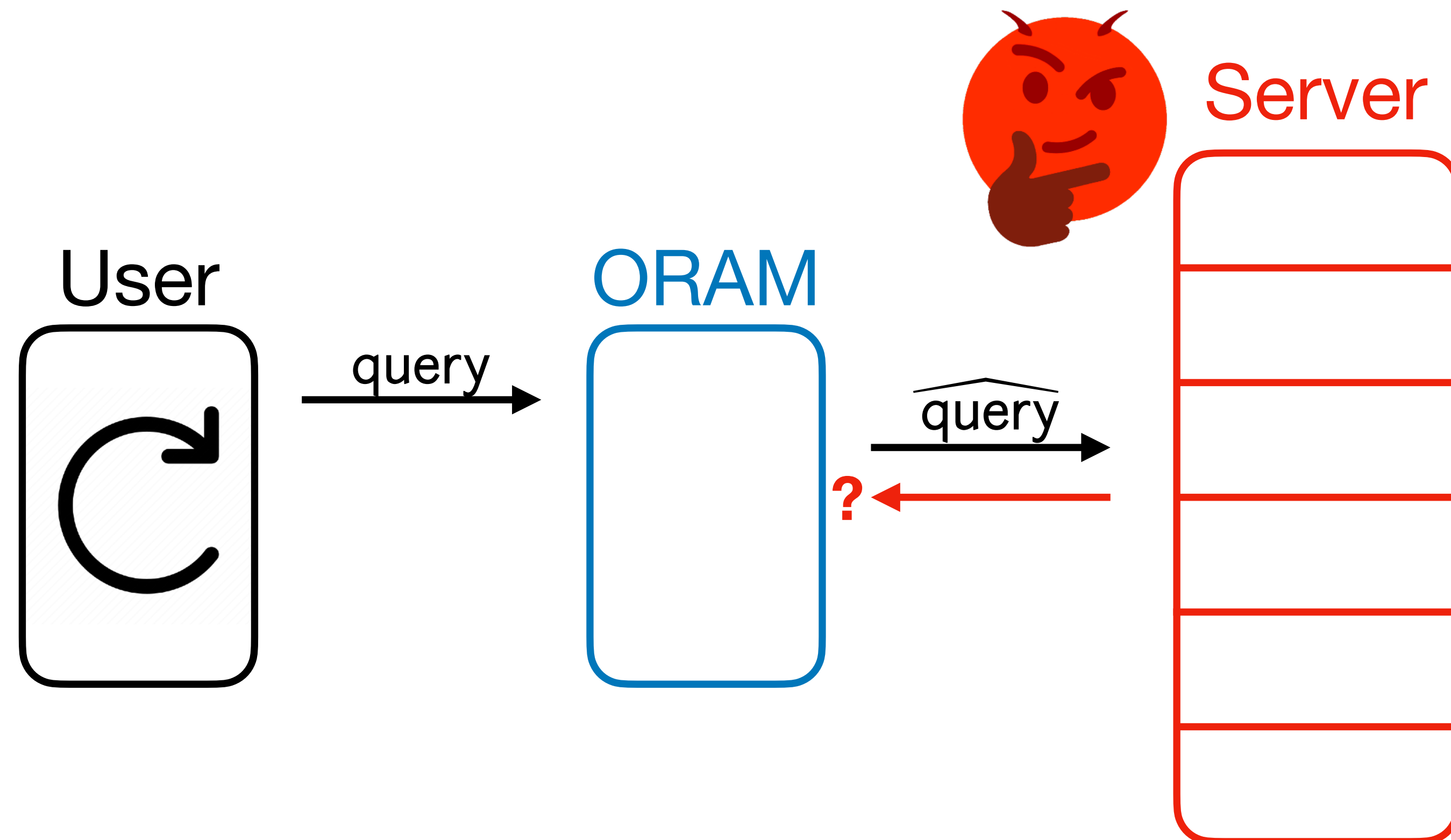
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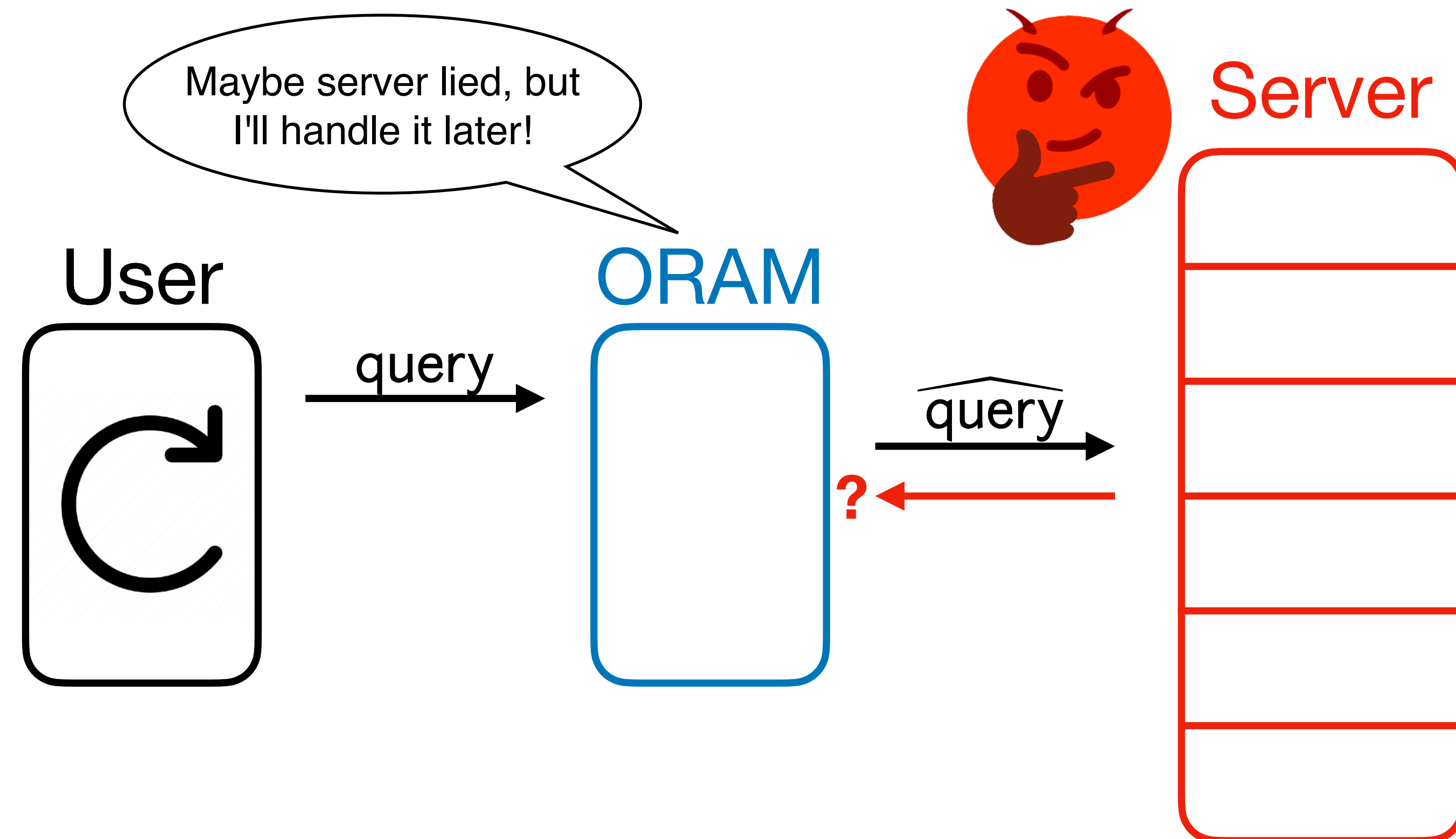
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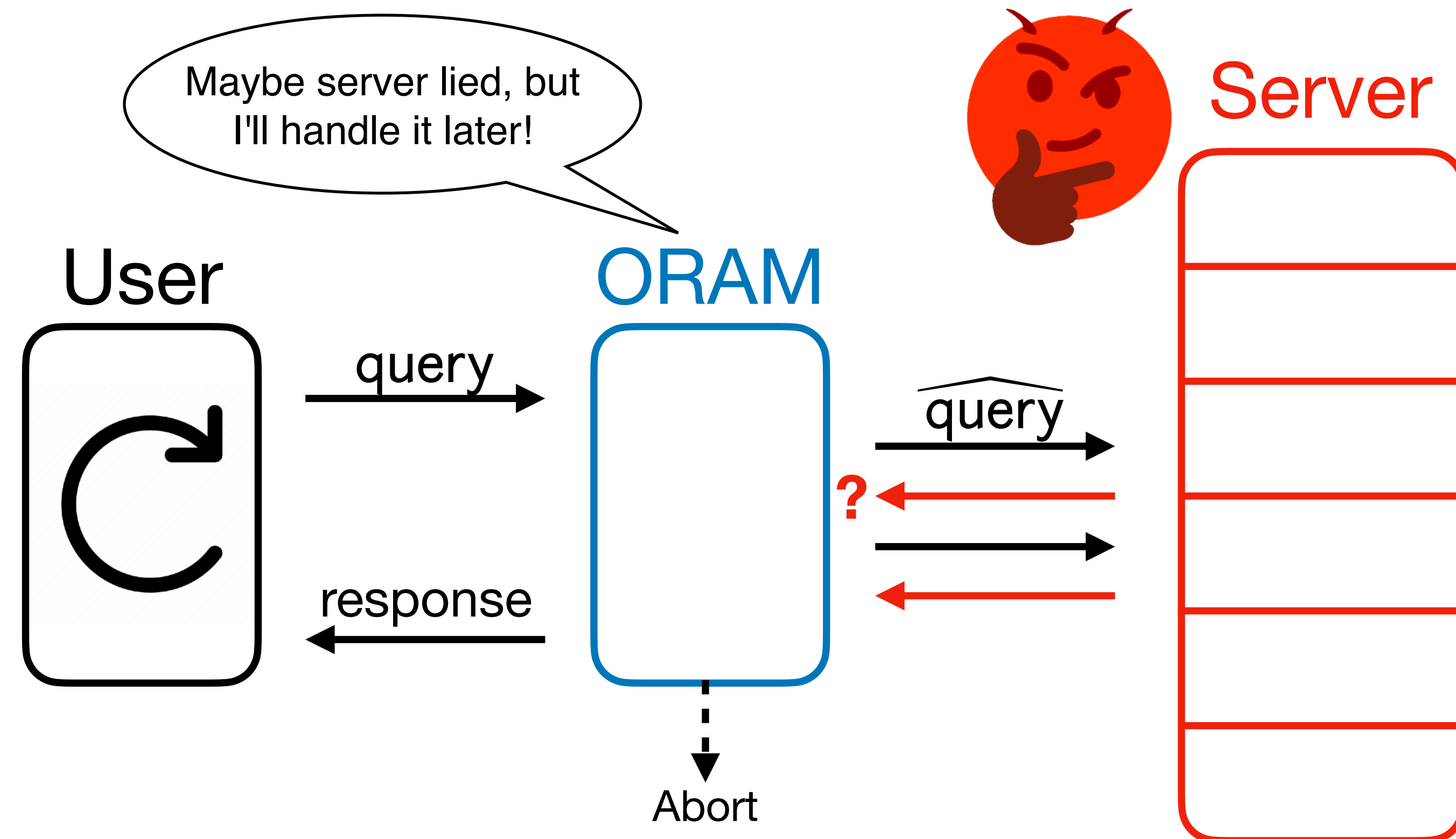
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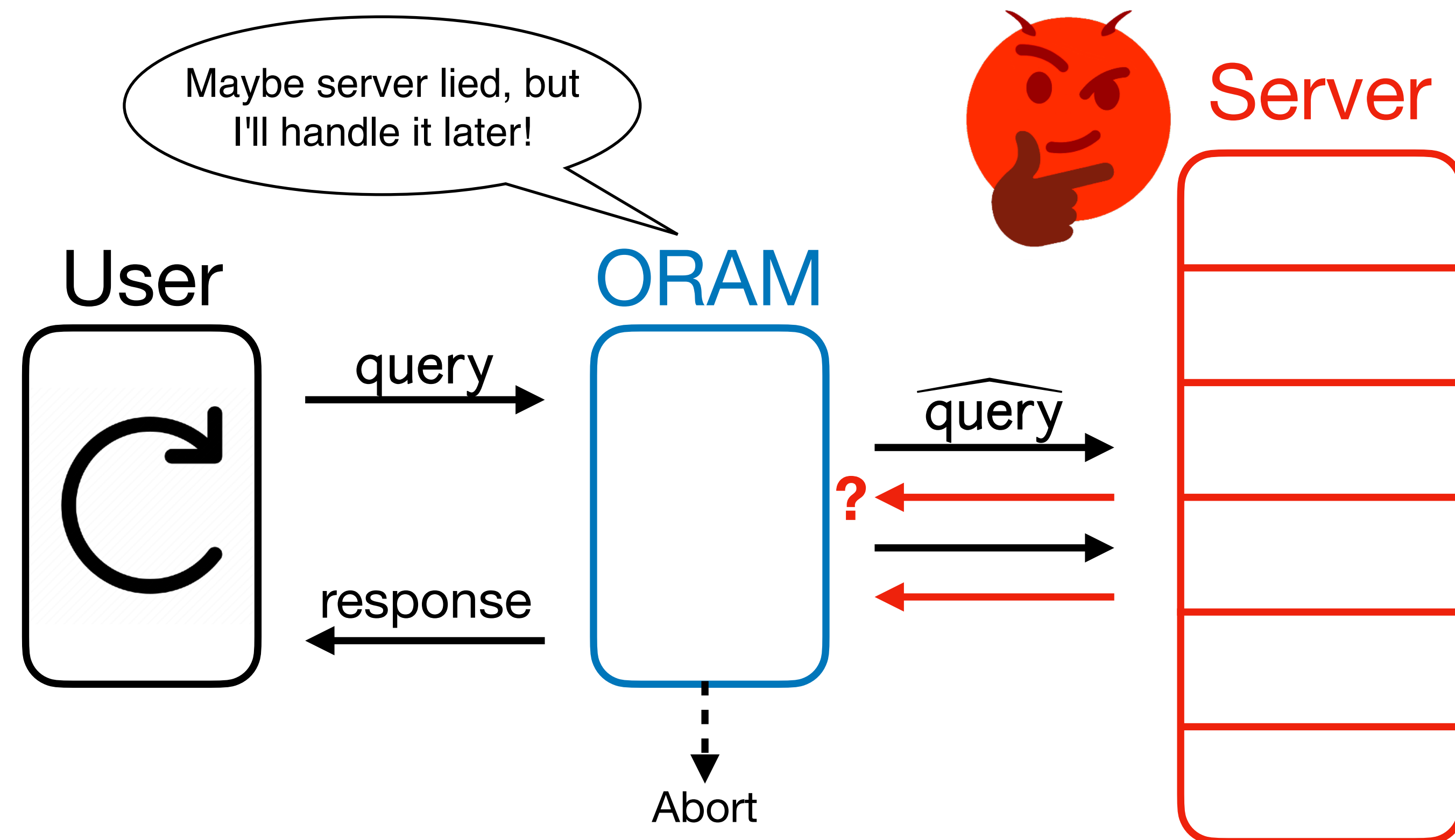
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- **Our Idea:** Use weaker, more efficient notion of memory checking to capitalize on this!



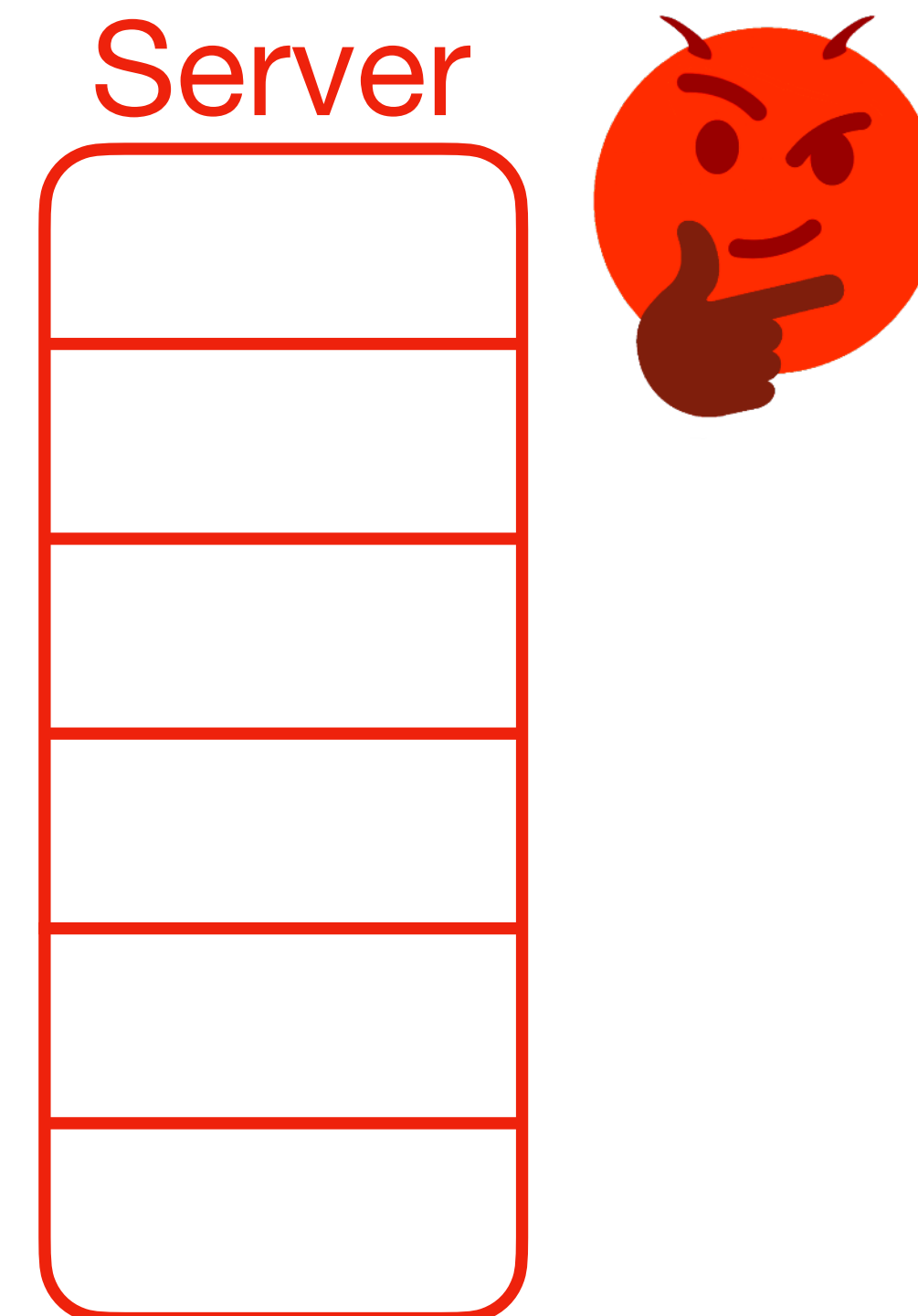
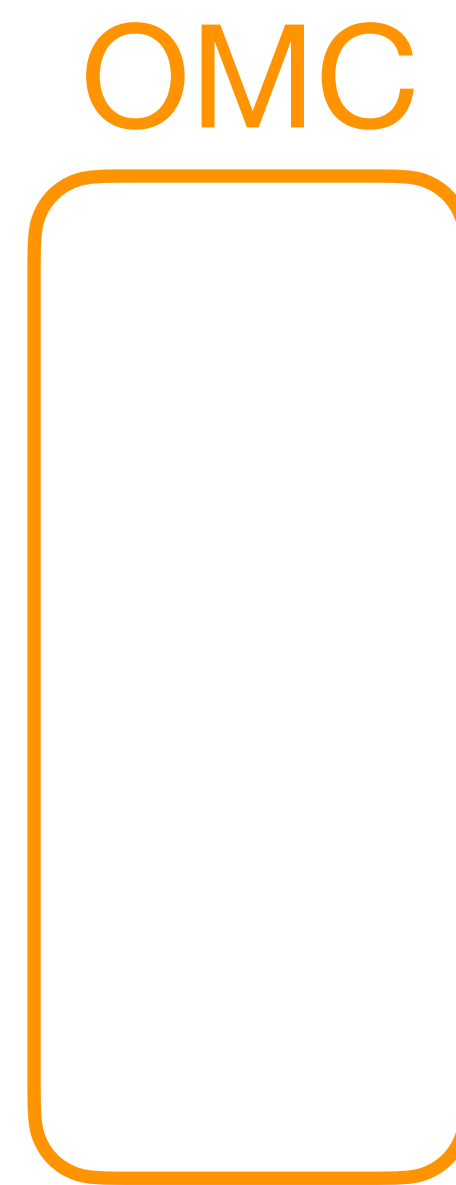
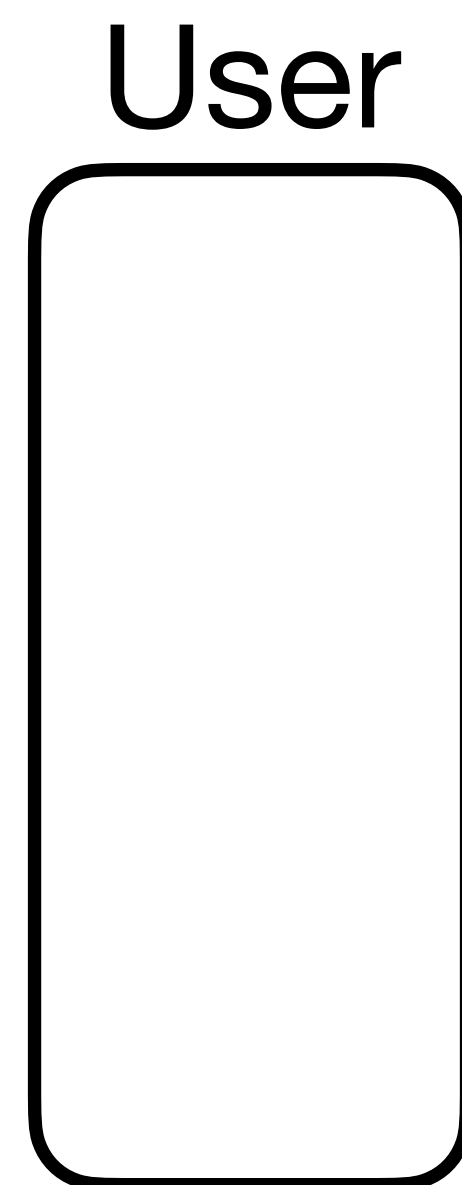
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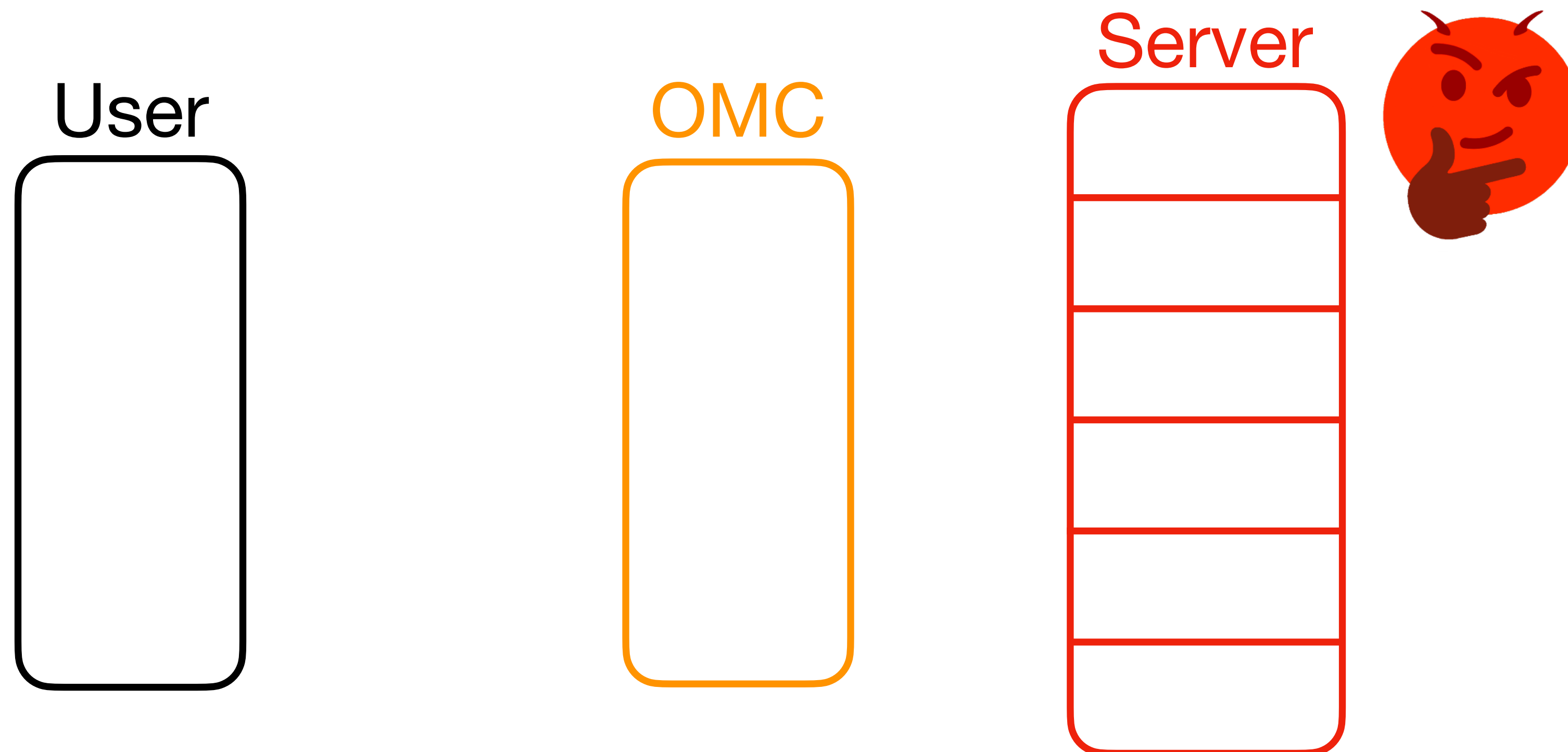
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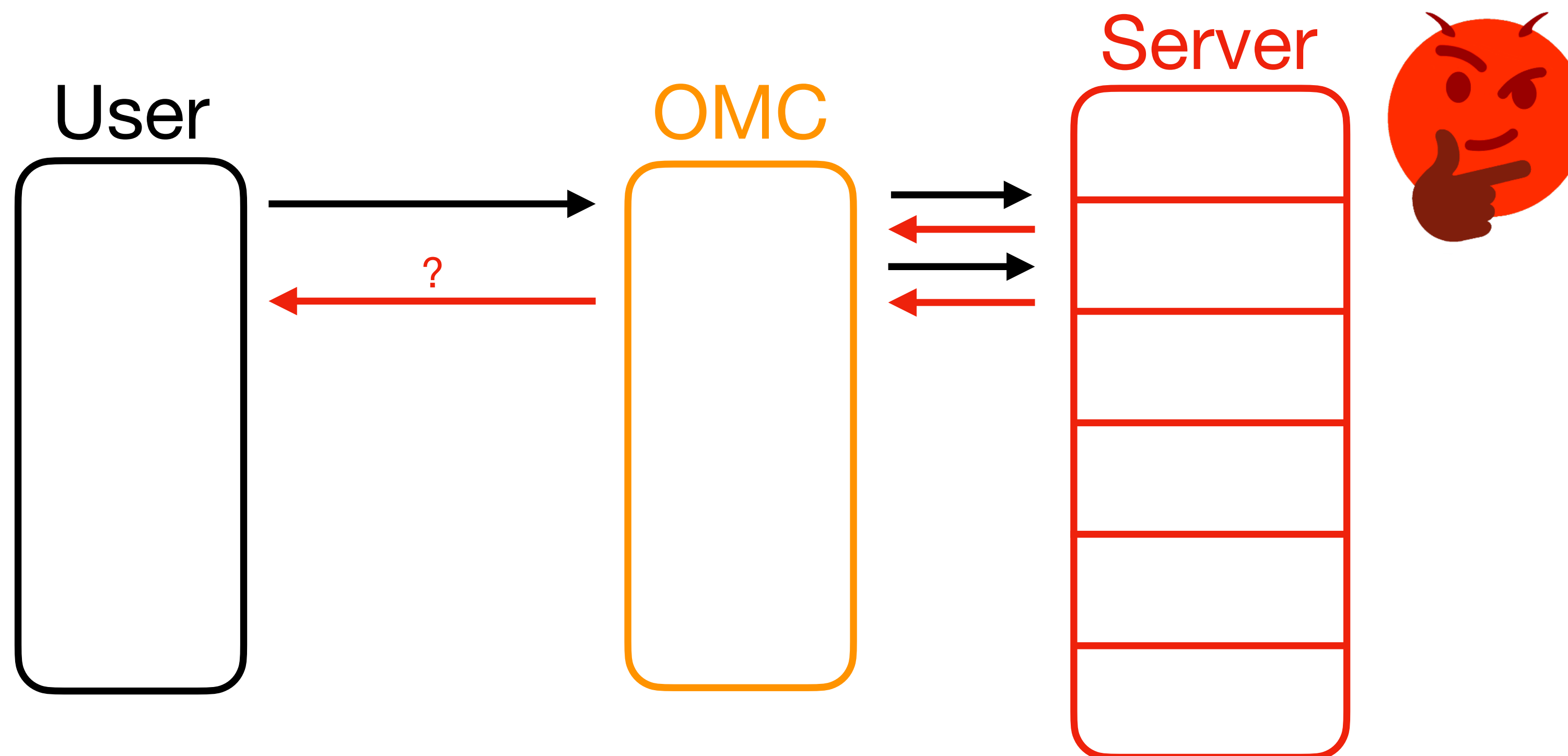
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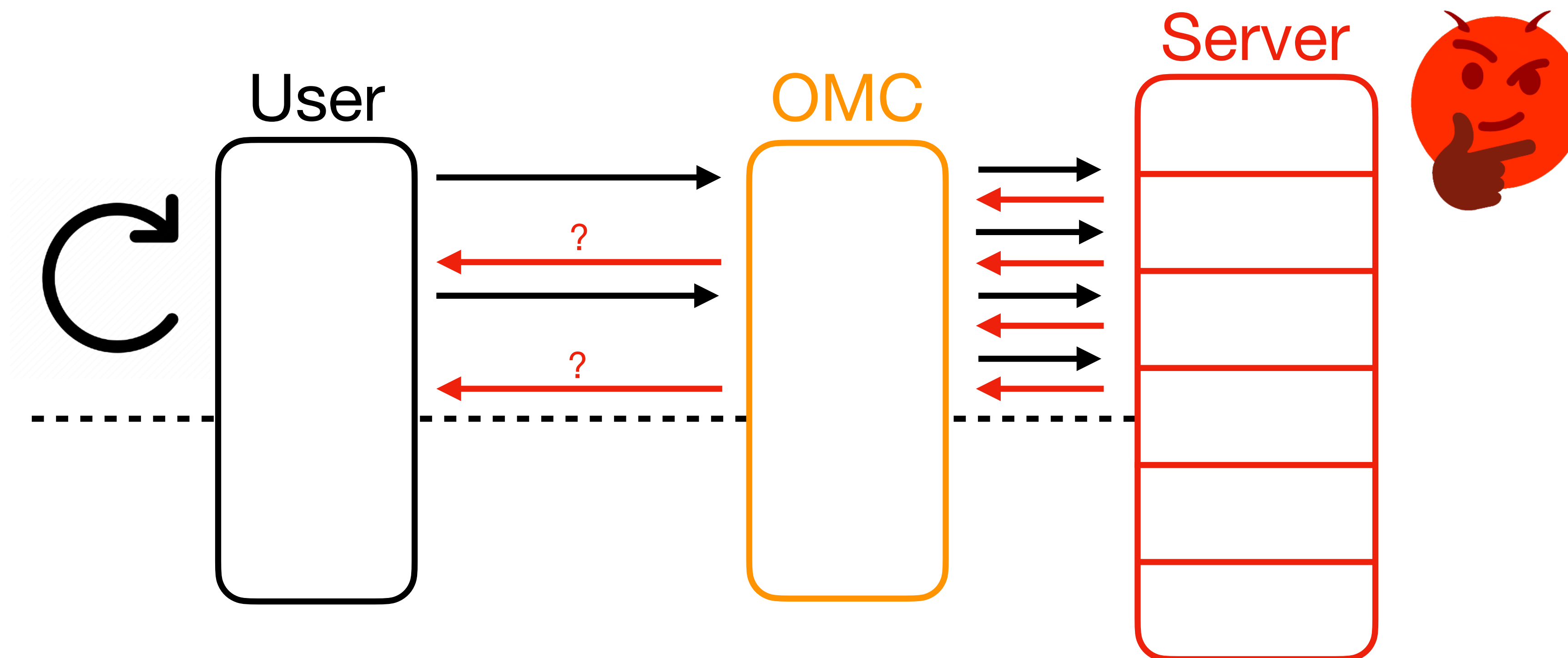
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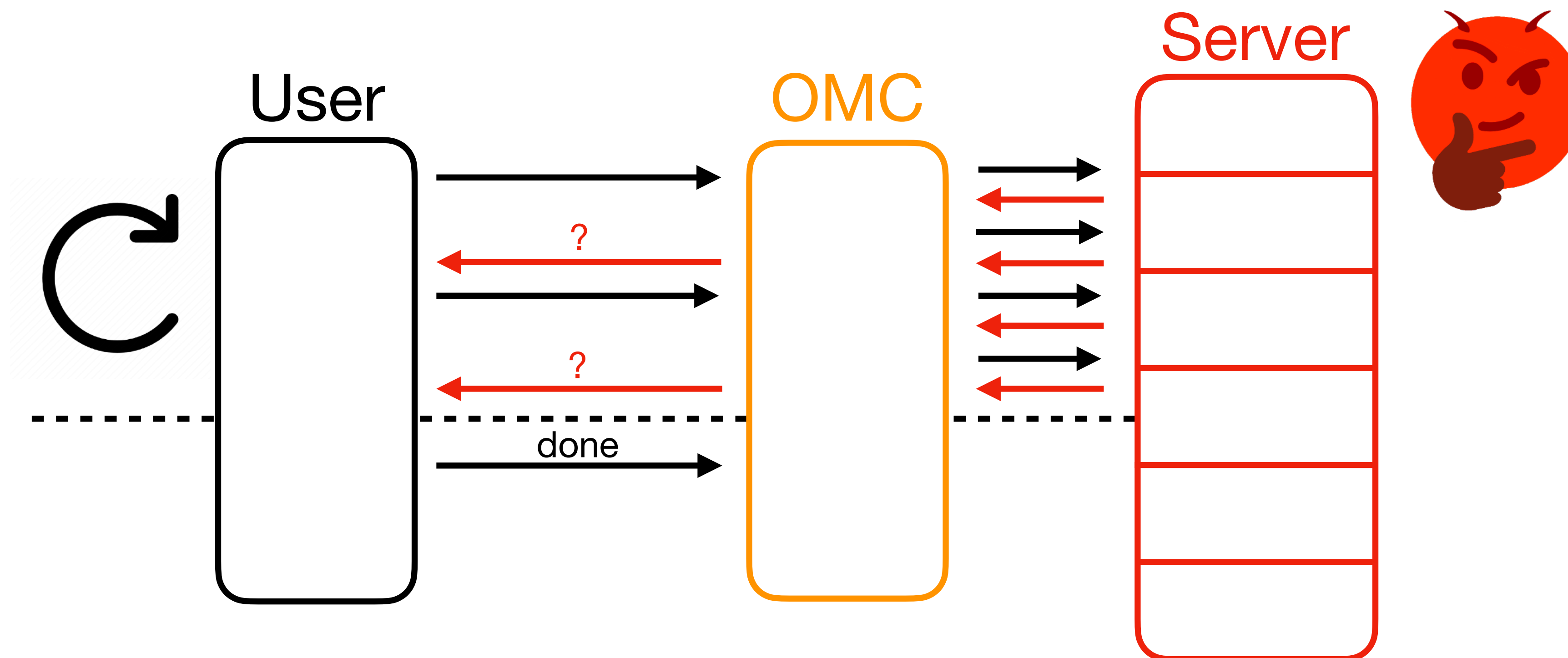
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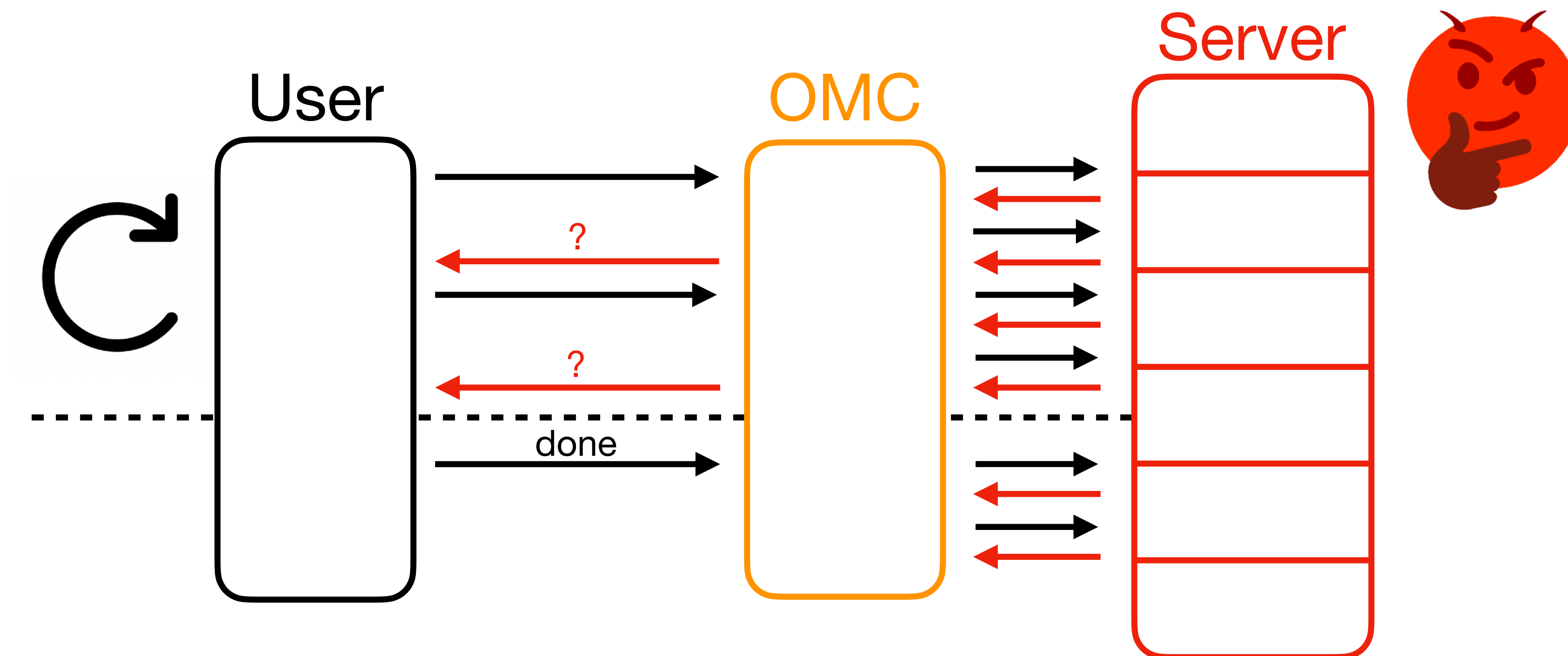
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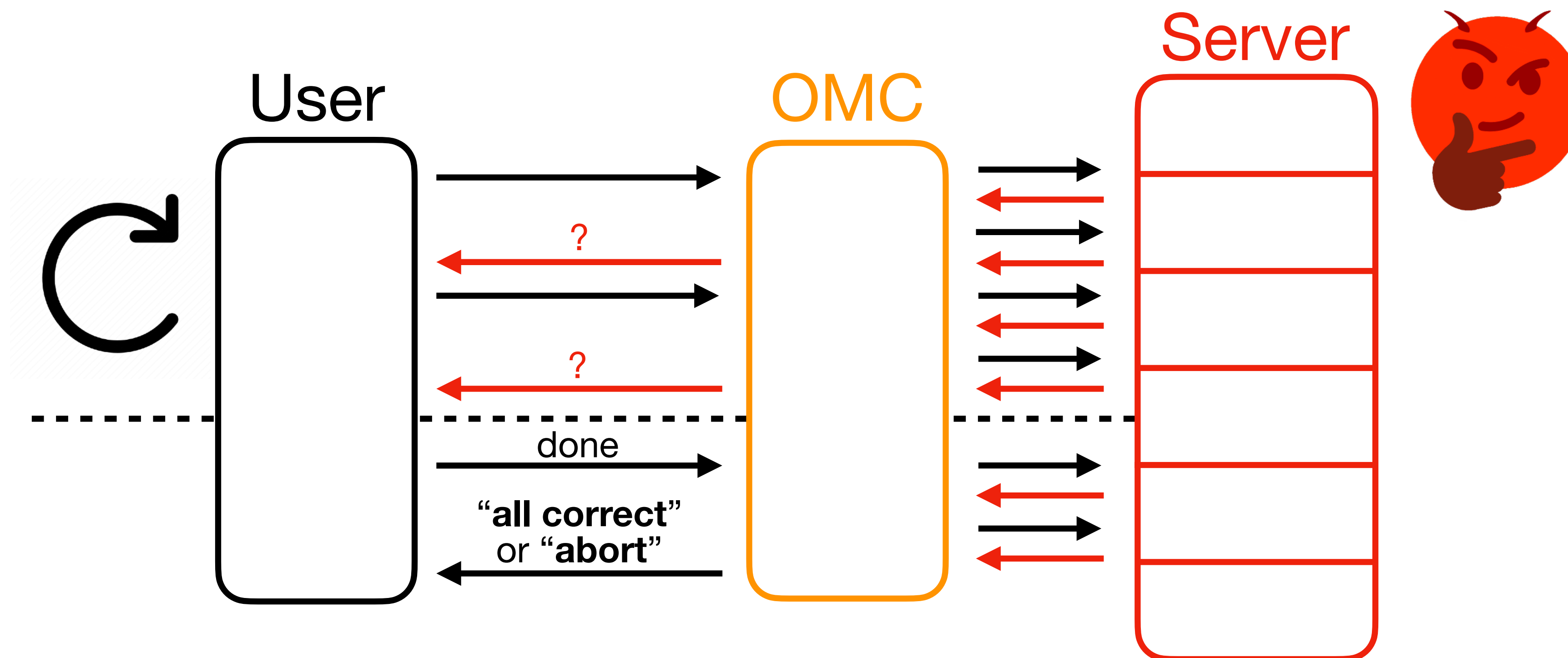
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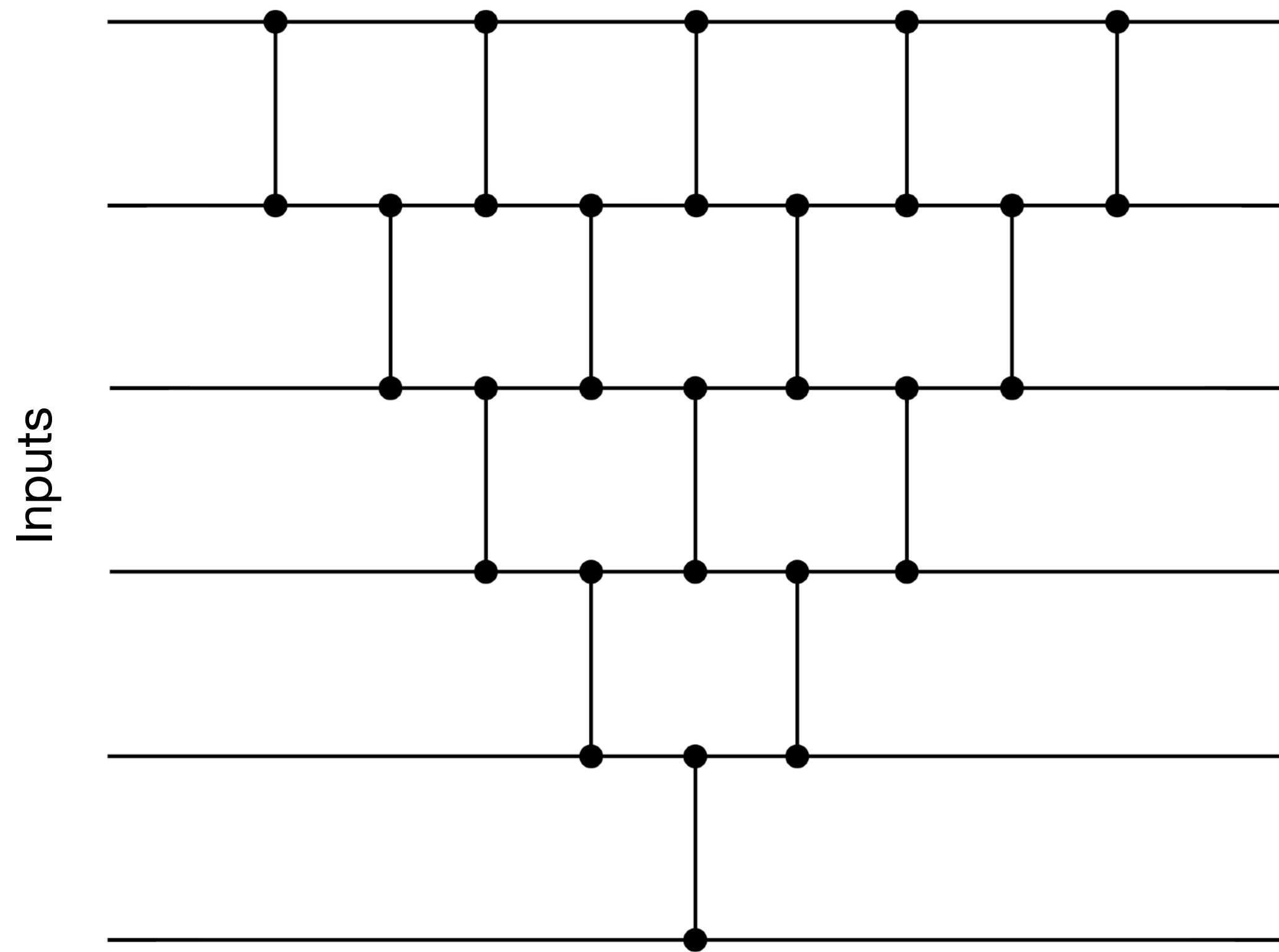
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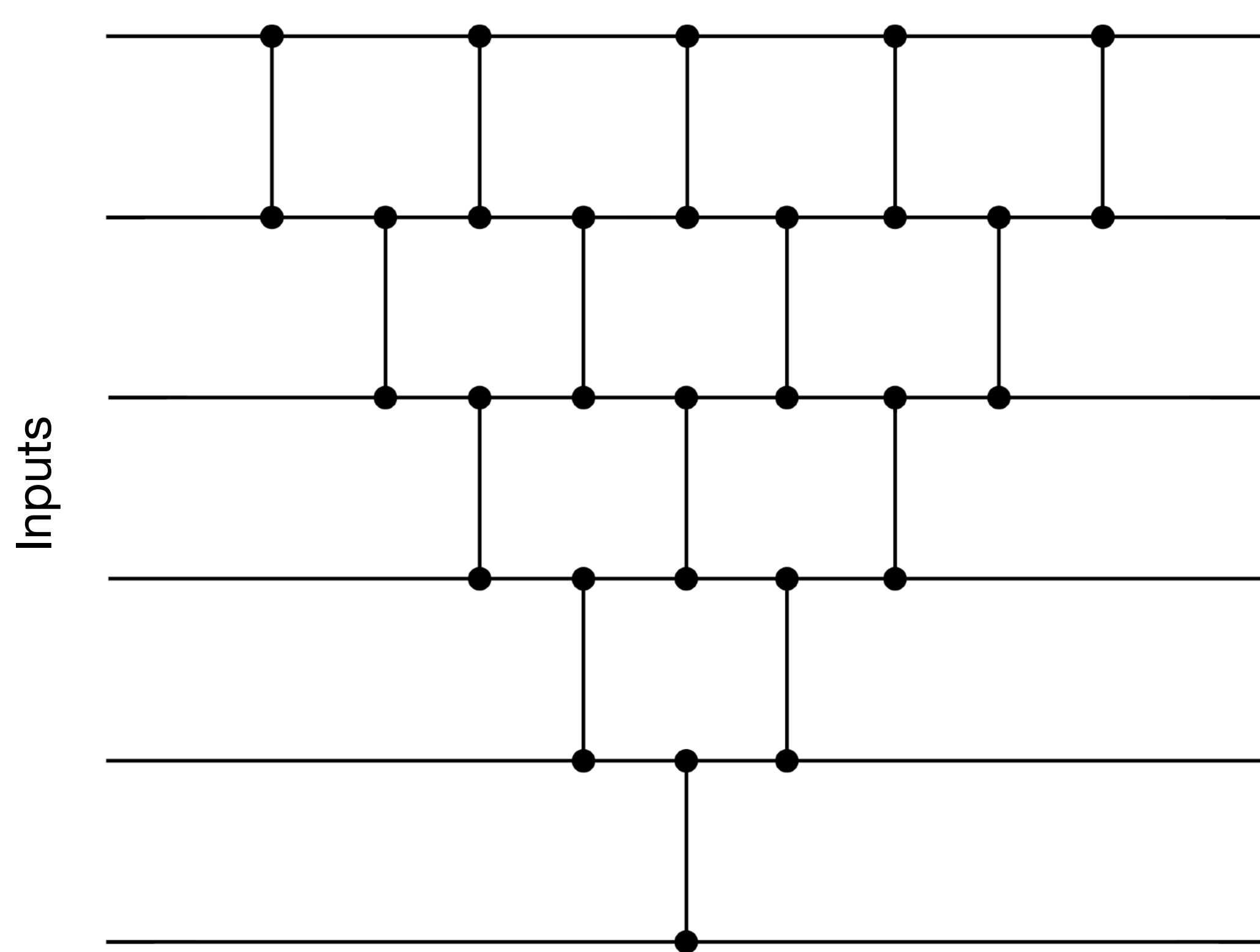
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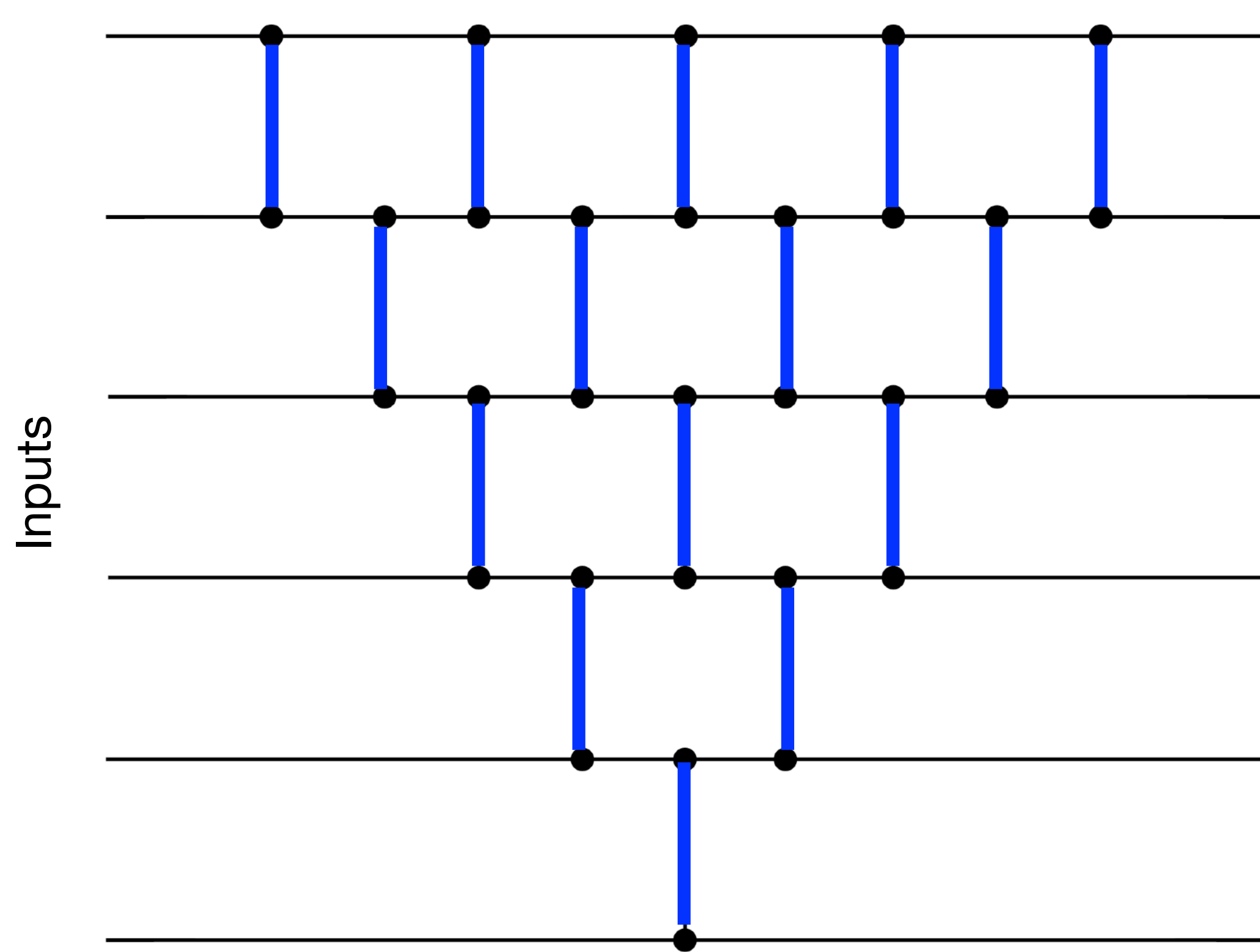


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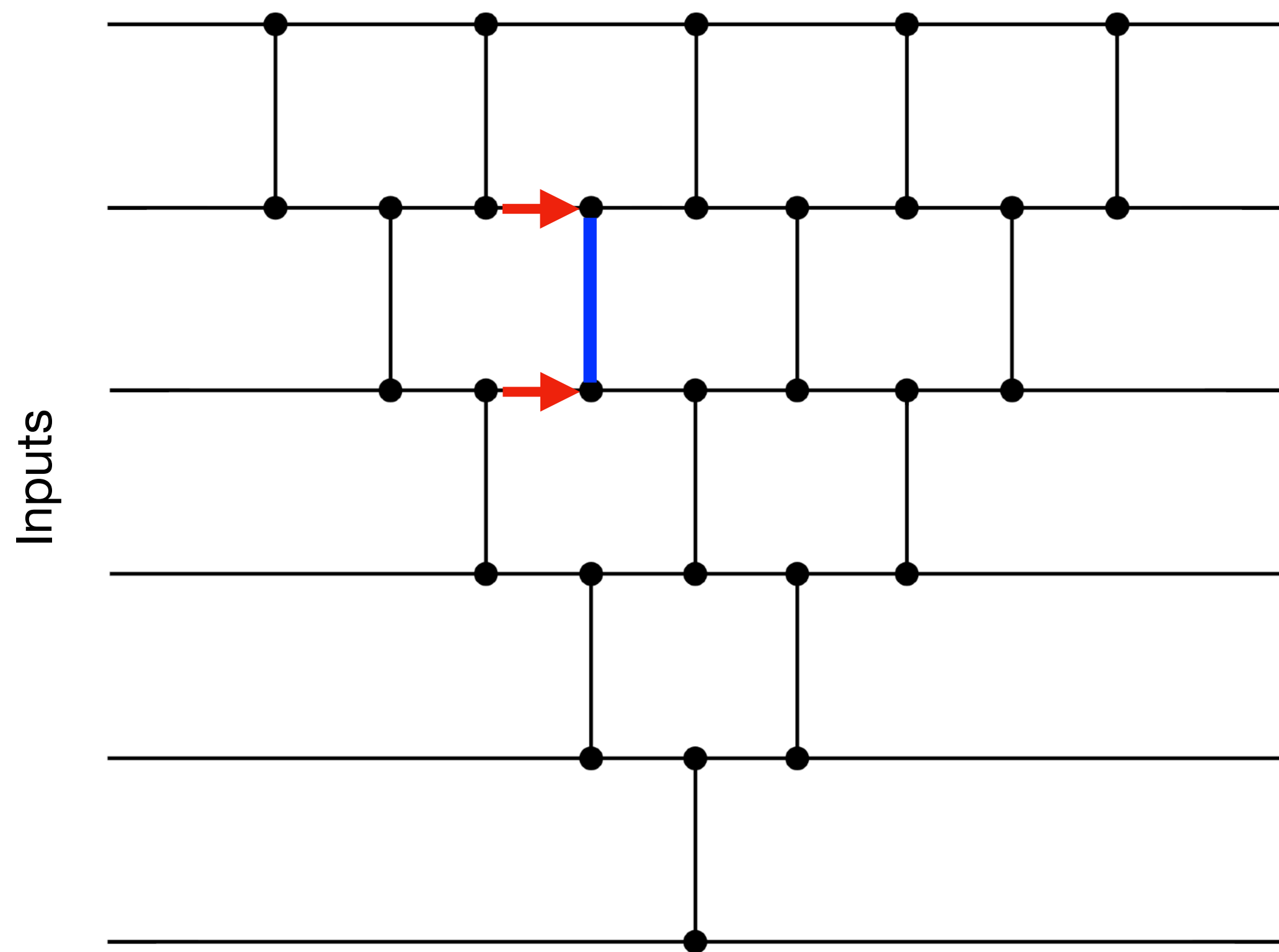
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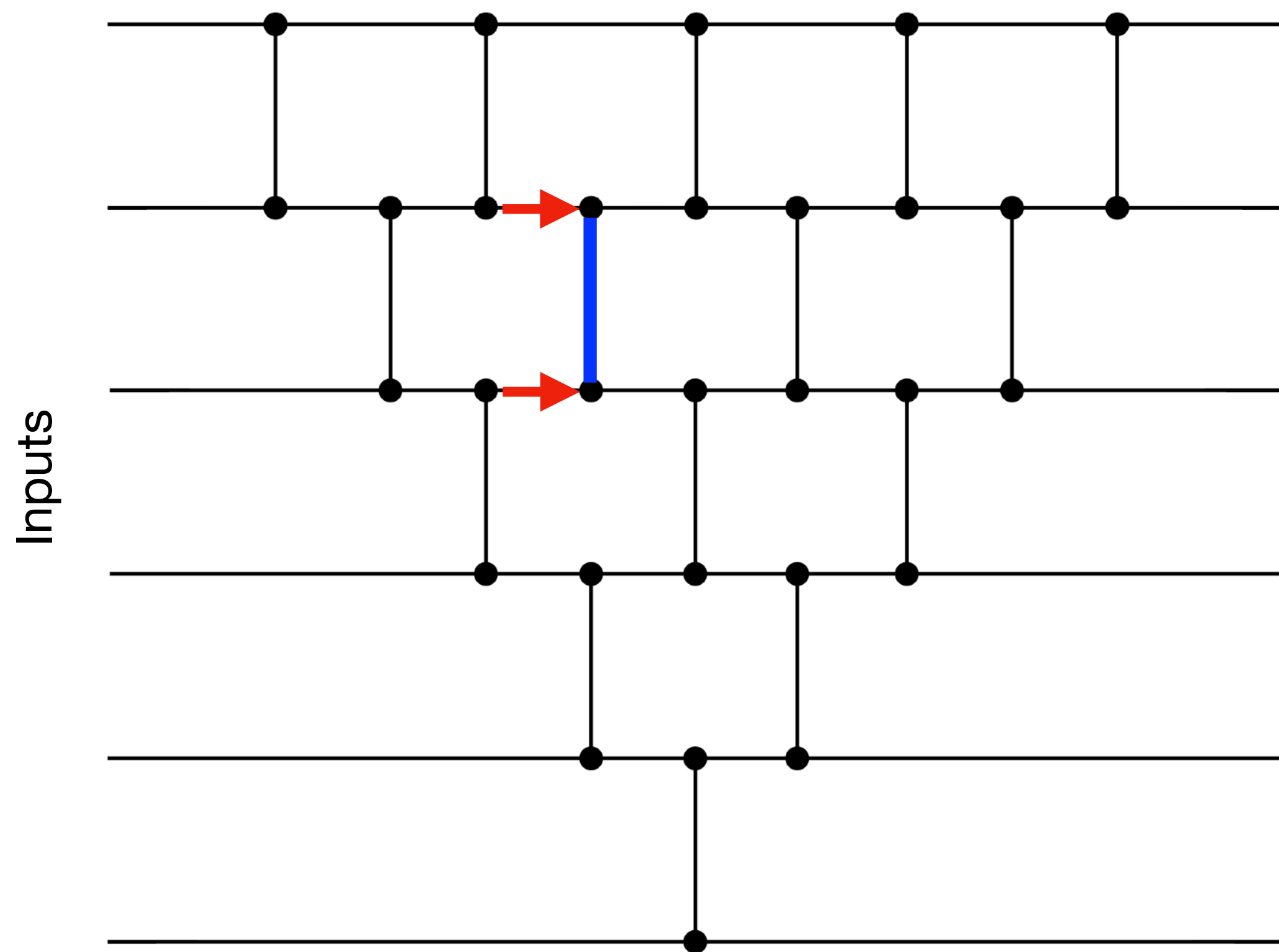
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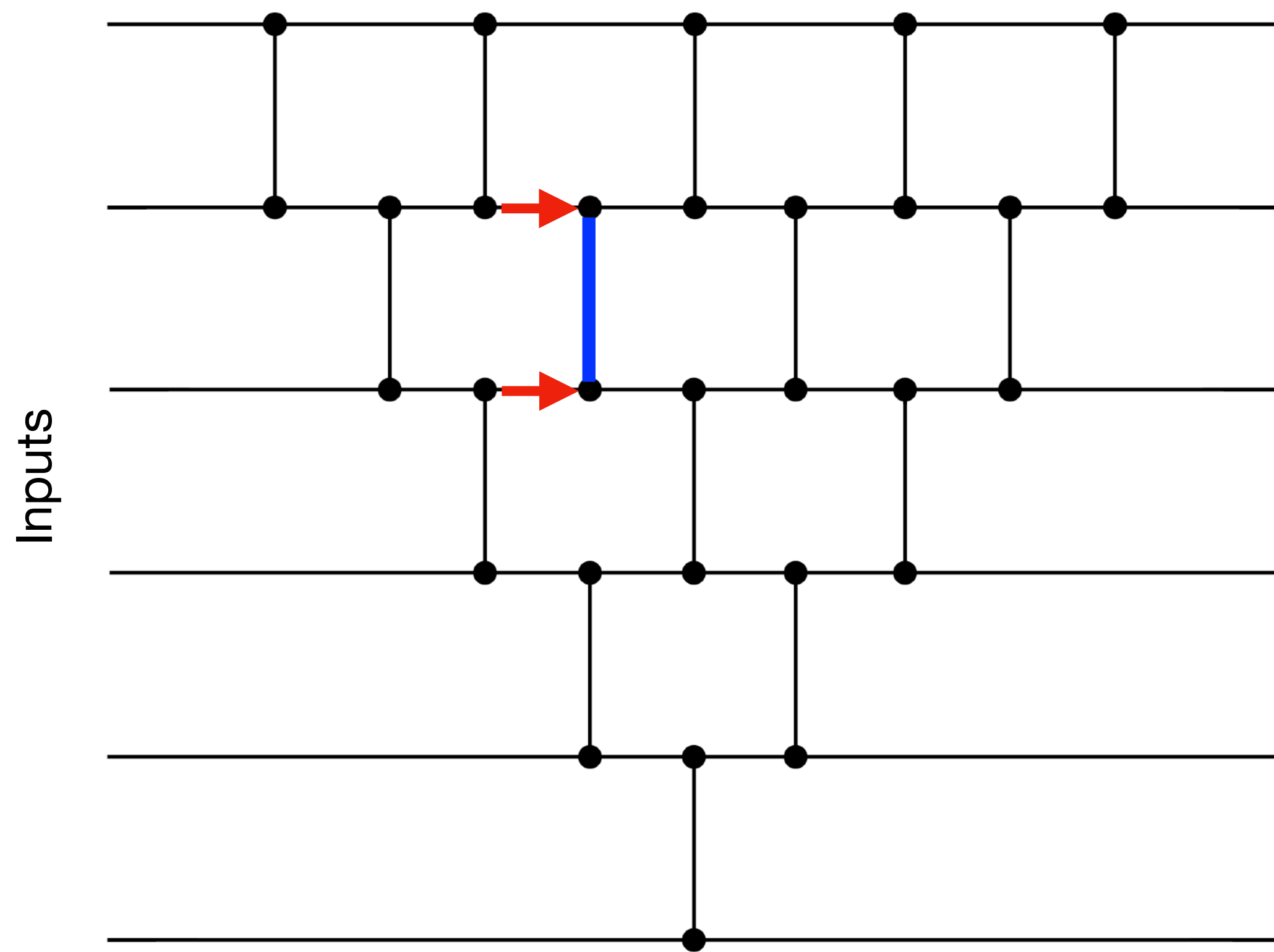
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- In our work, we generalise this further to capture more classes of algorithms.

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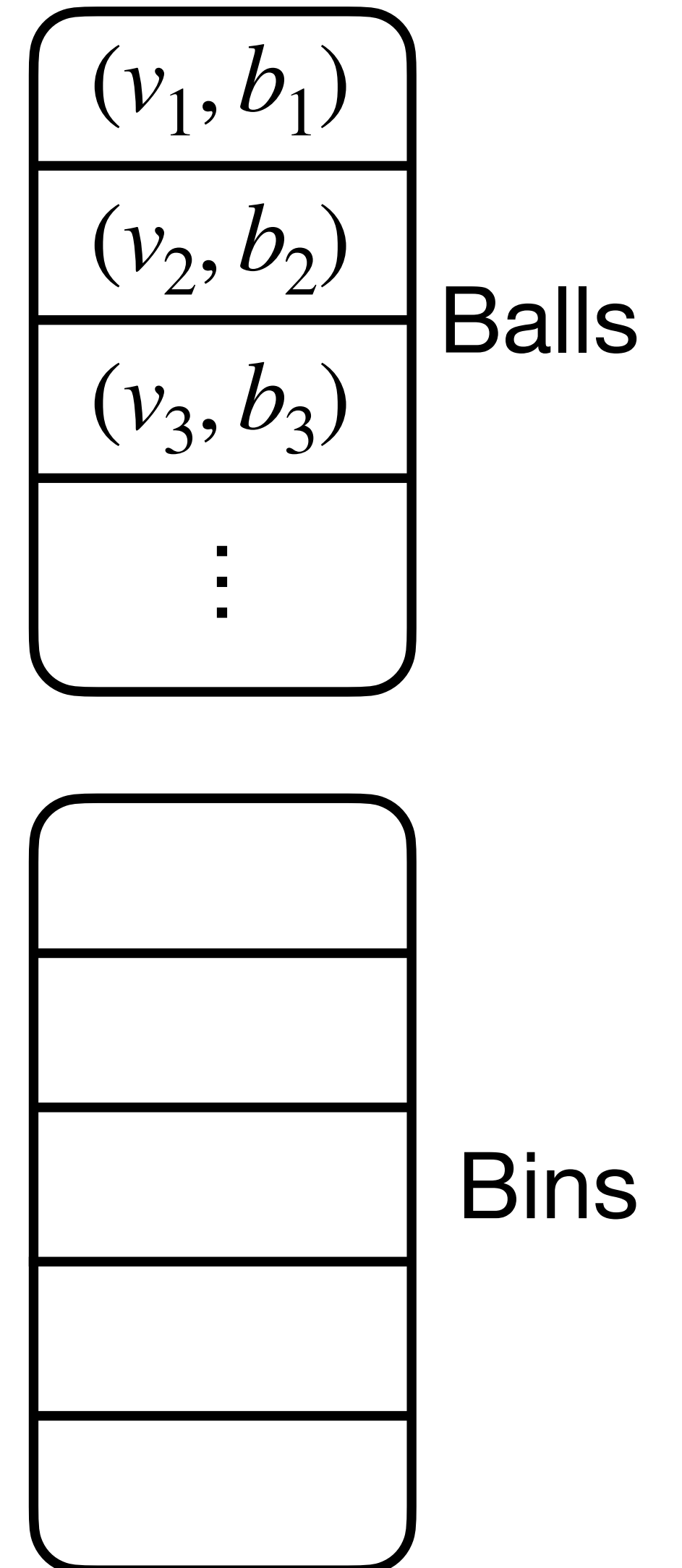
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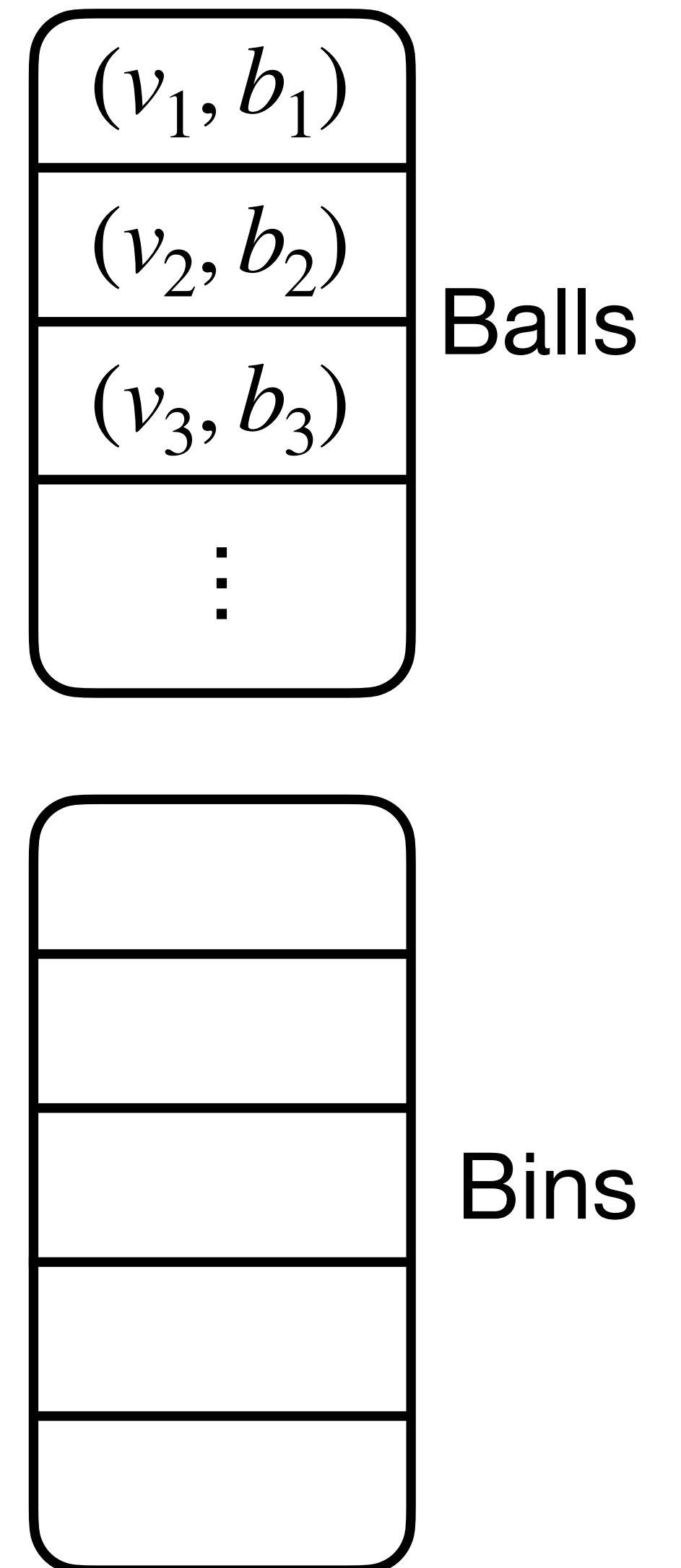
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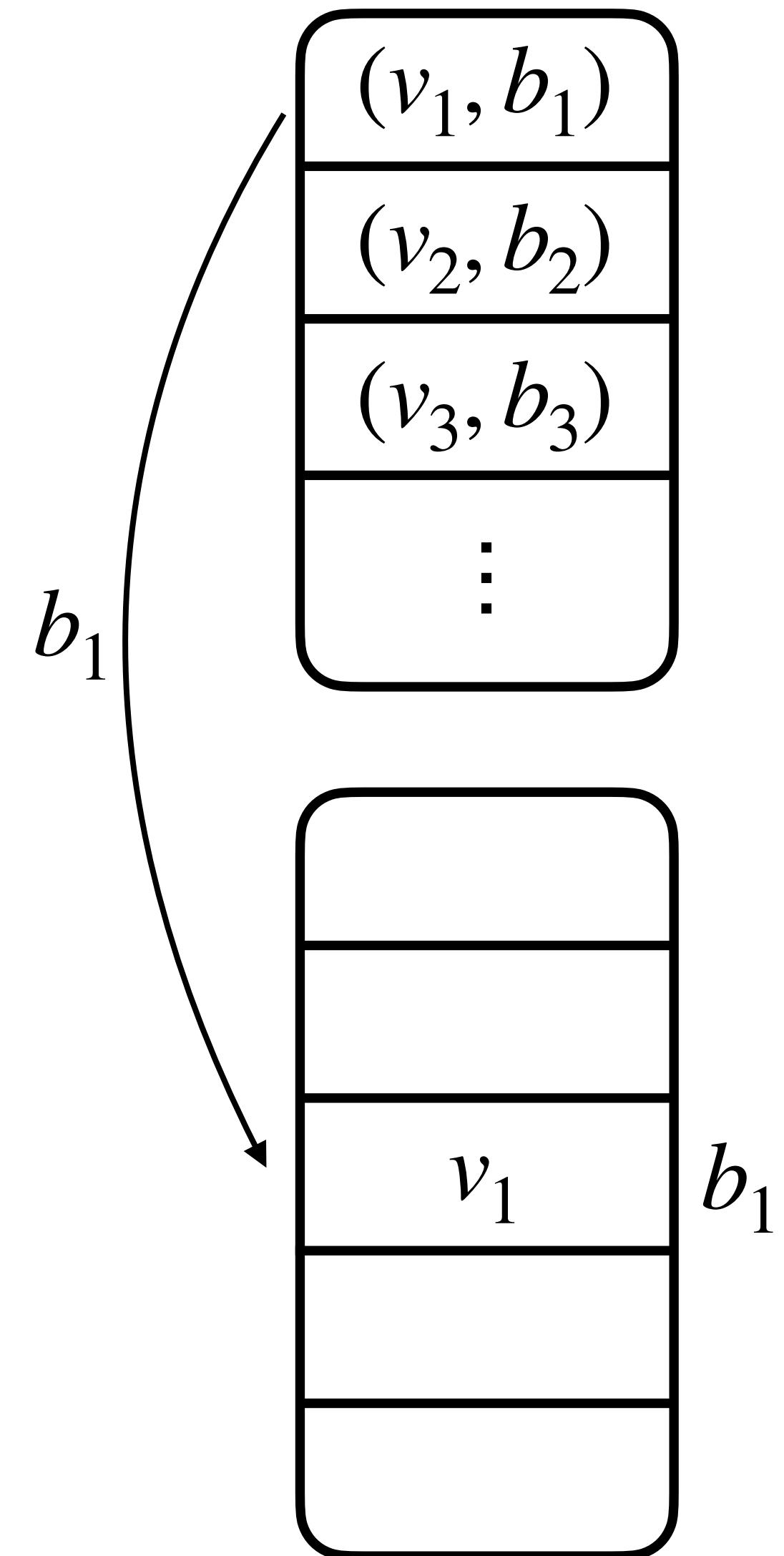
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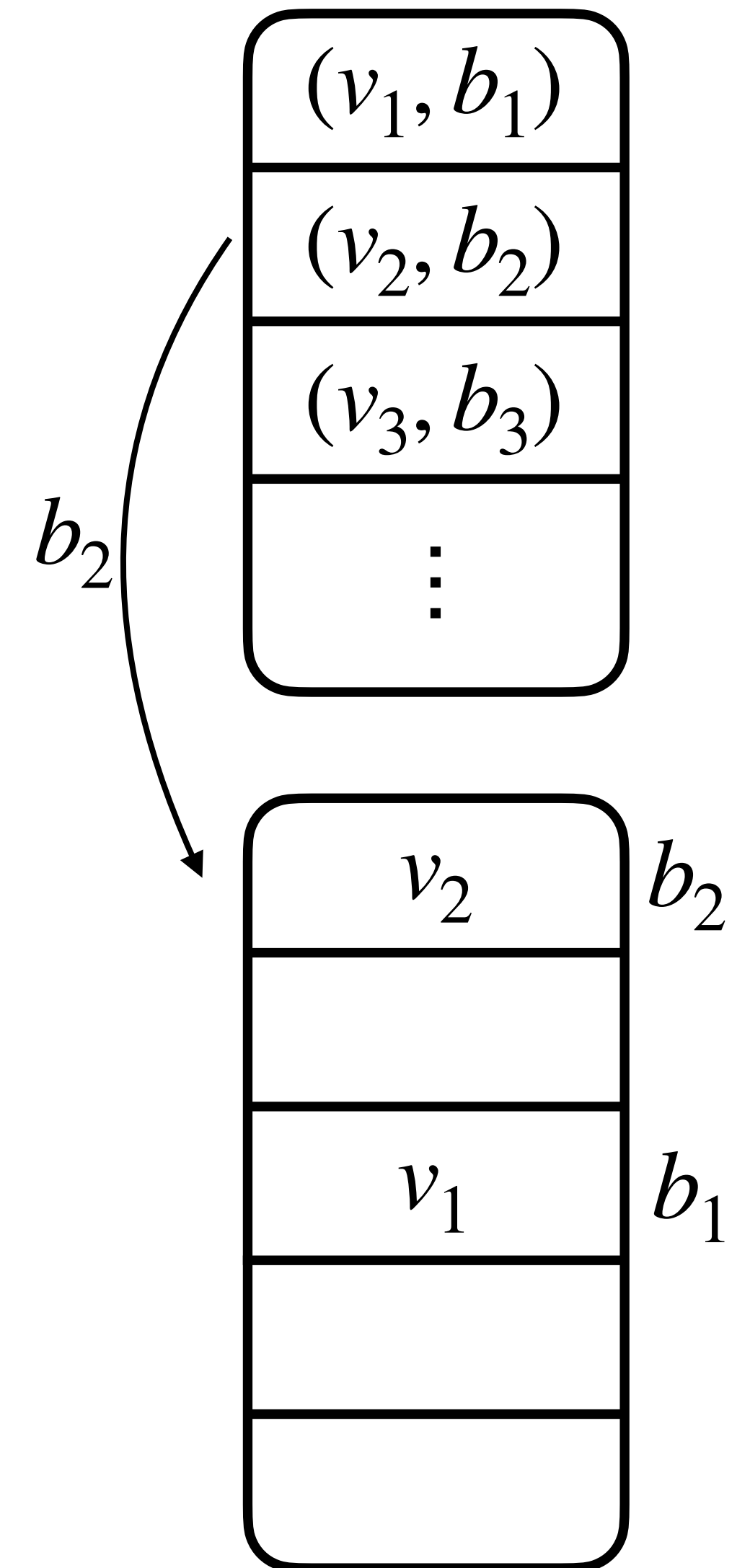
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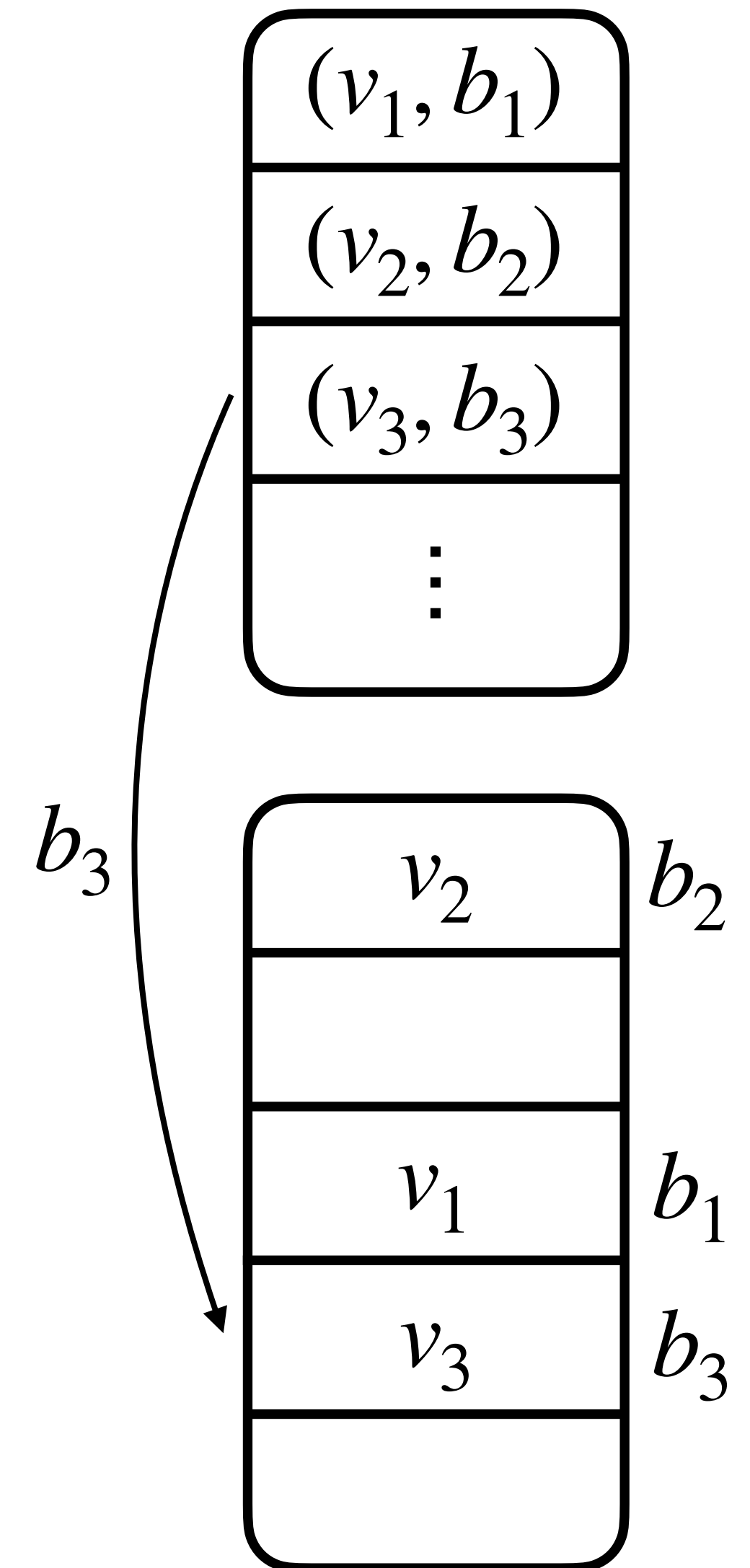
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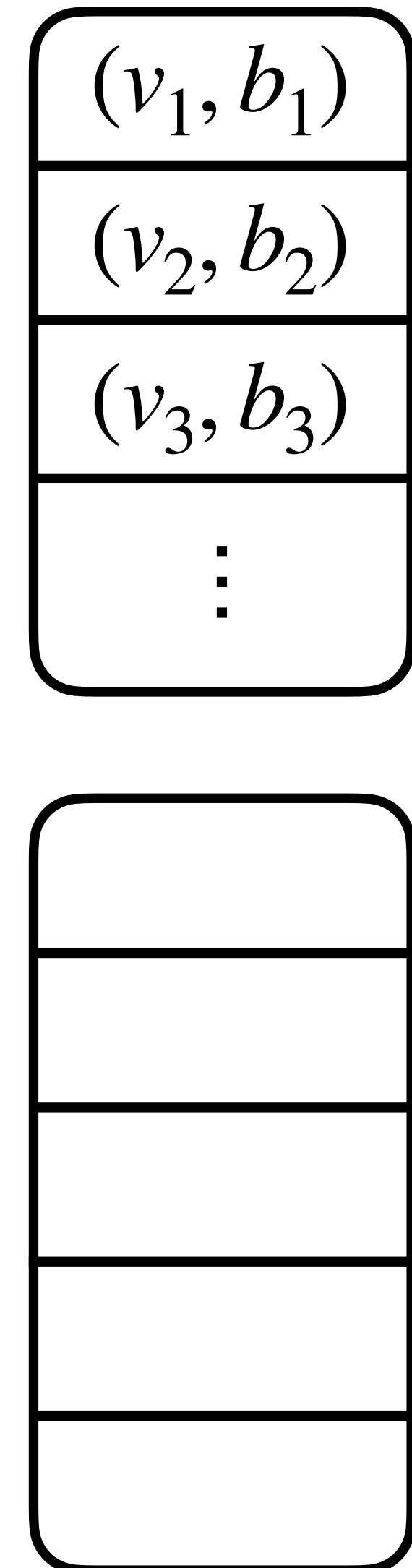
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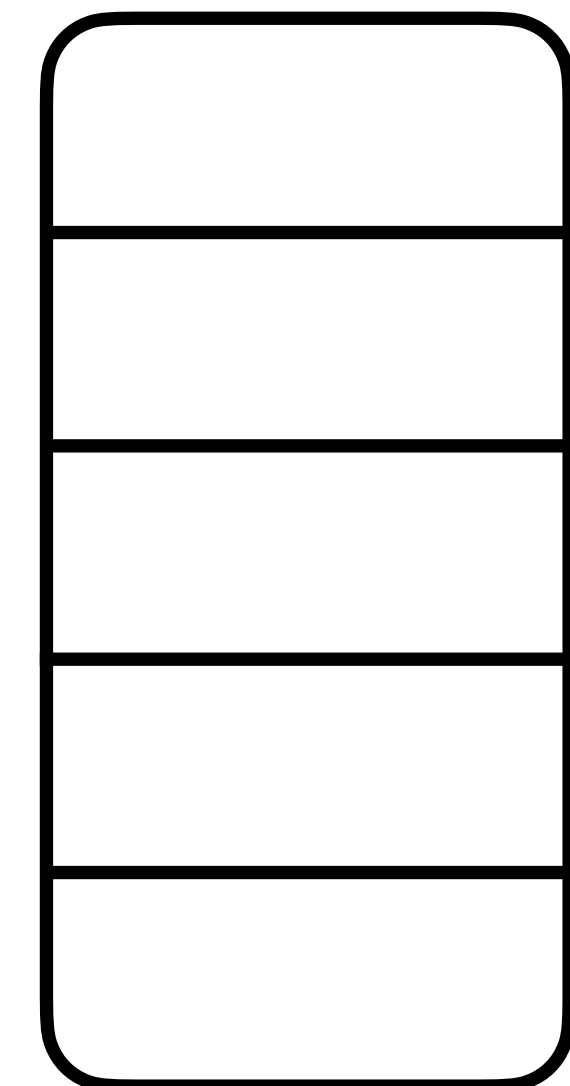
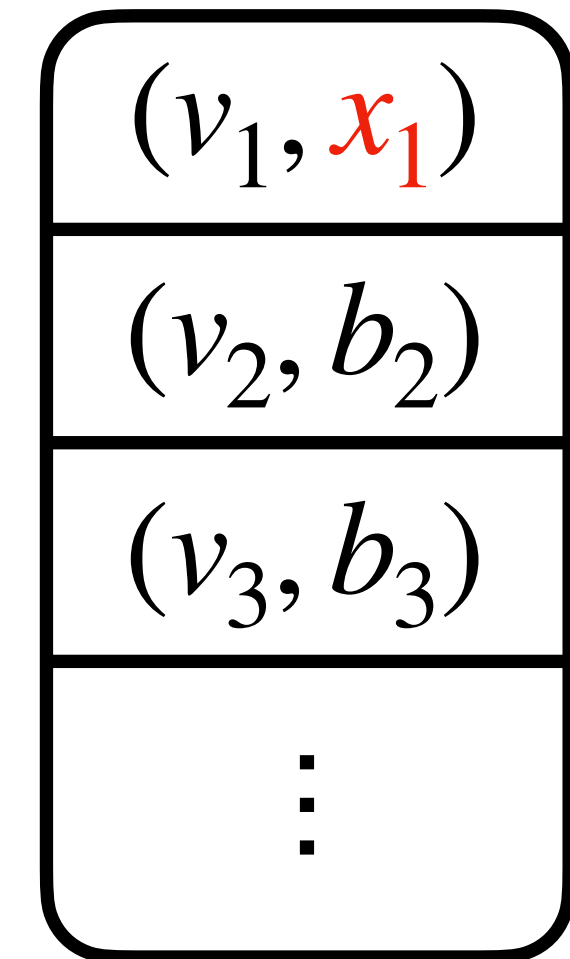
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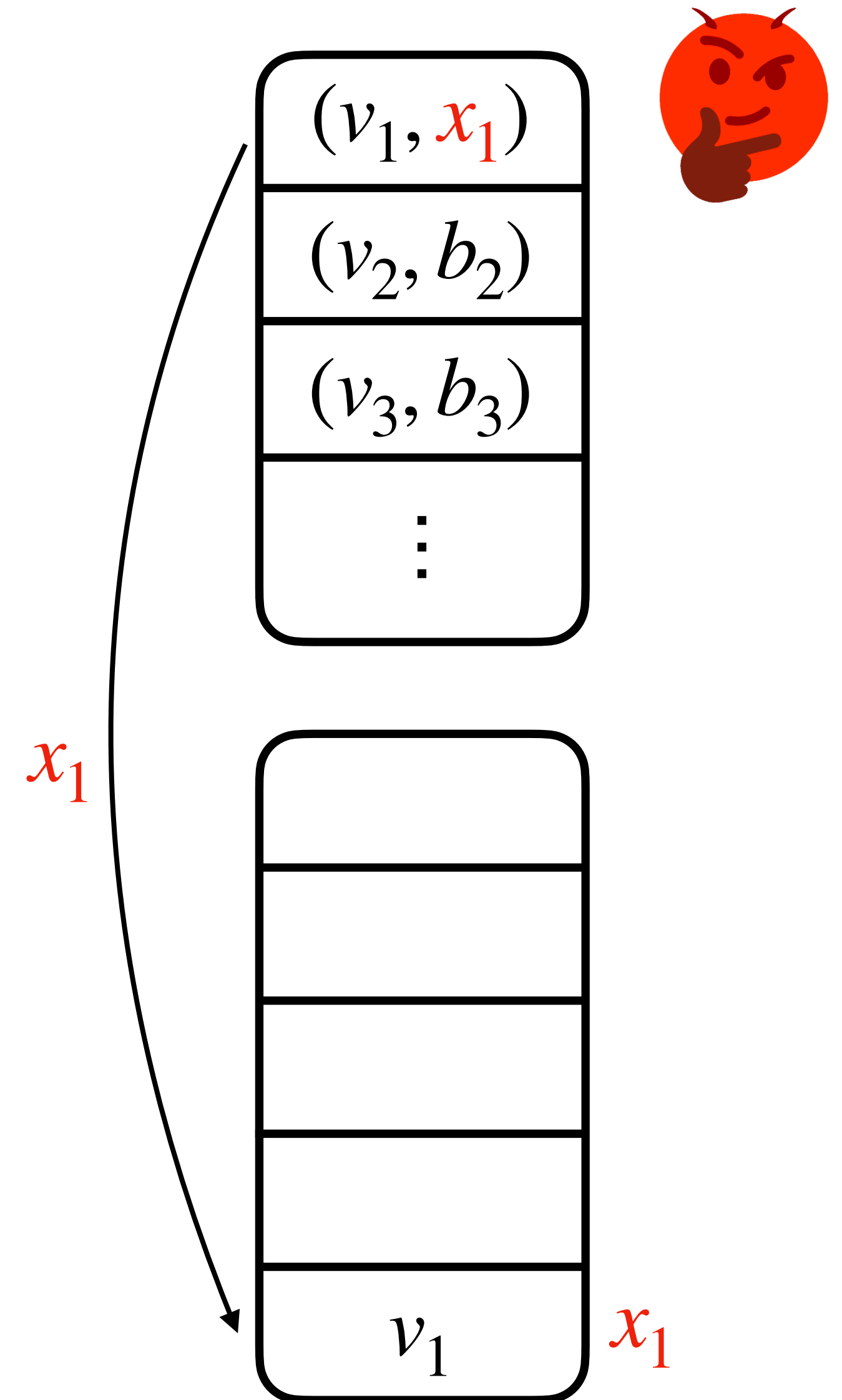
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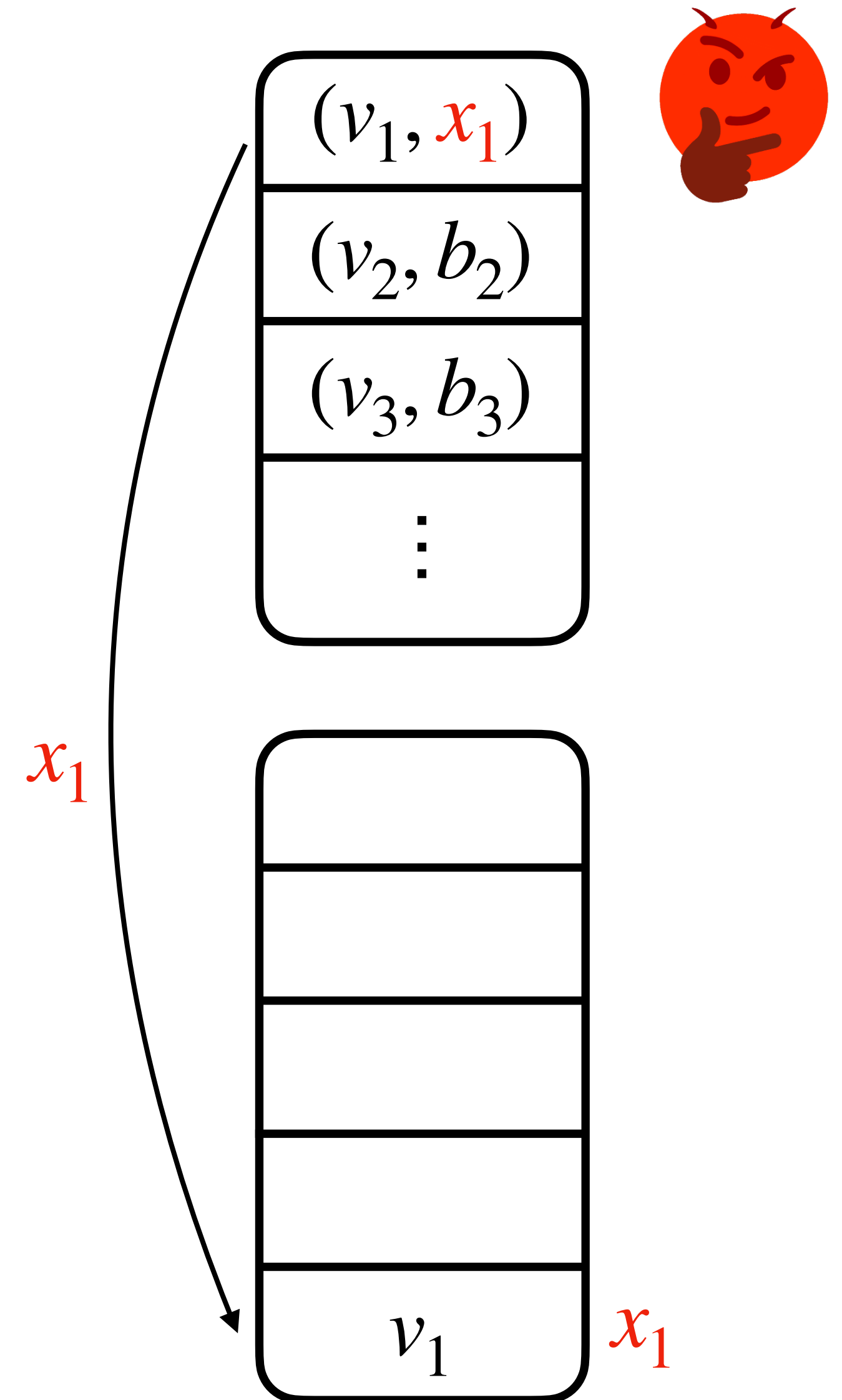
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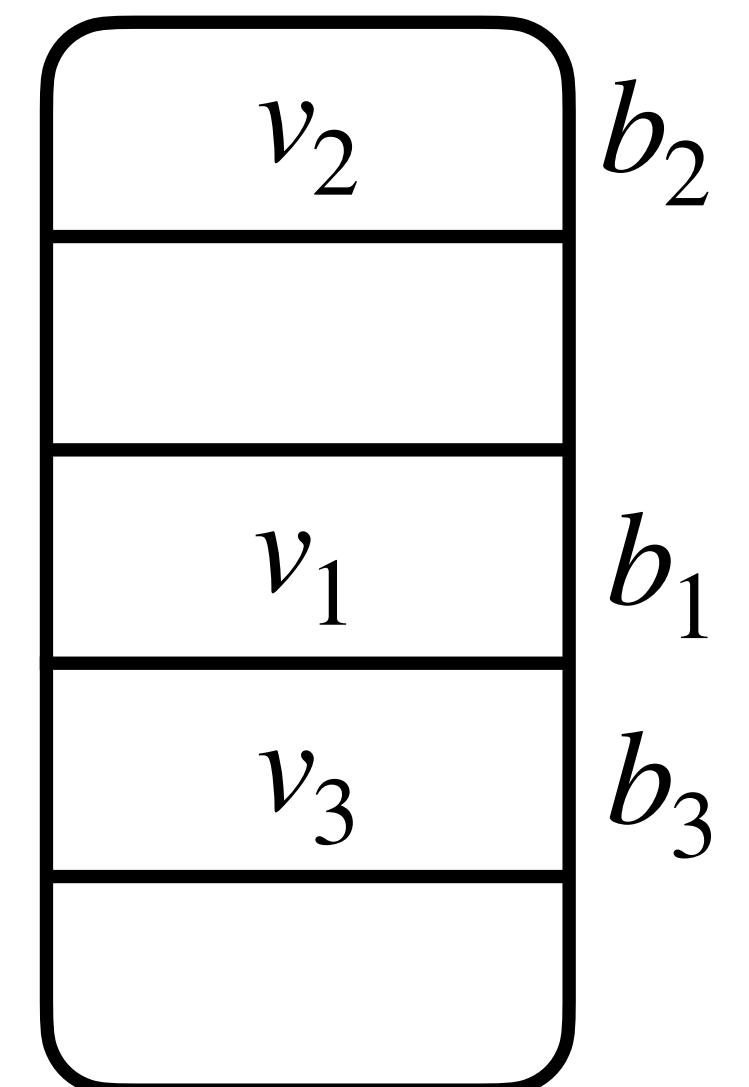
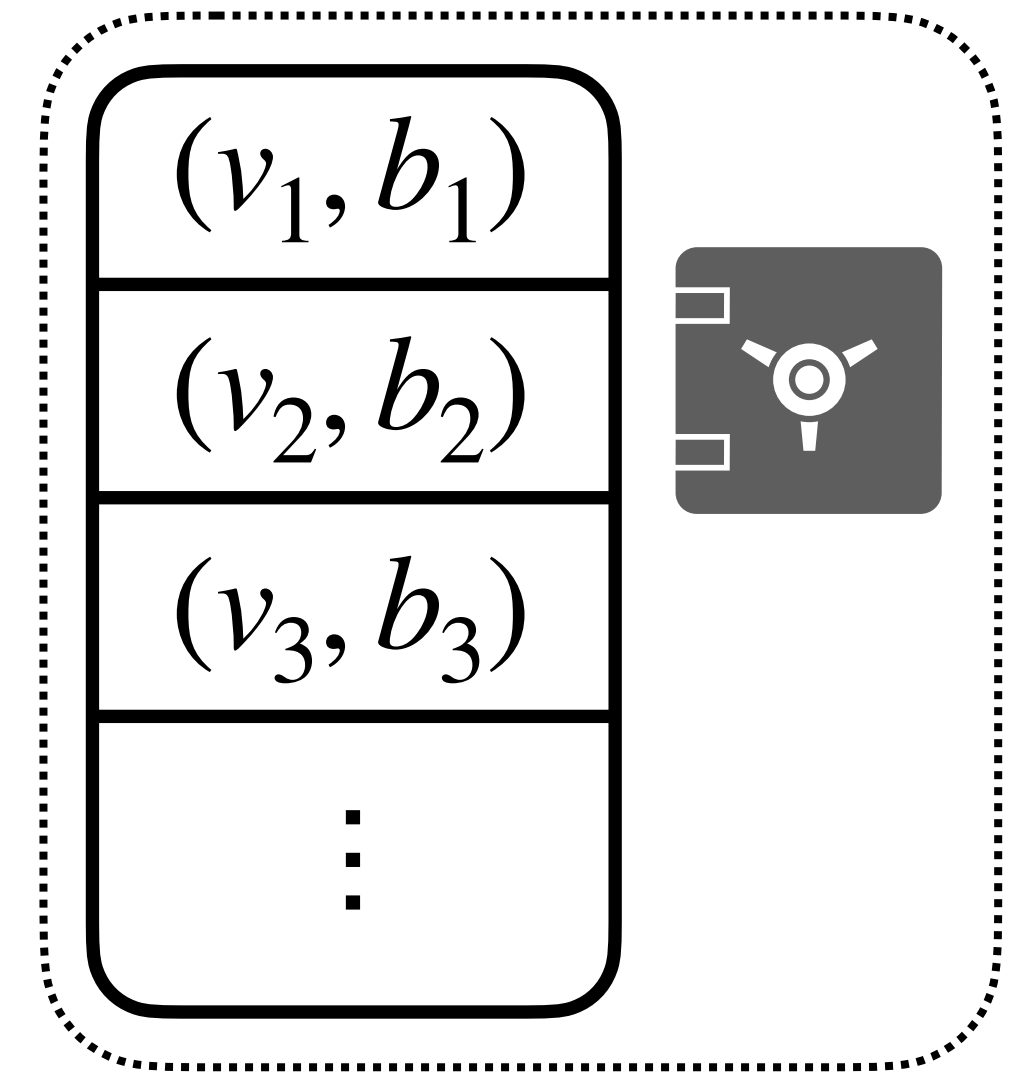


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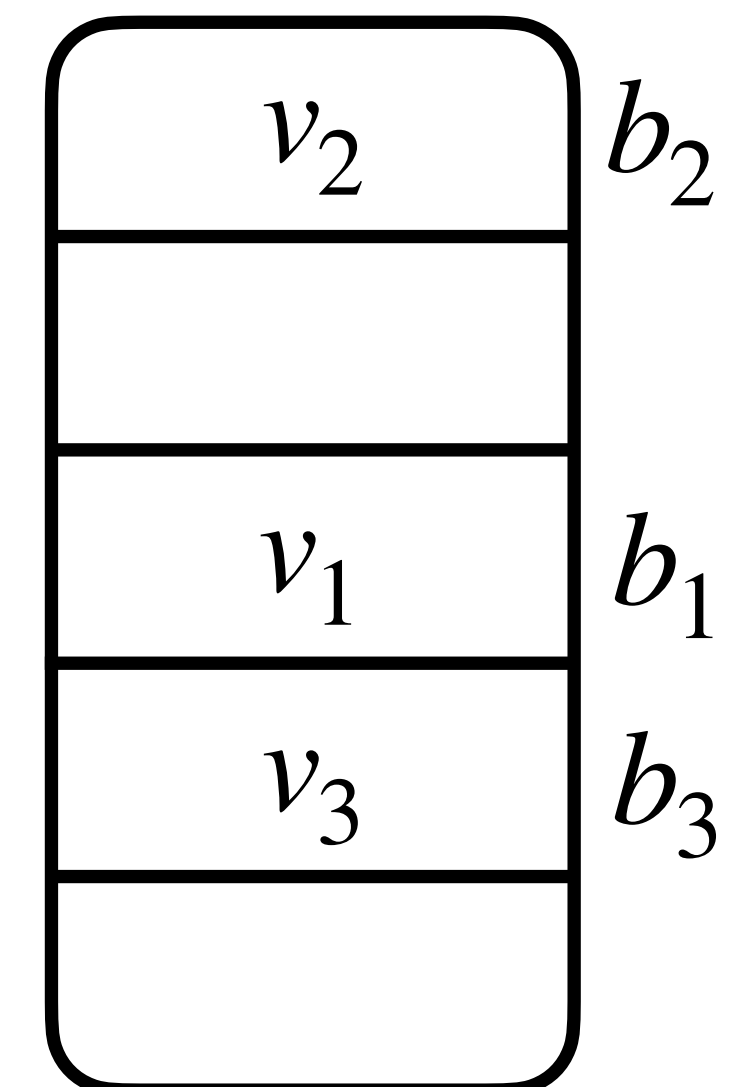
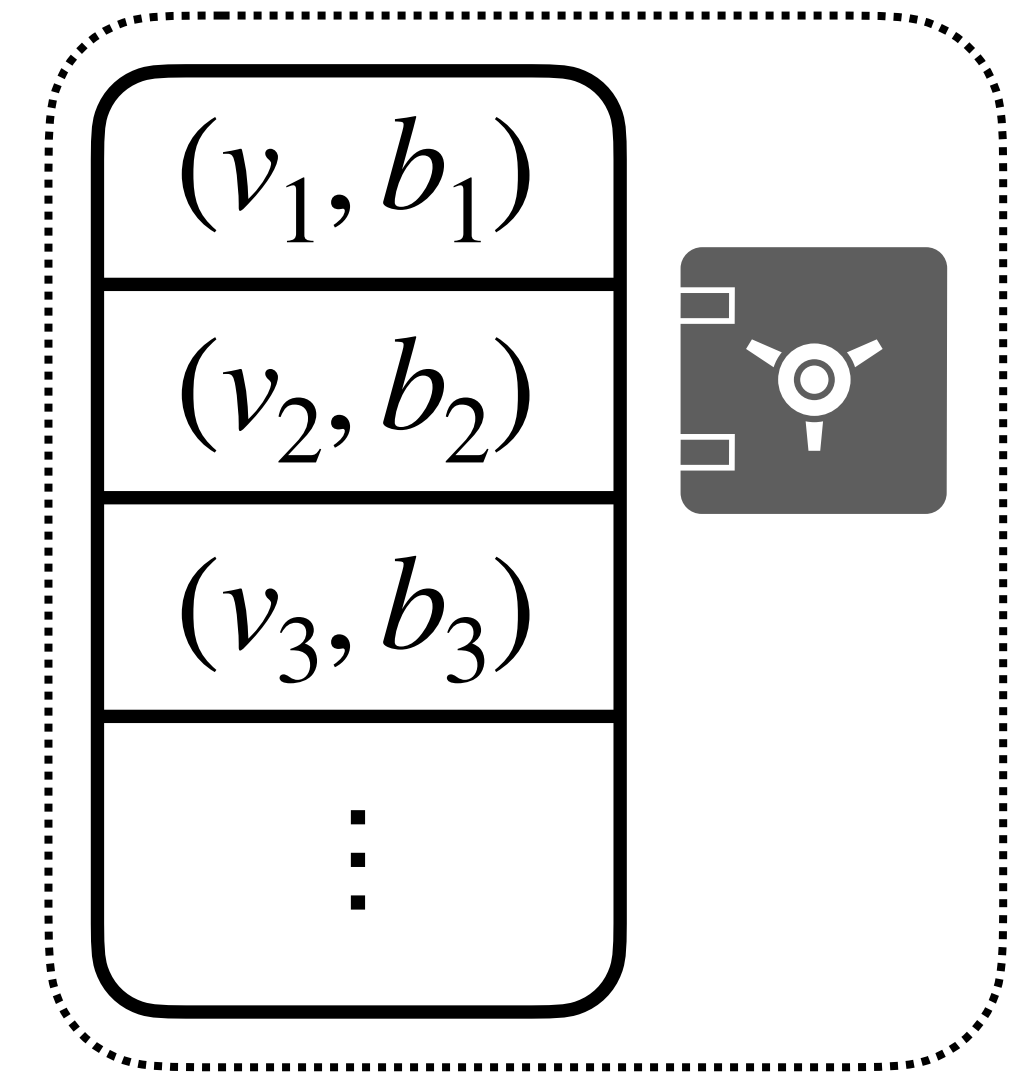


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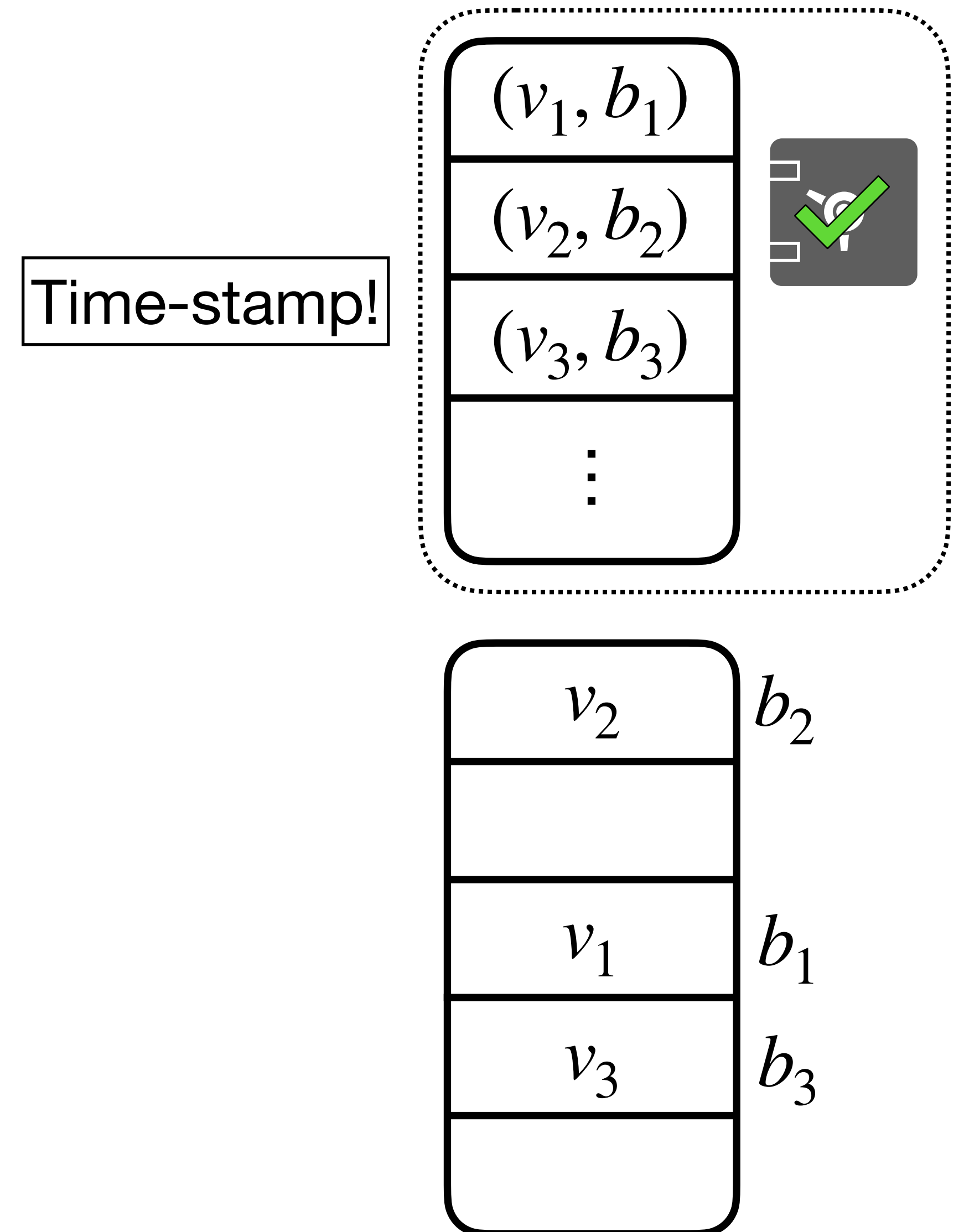
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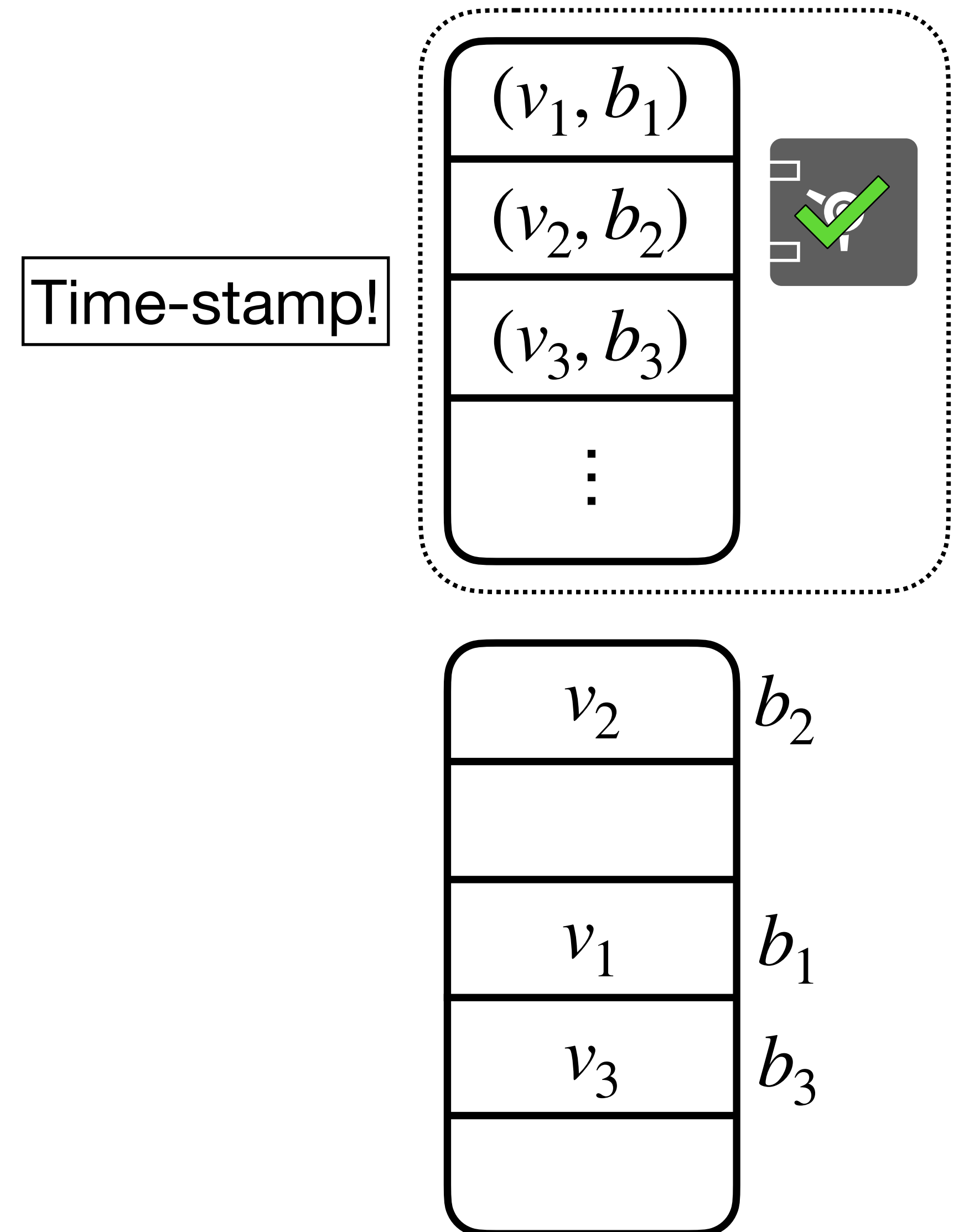
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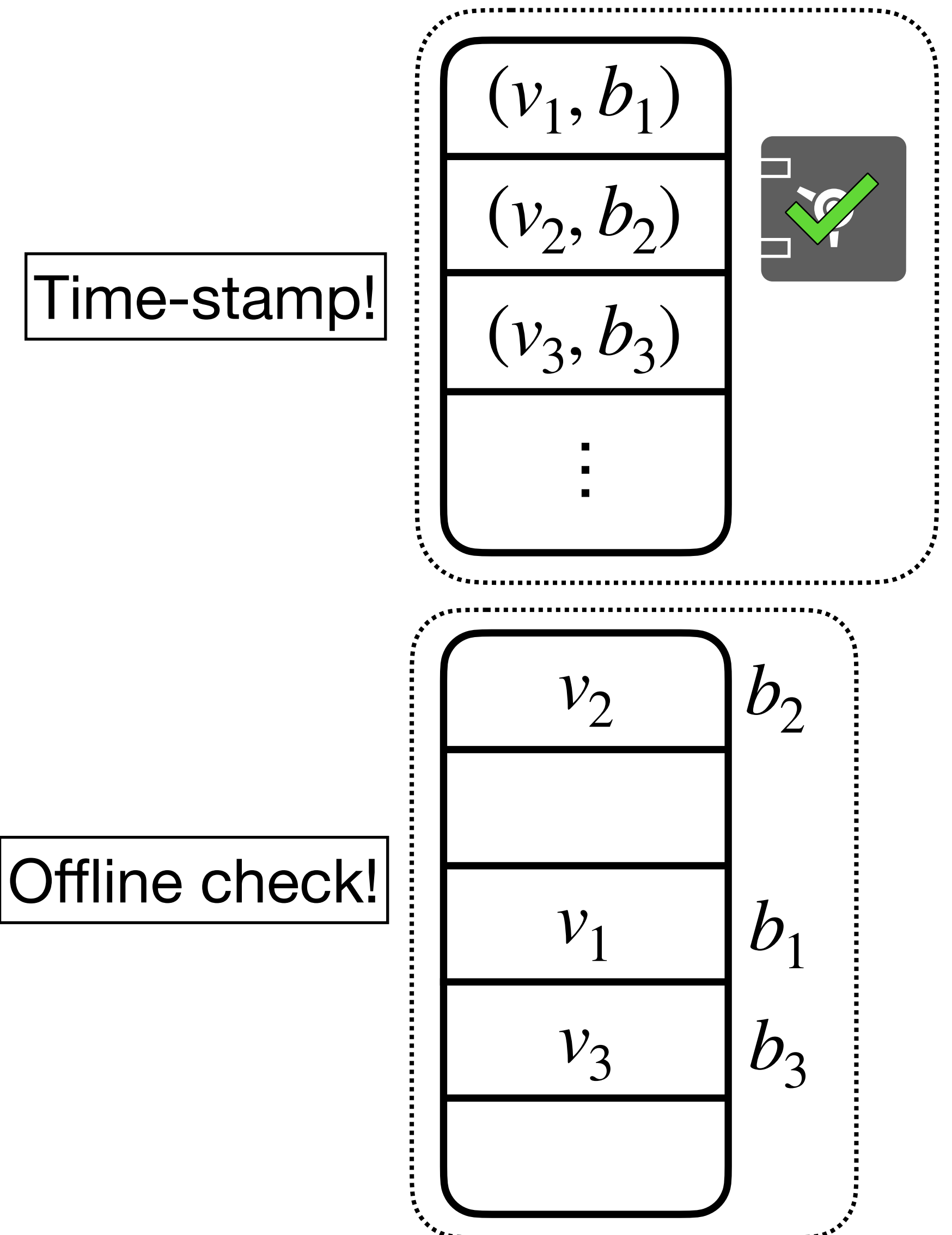
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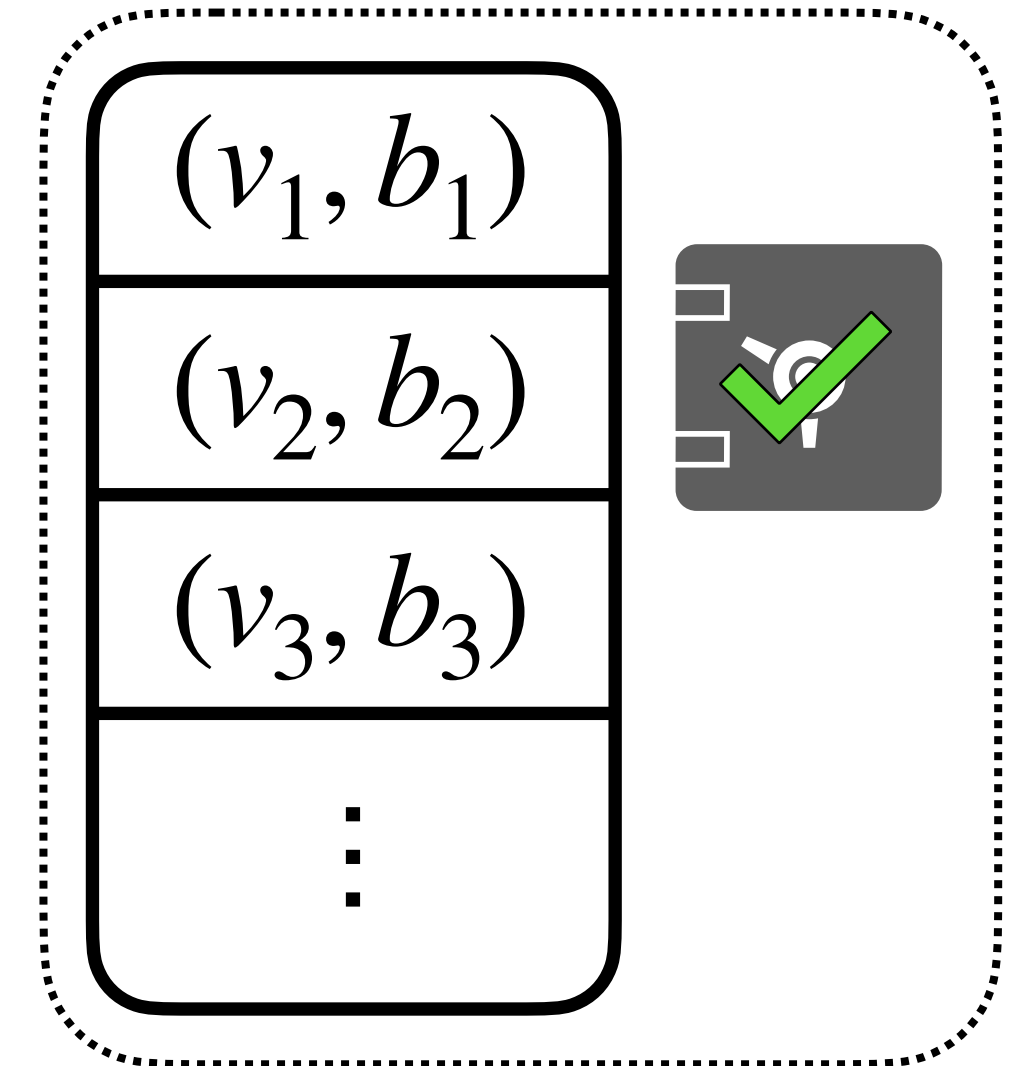




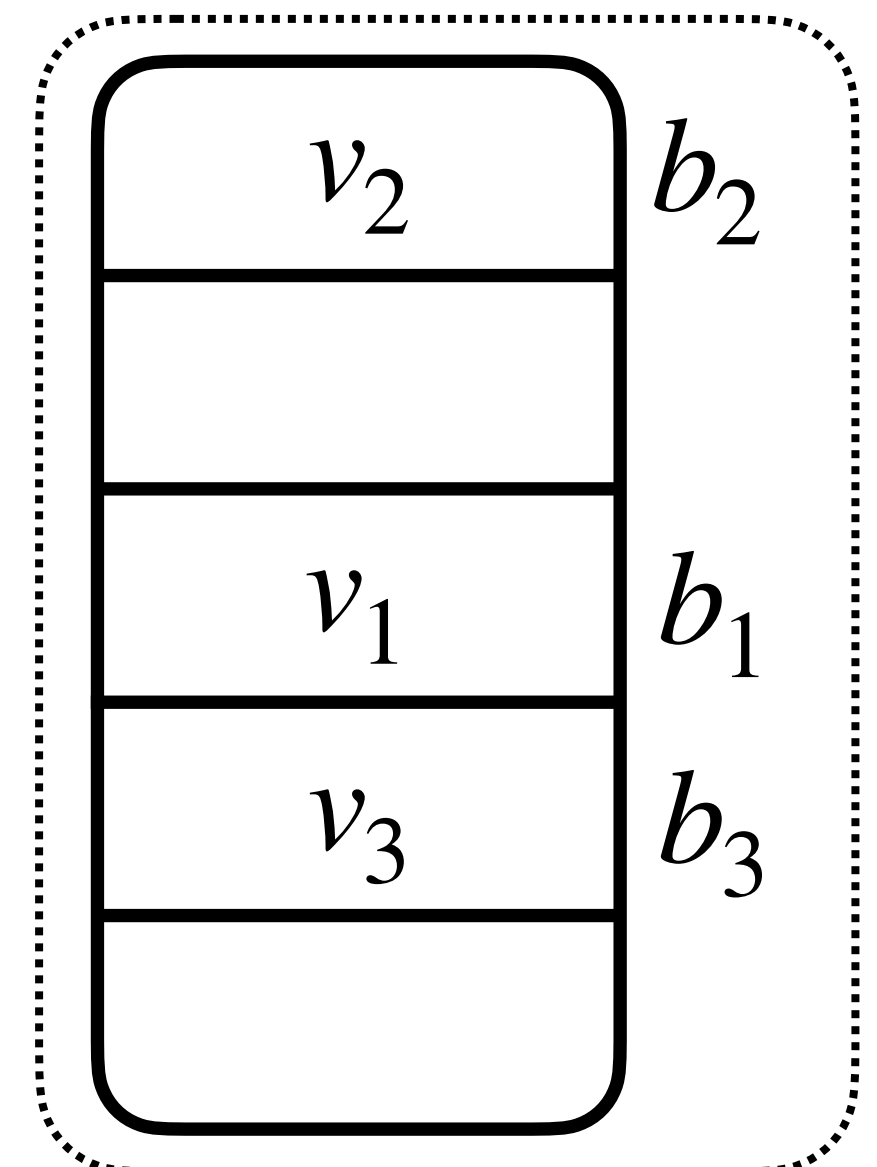
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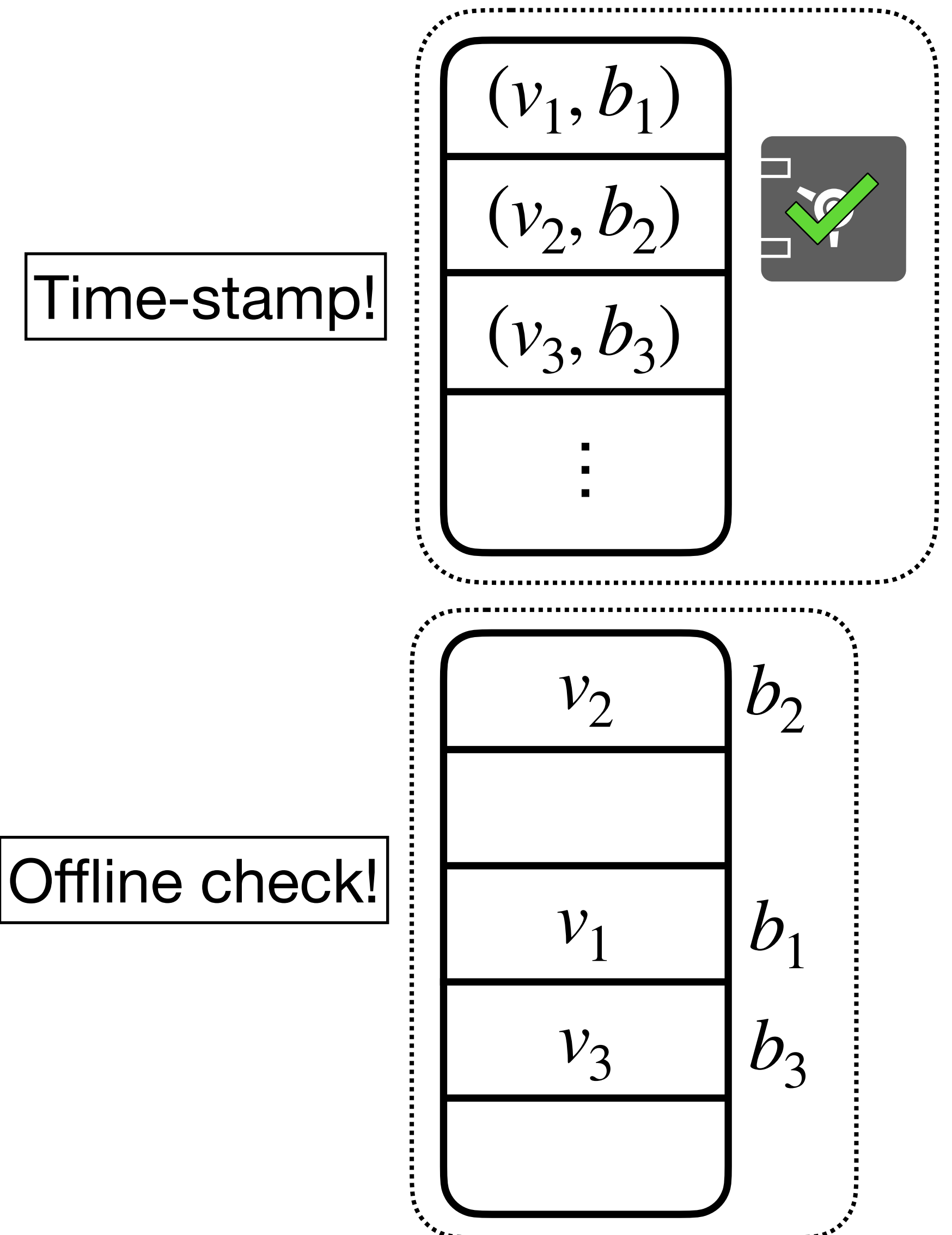


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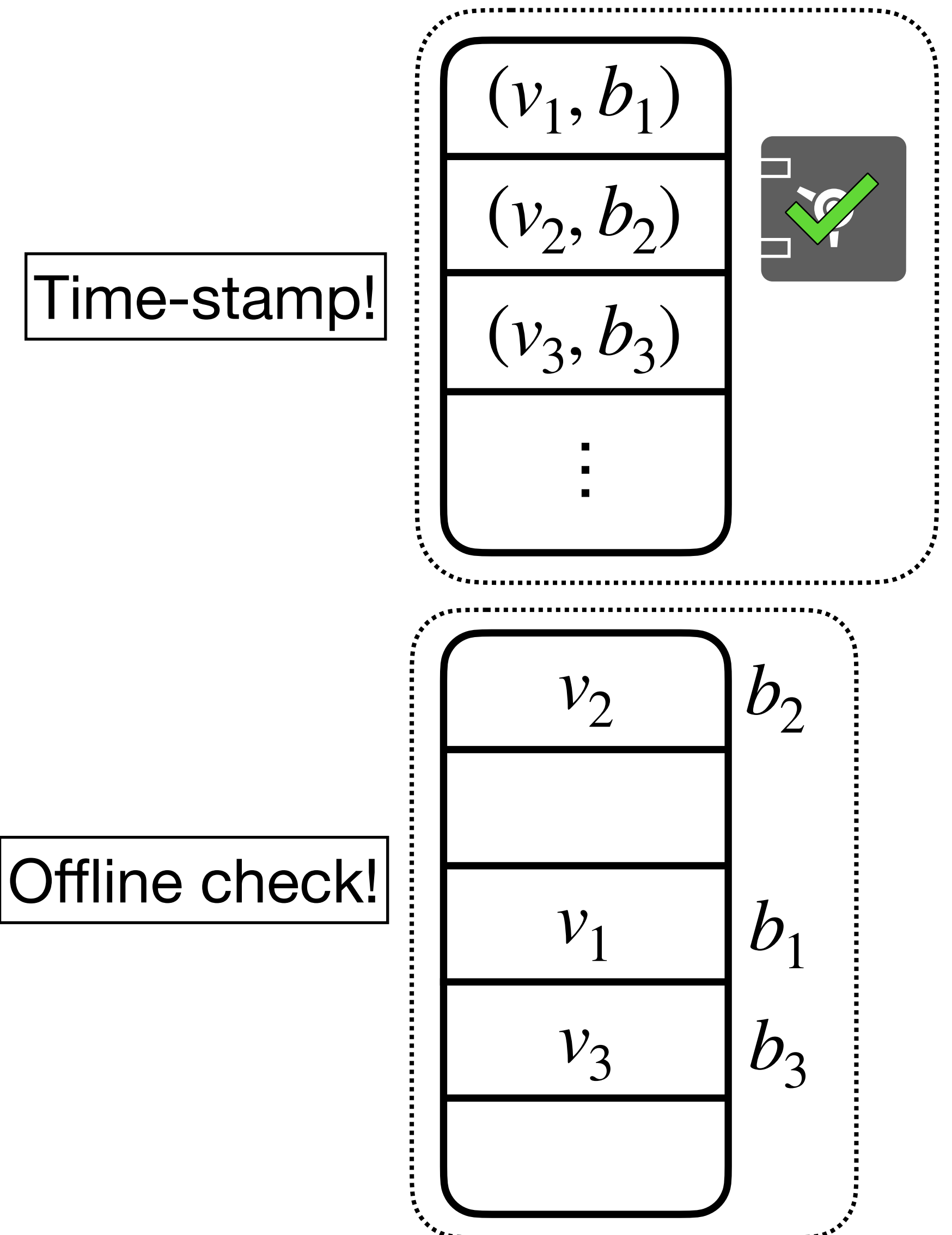
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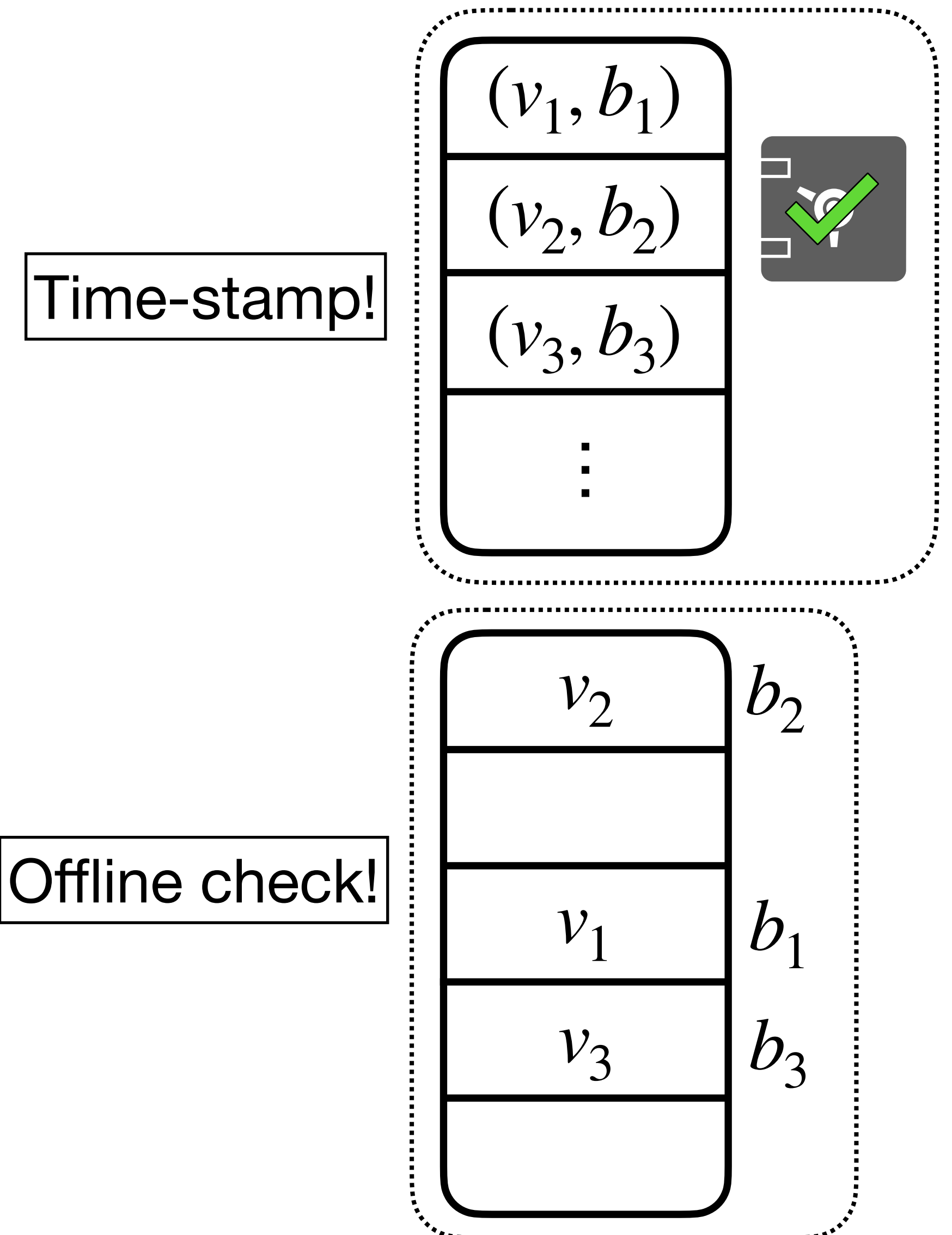
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- Instead, we develop **memory checking** techniques in the ORAM setting that should generalize to future constructions.

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- Any memory checker with  $O(1)$  overhead? Any lower bounds? (Best constructions have  $O(\log N)$  overhead.)

**Thank you!**

# Bonus Slides

# Ideal Malicious Security



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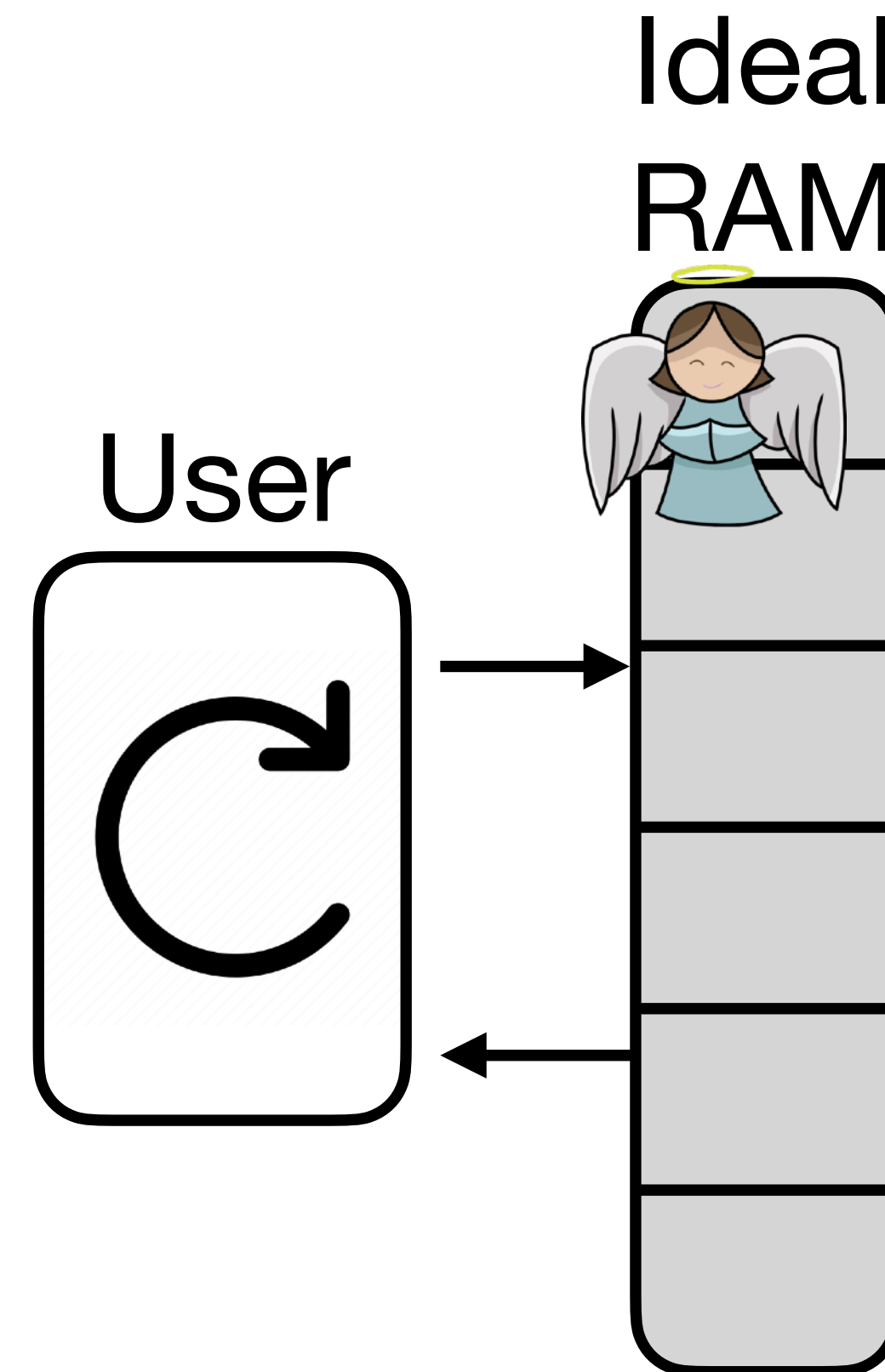
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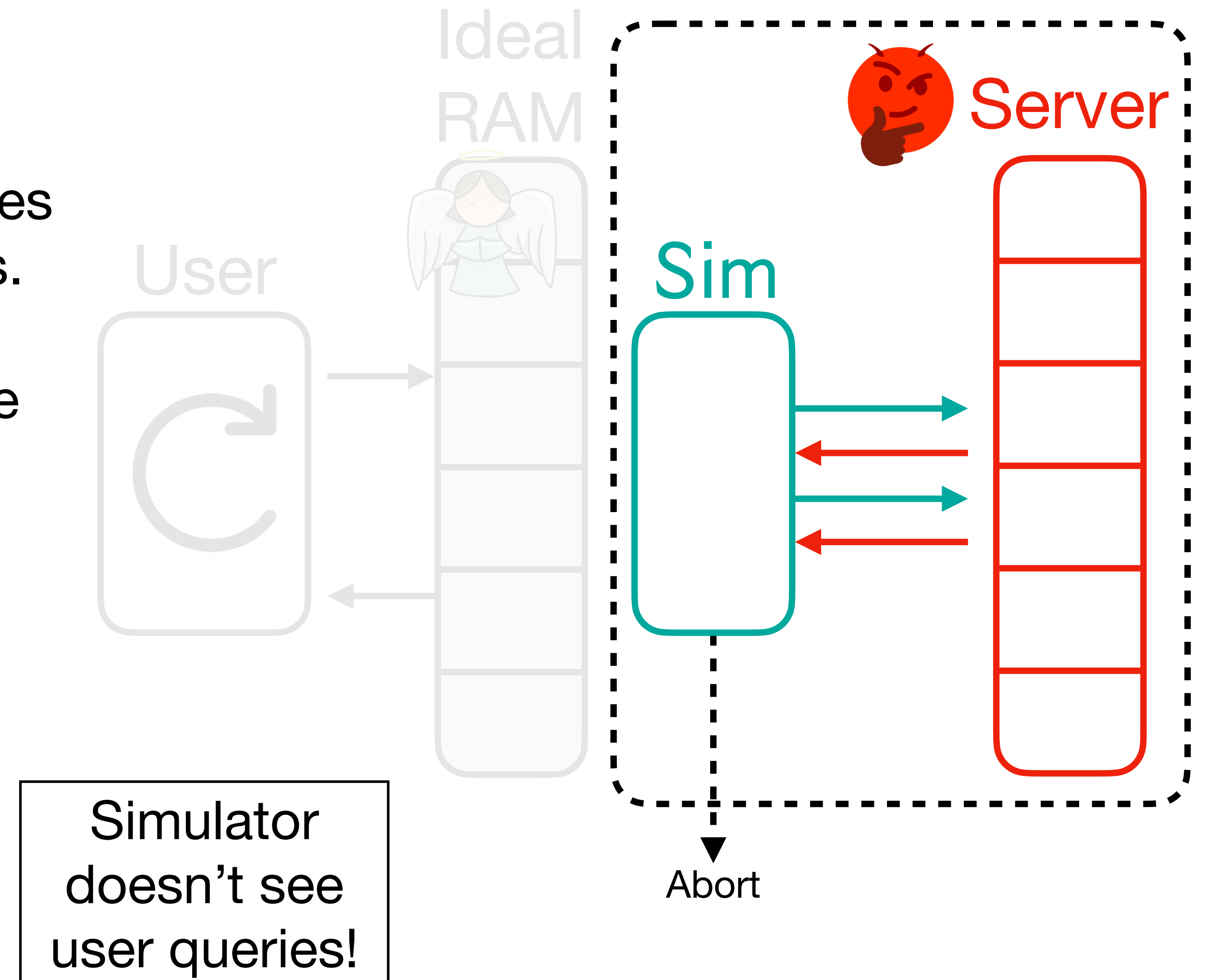
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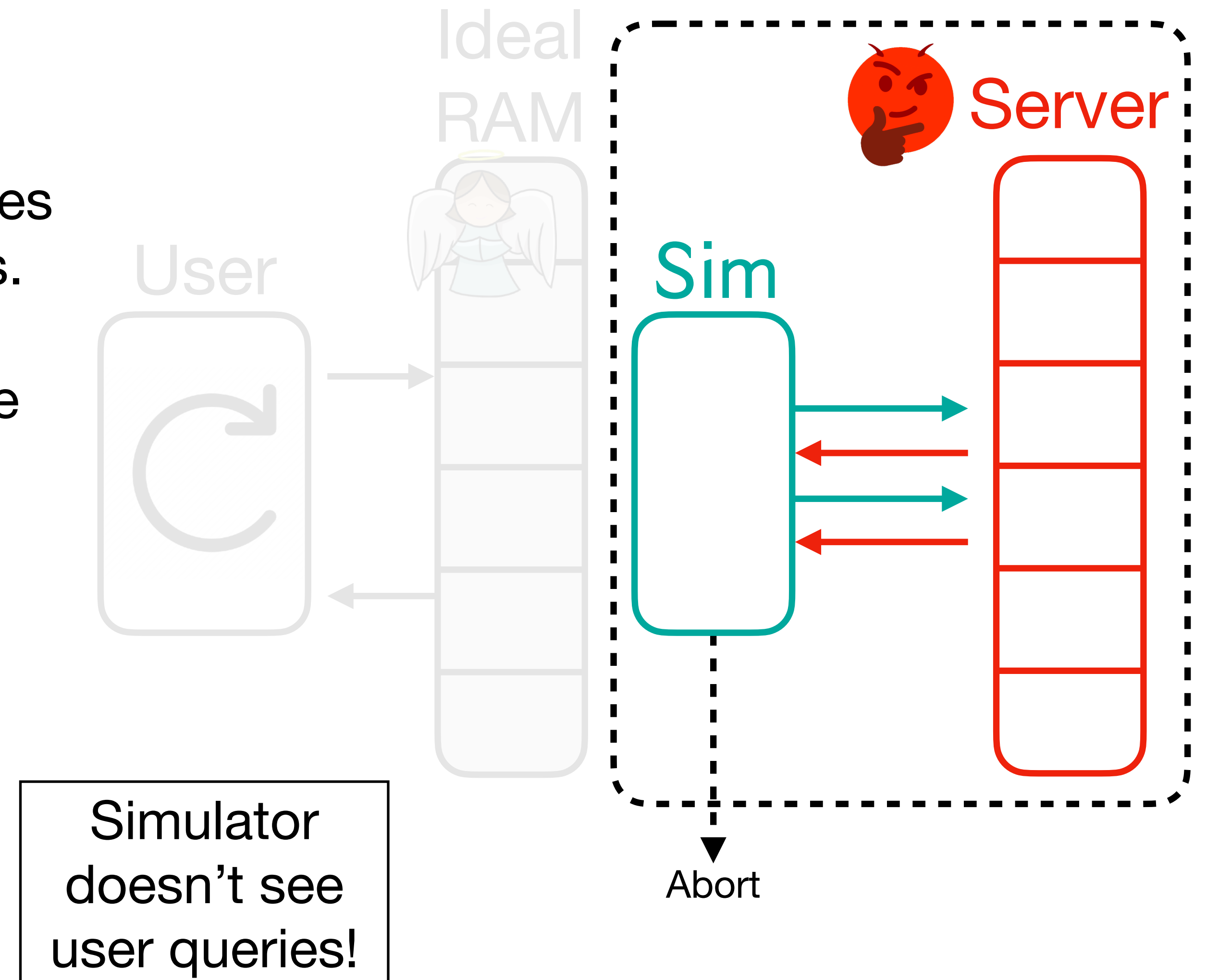
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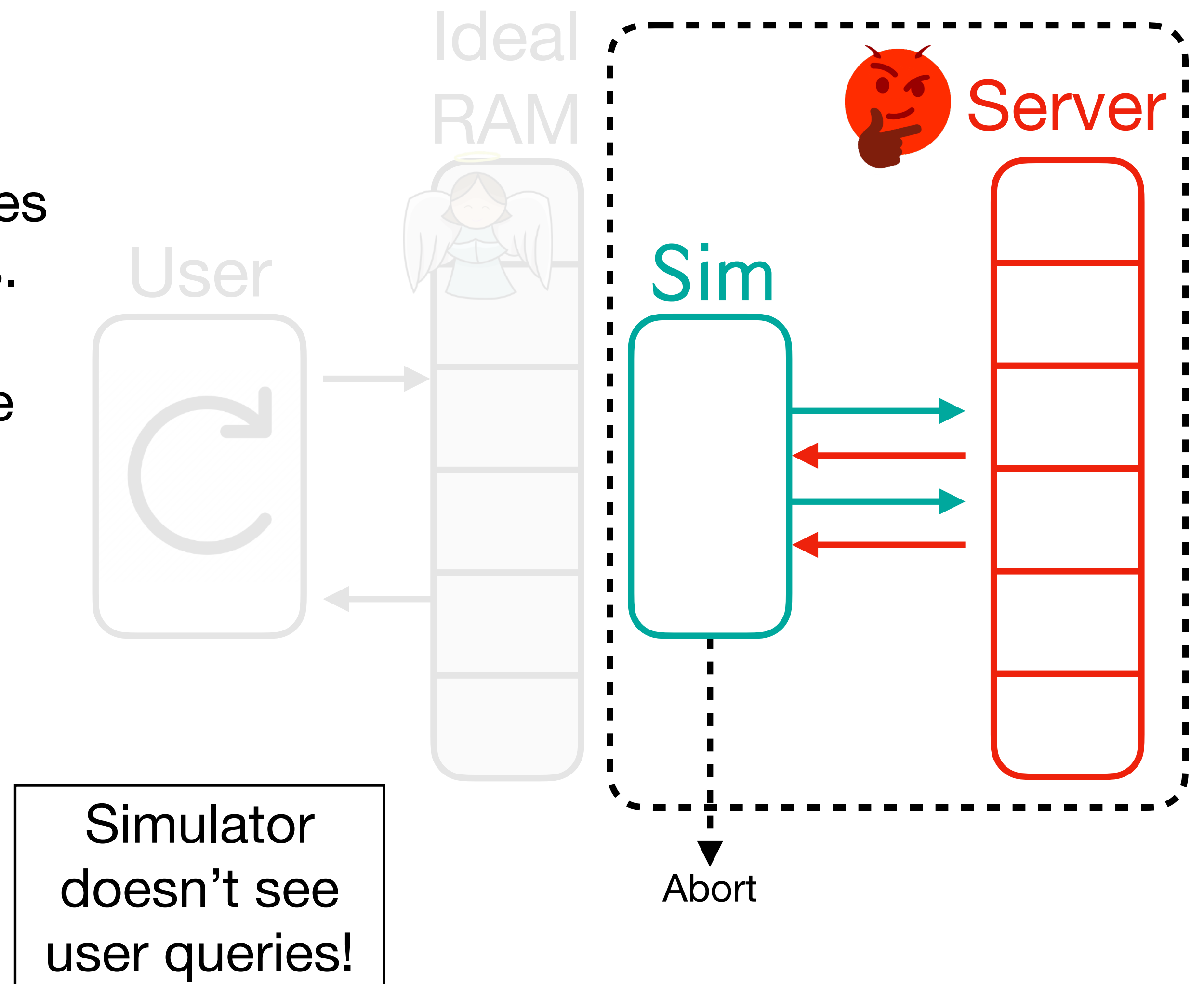
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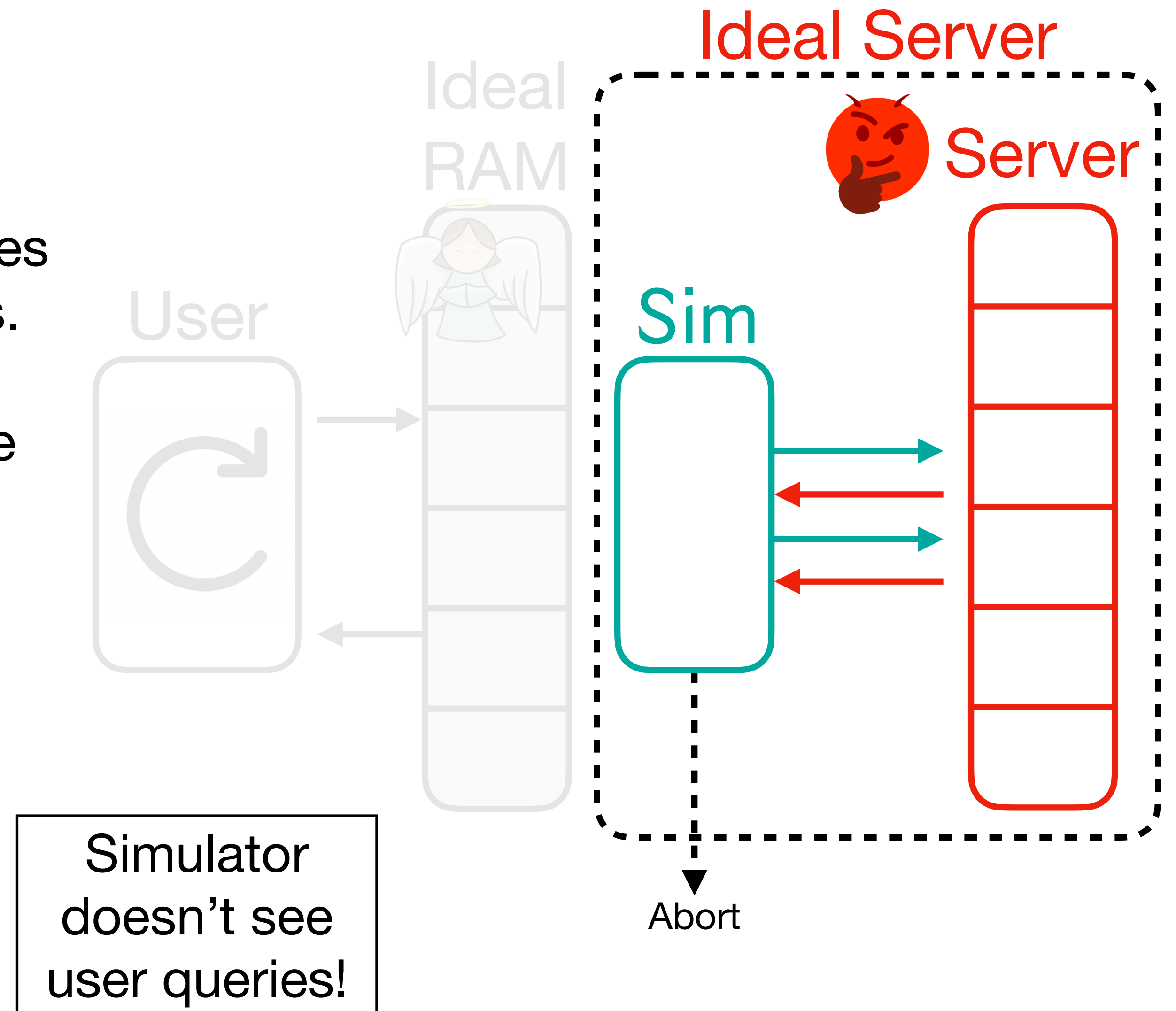
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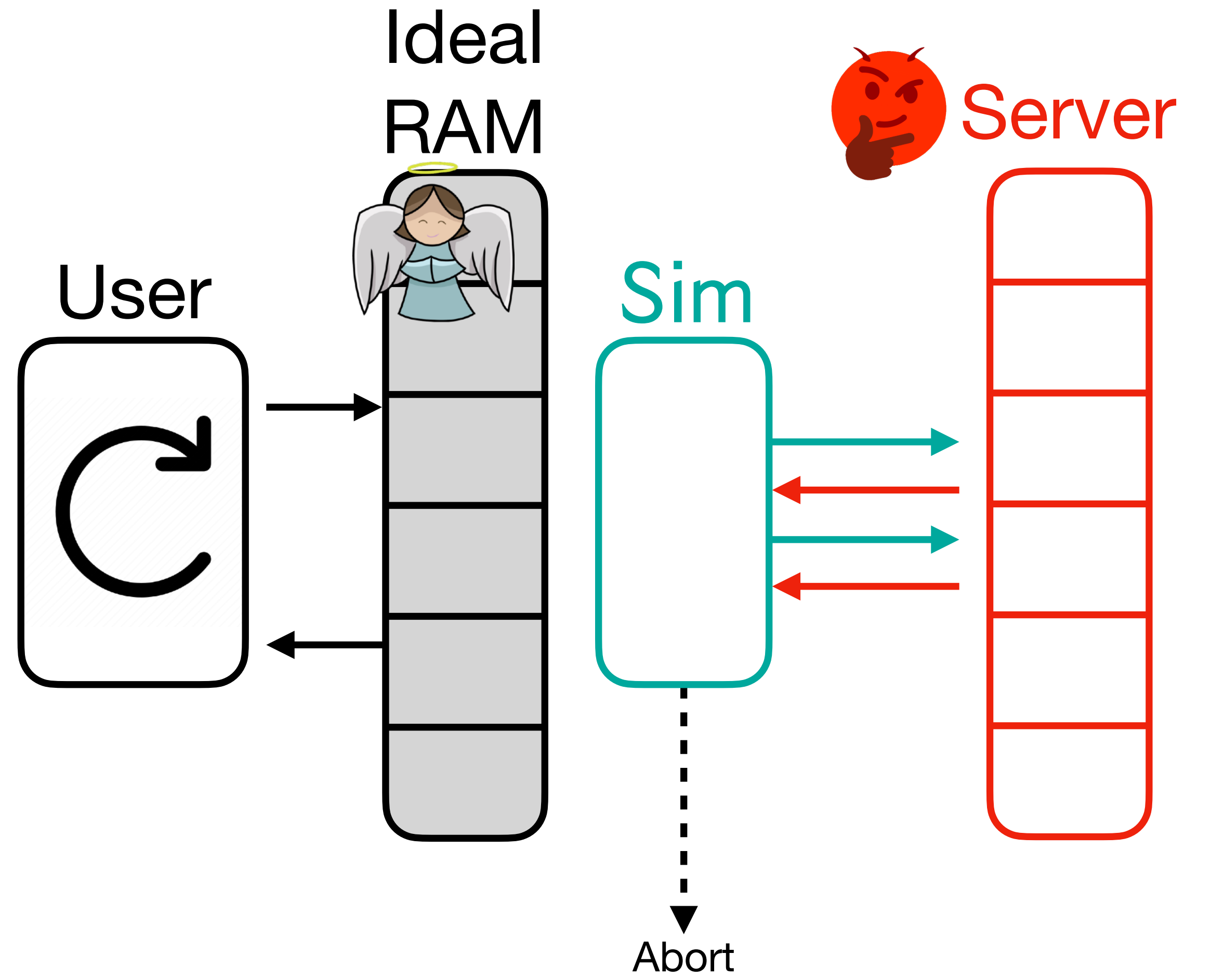
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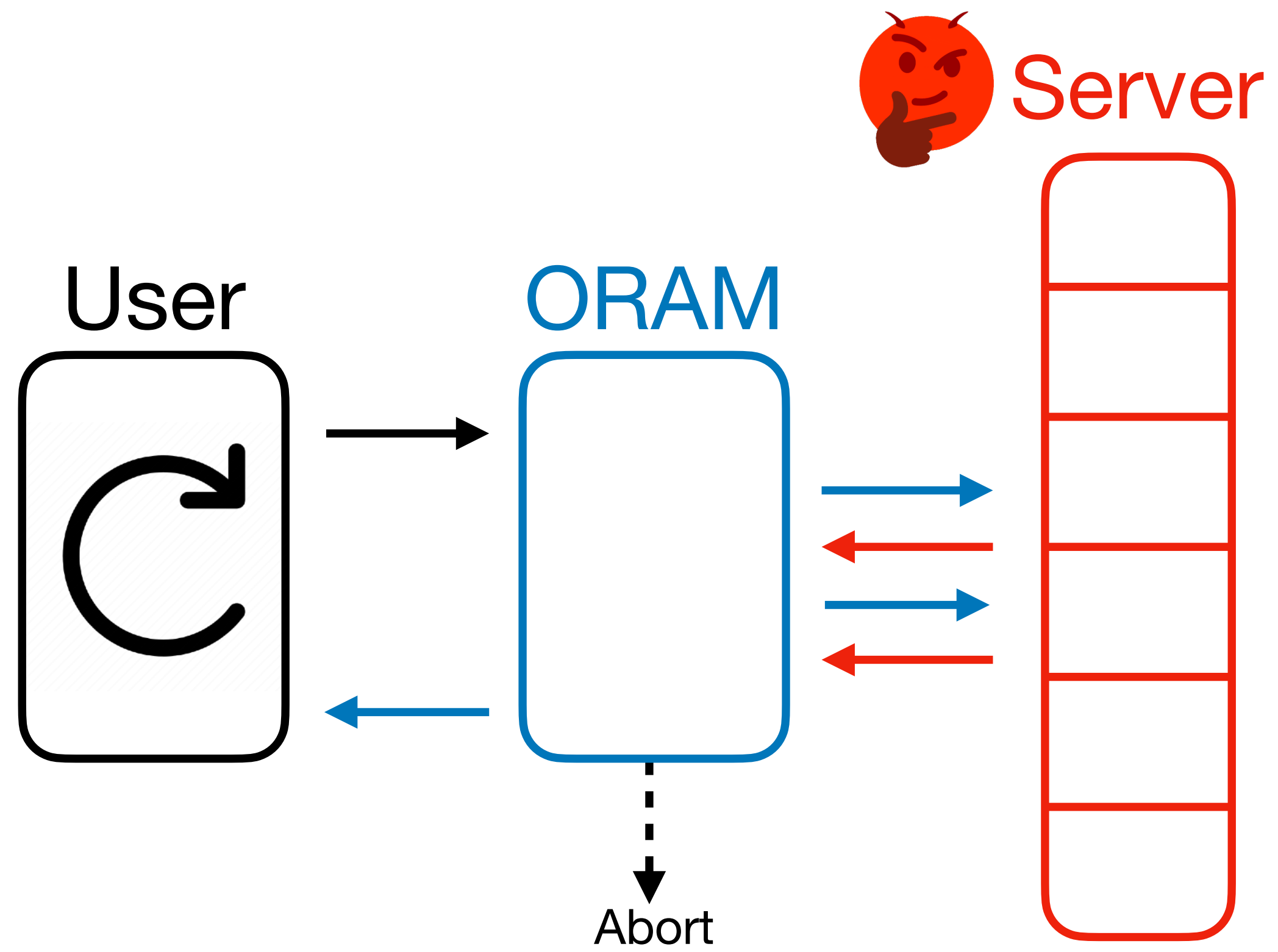
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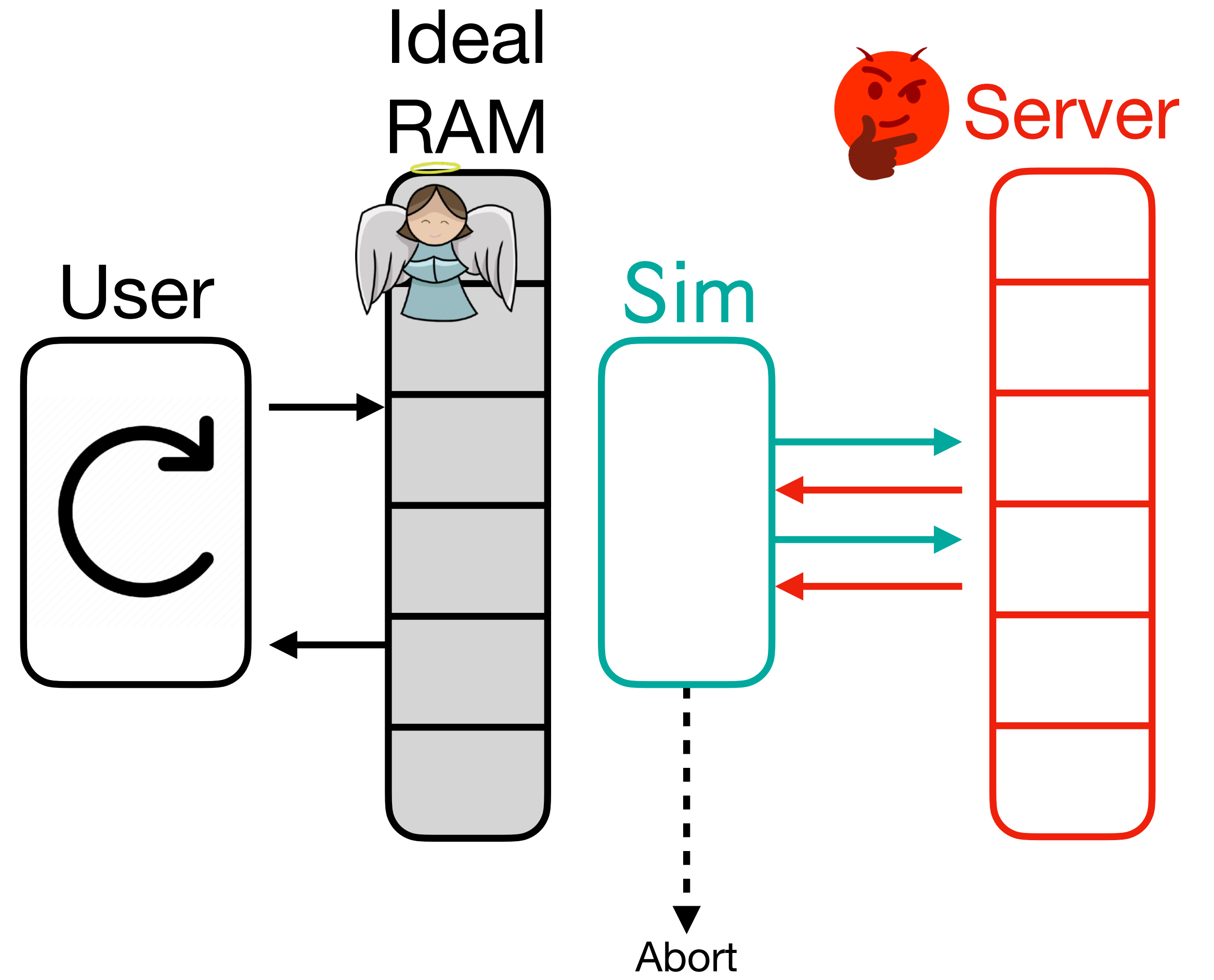




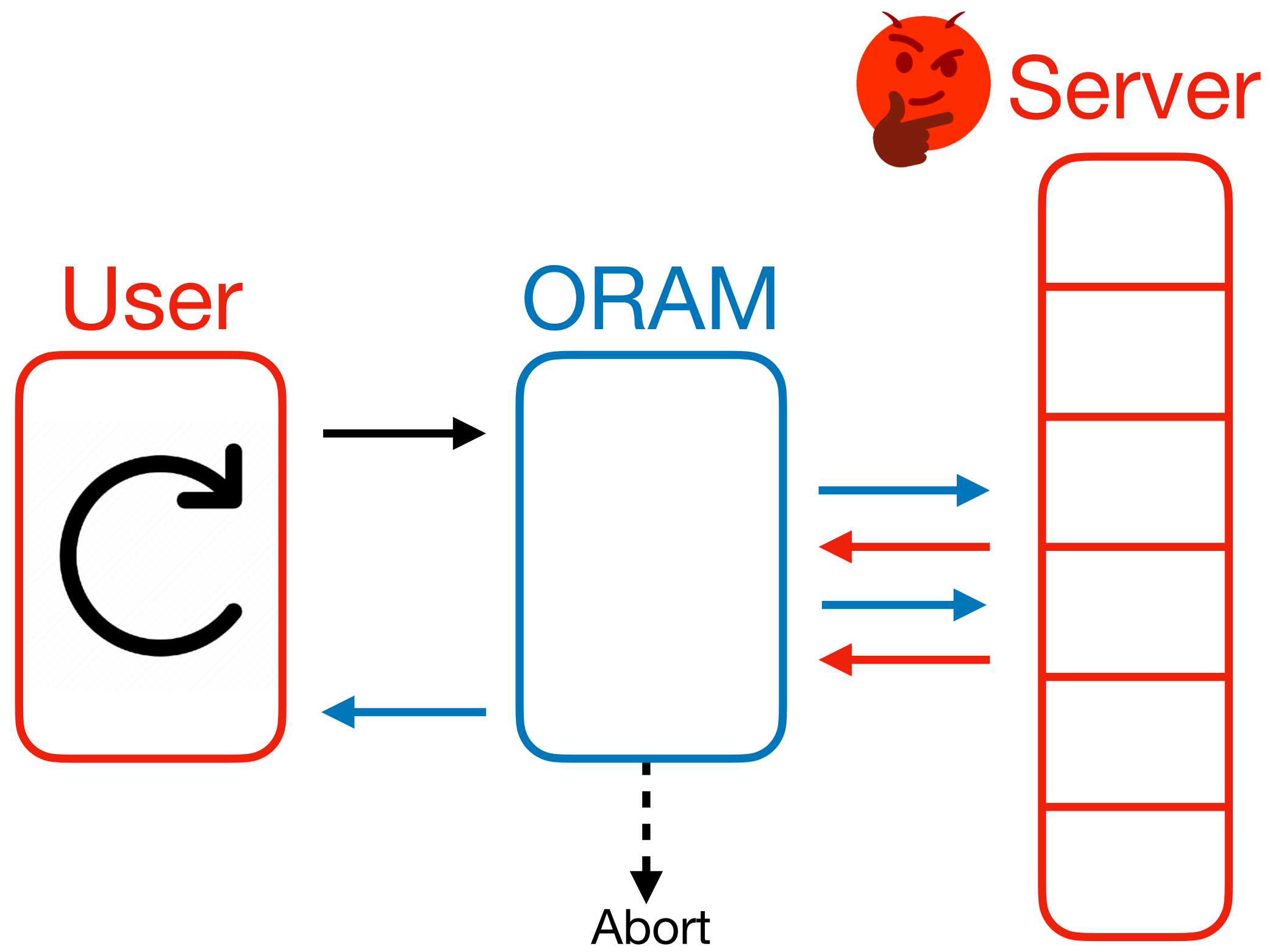
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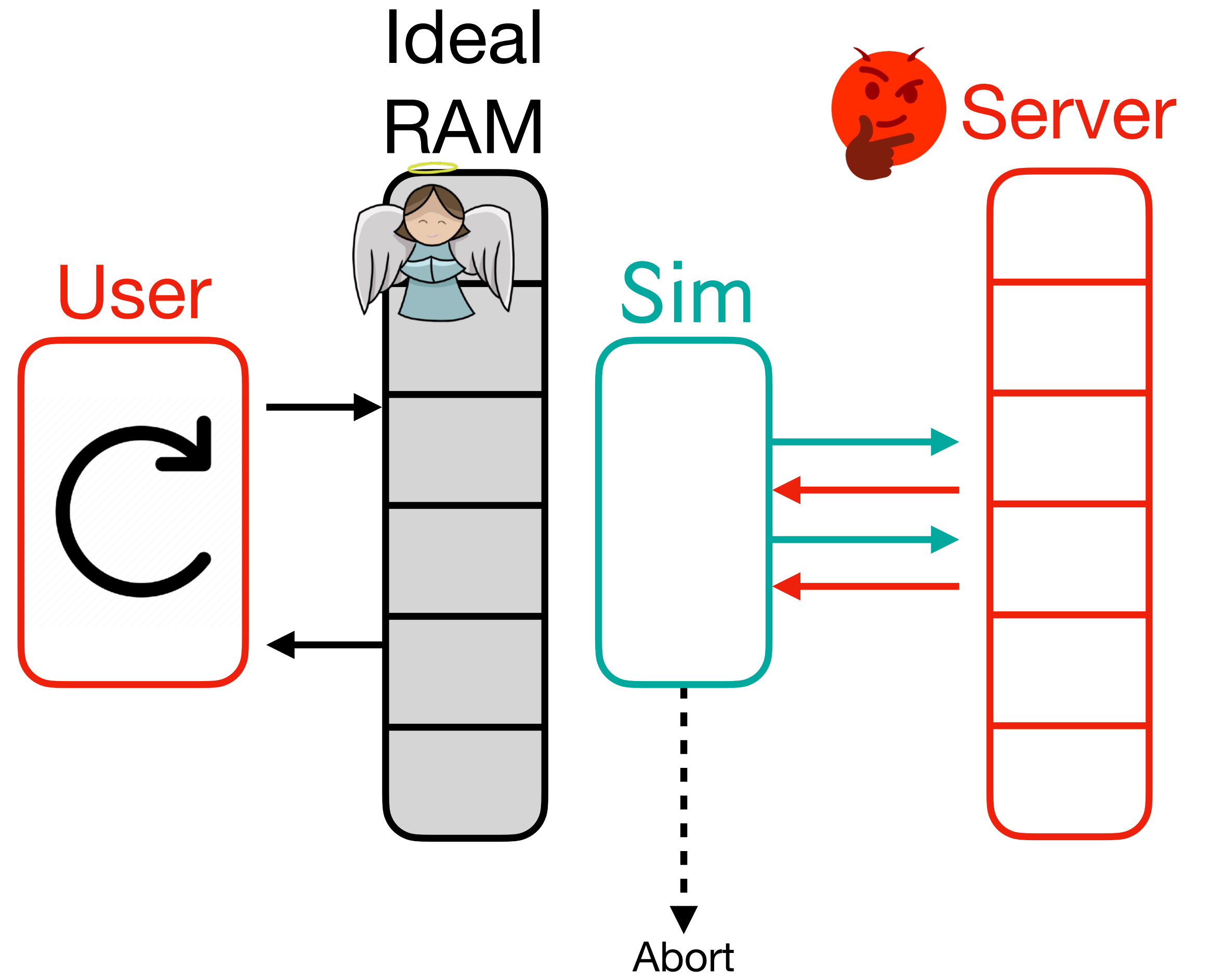
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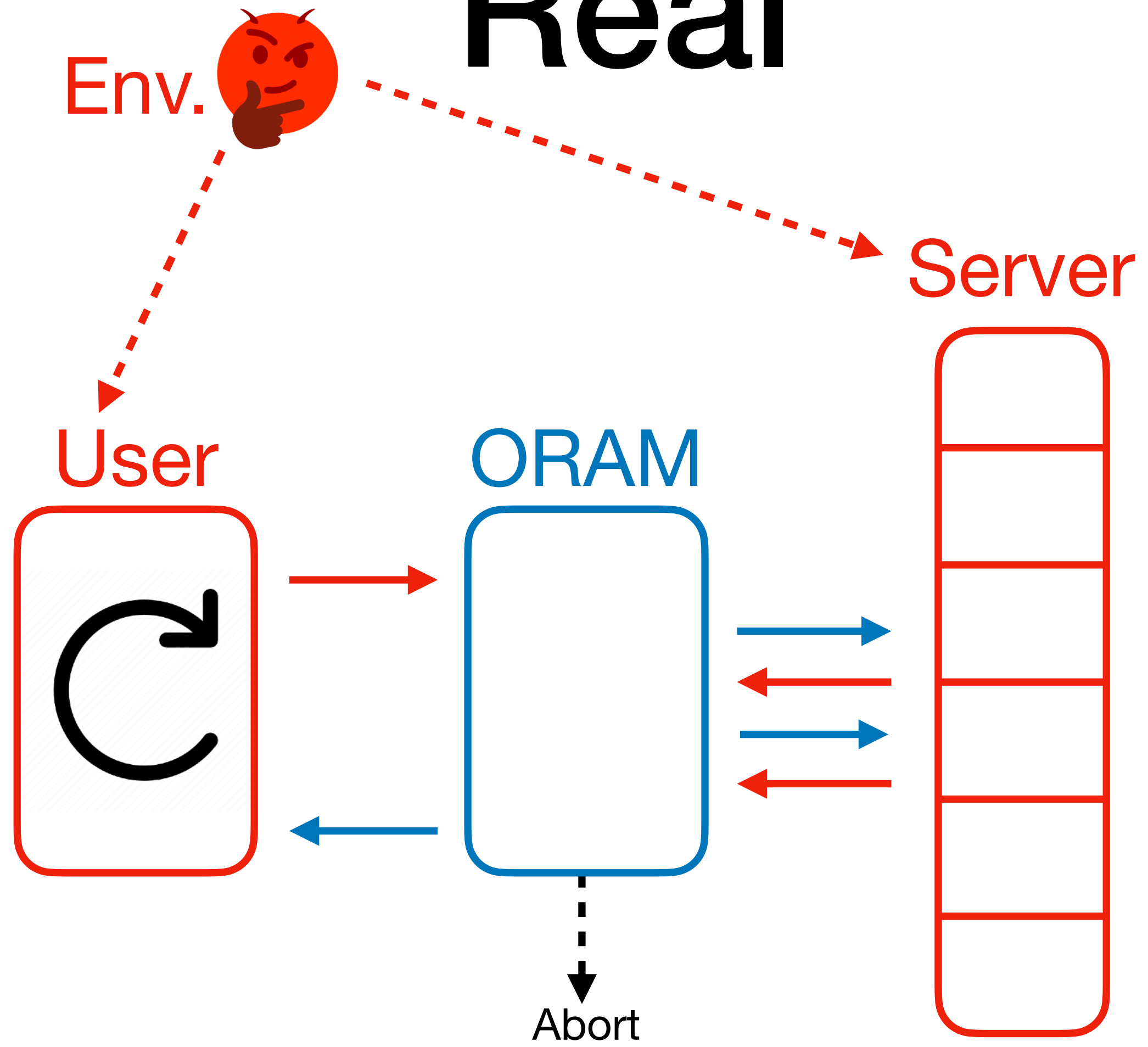
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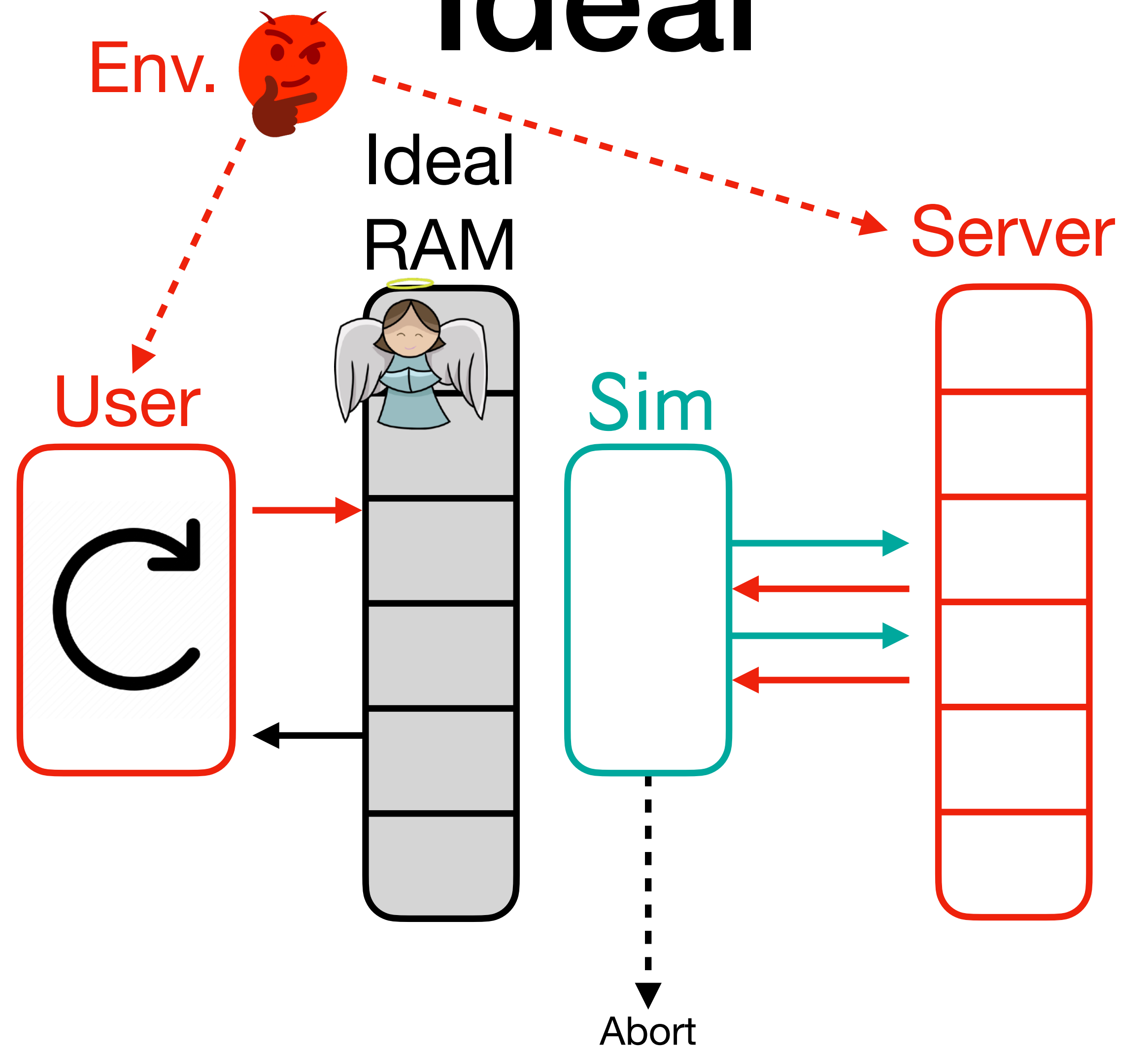
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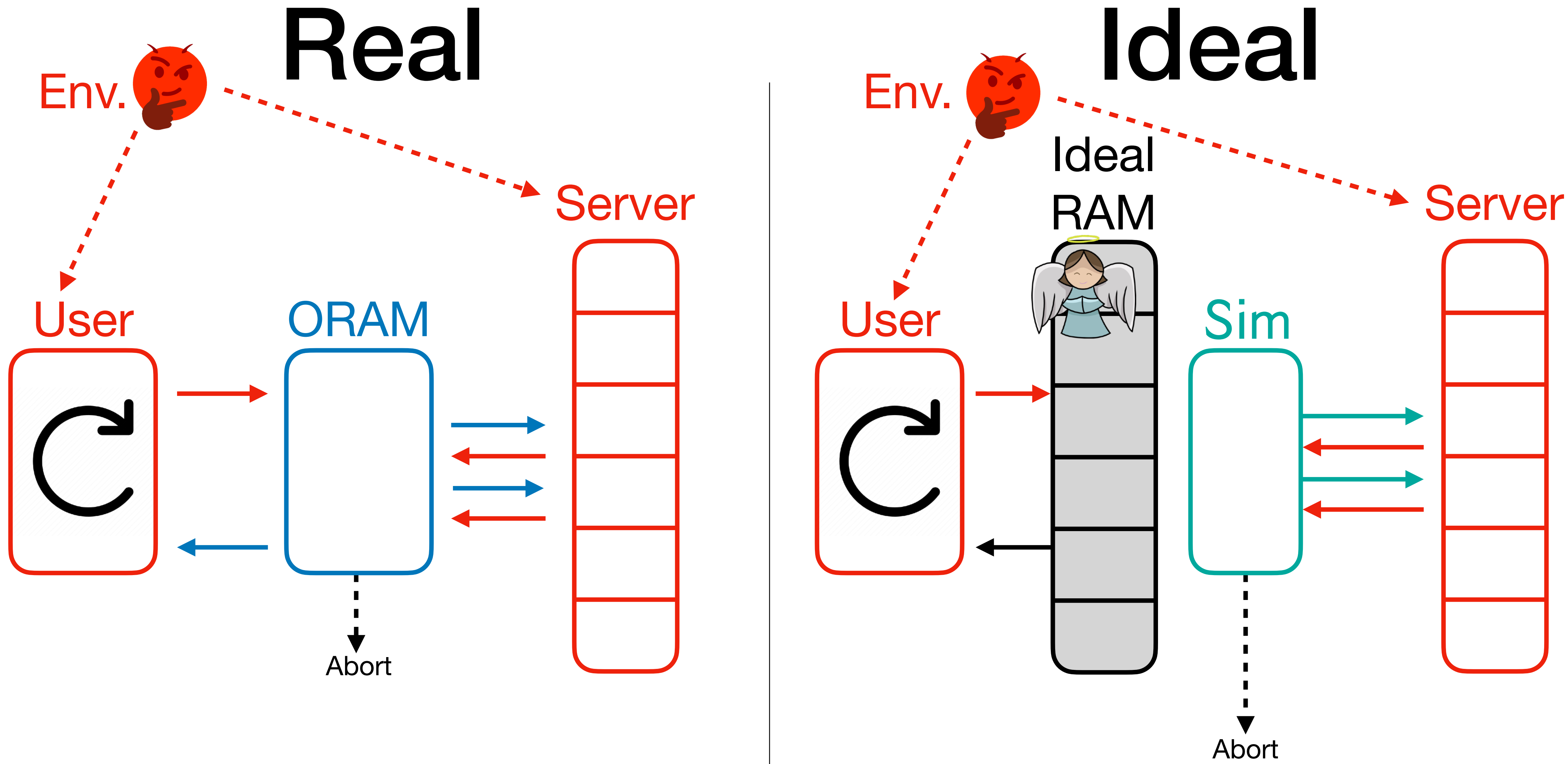



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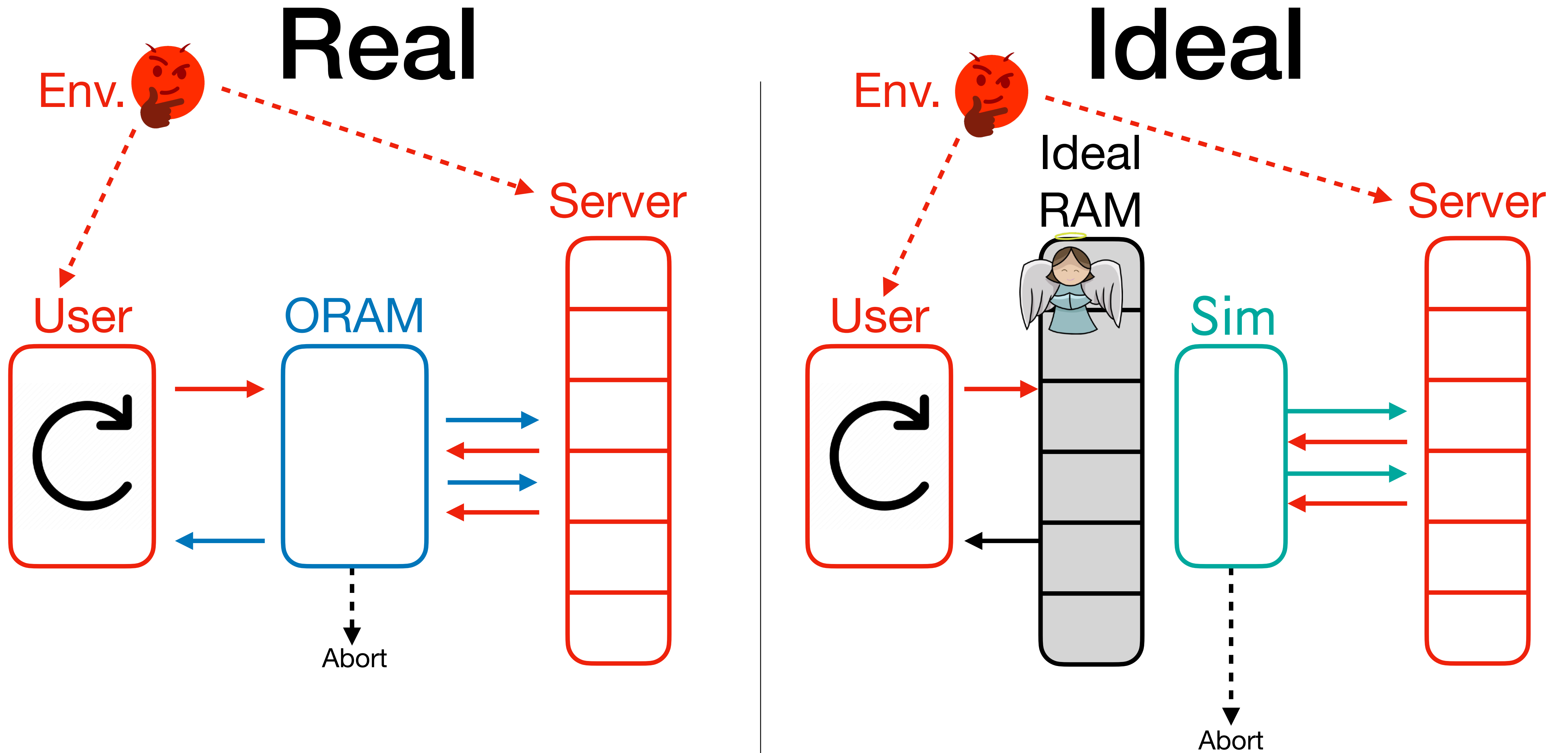



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*Quite difficult! Long line of work to get this efficiency.*

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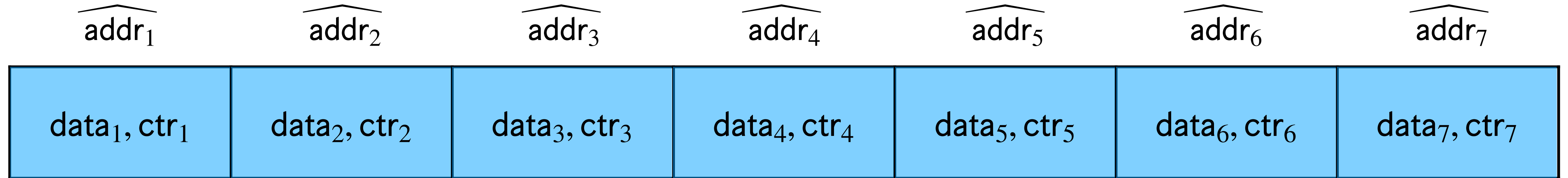
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- Can be viewed as a strengthening of Goldreich-Ostrovsky's time-stamping theorem!

# Why Access-Deterministic Algorithms May Not Be Offline-Safe

- Consider the following implementation of an AKS sort.
  1. Use server space to compute and store a bipartite expander  $G = (V, E)$ .
  2. Iterate over edge set  $E$ , and make comparisons according to  $E$ .
- If the contents of  $E$  are **replaced with secret data**, the secret data will be leaked!

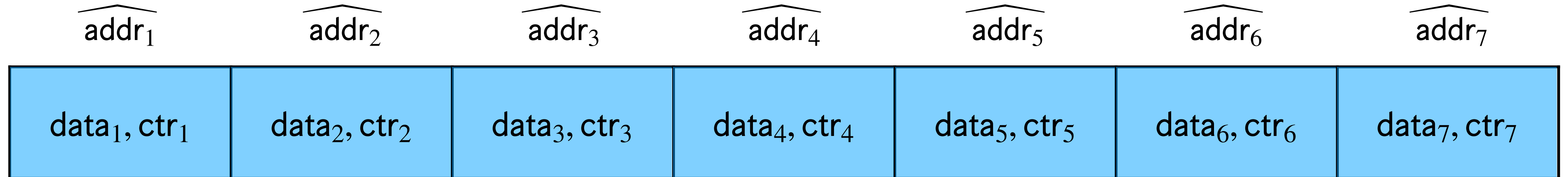
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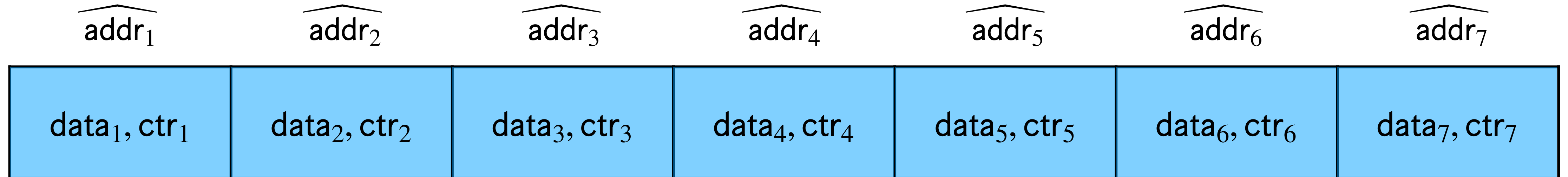
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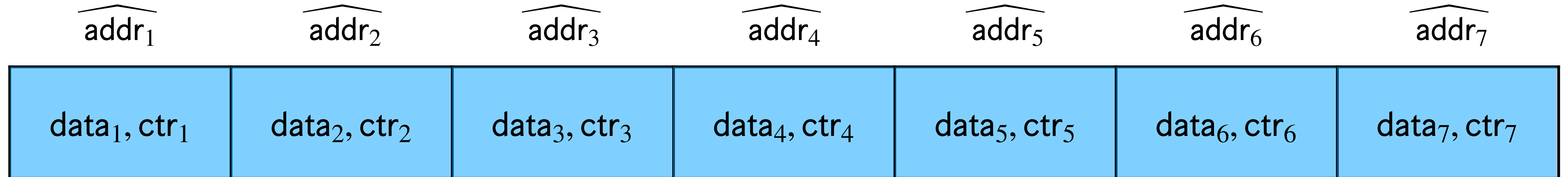
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- At the end of the execution, iterate over the array and accept if and only if  $\sum_i ctr_i = T$ .